Double Integrals (5A)

- Double Integral
- Double Integrals in Polar Coordinates
- Green's Theorem

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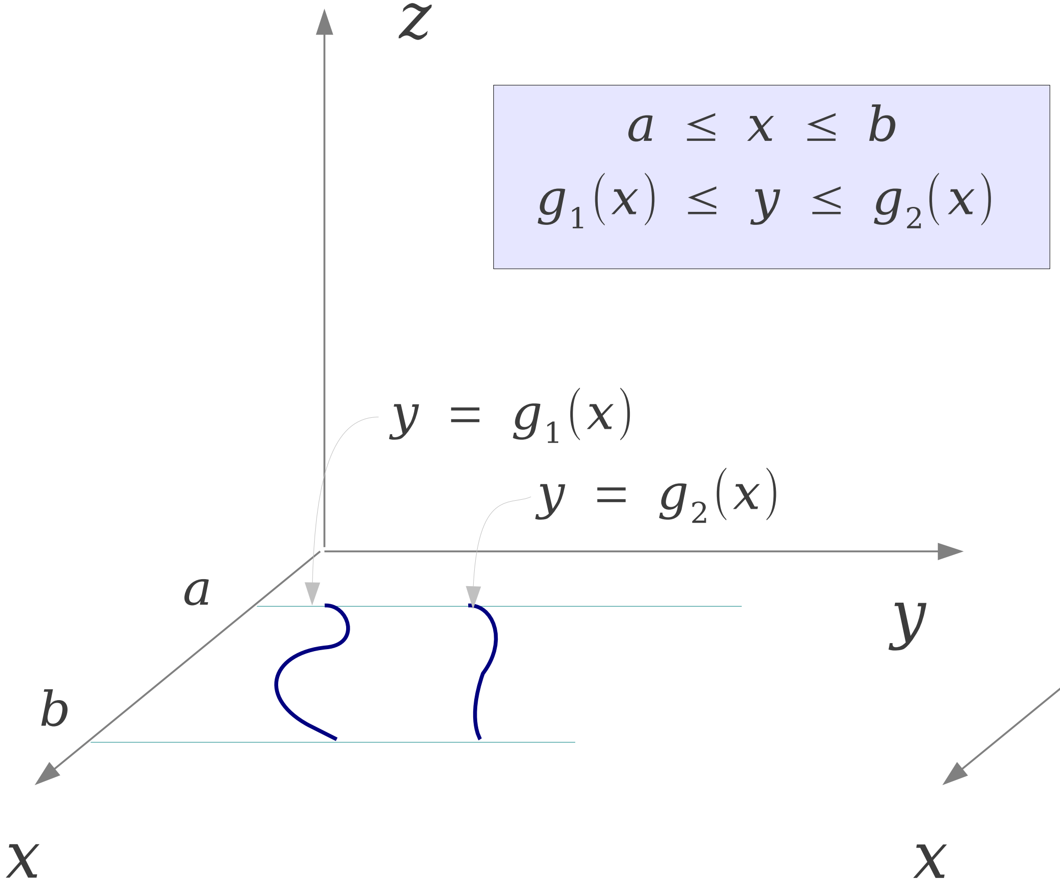
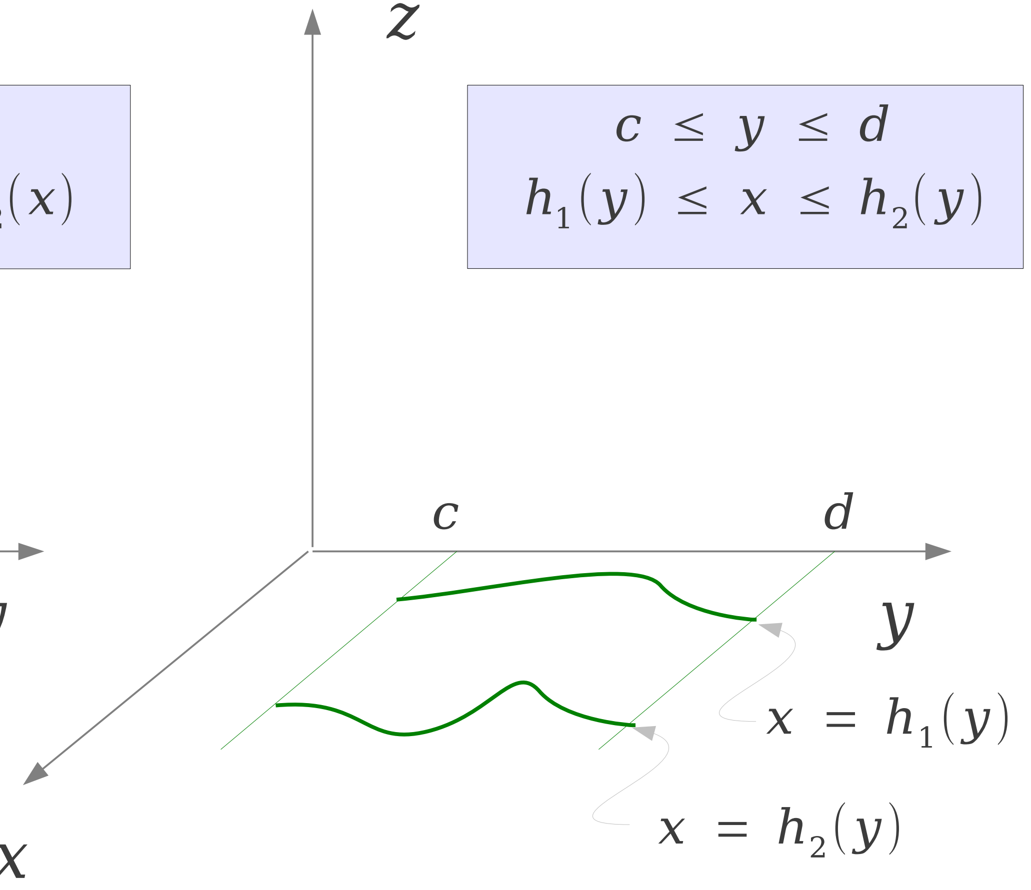
Area and Volume

$$A = \iint_R dA$$

$$V = \iint_R f(x, y)\, dA$$

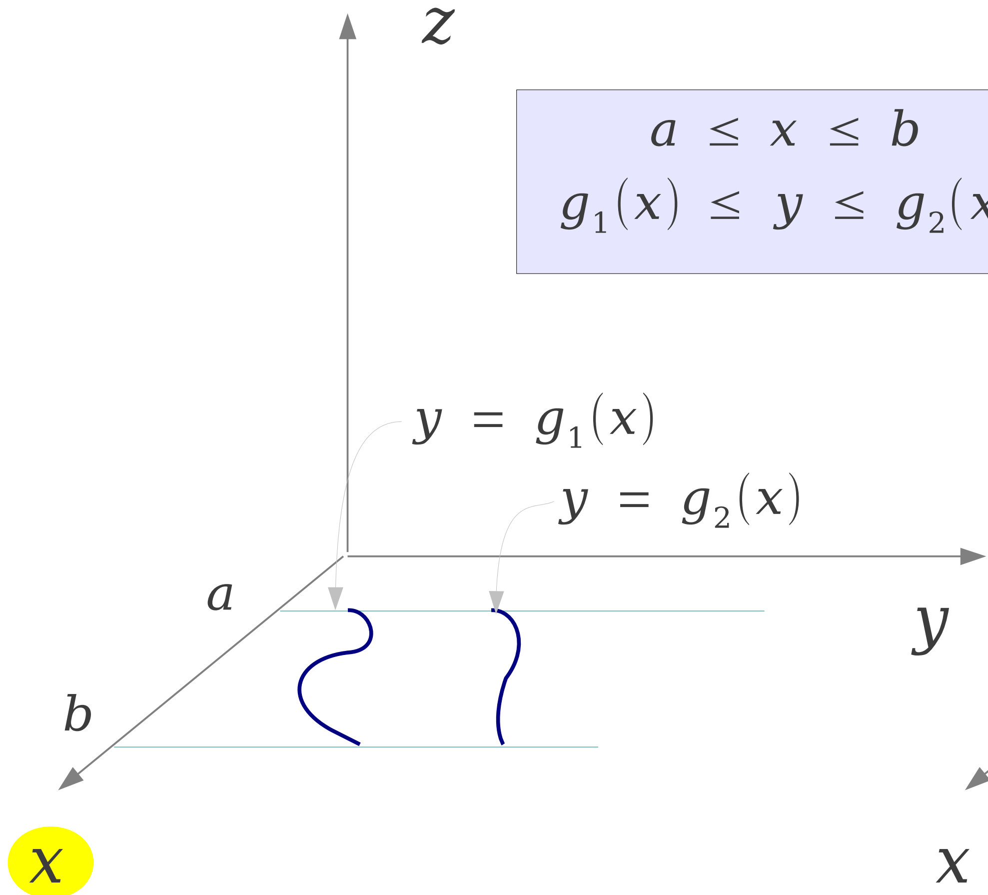
Type I and Type II





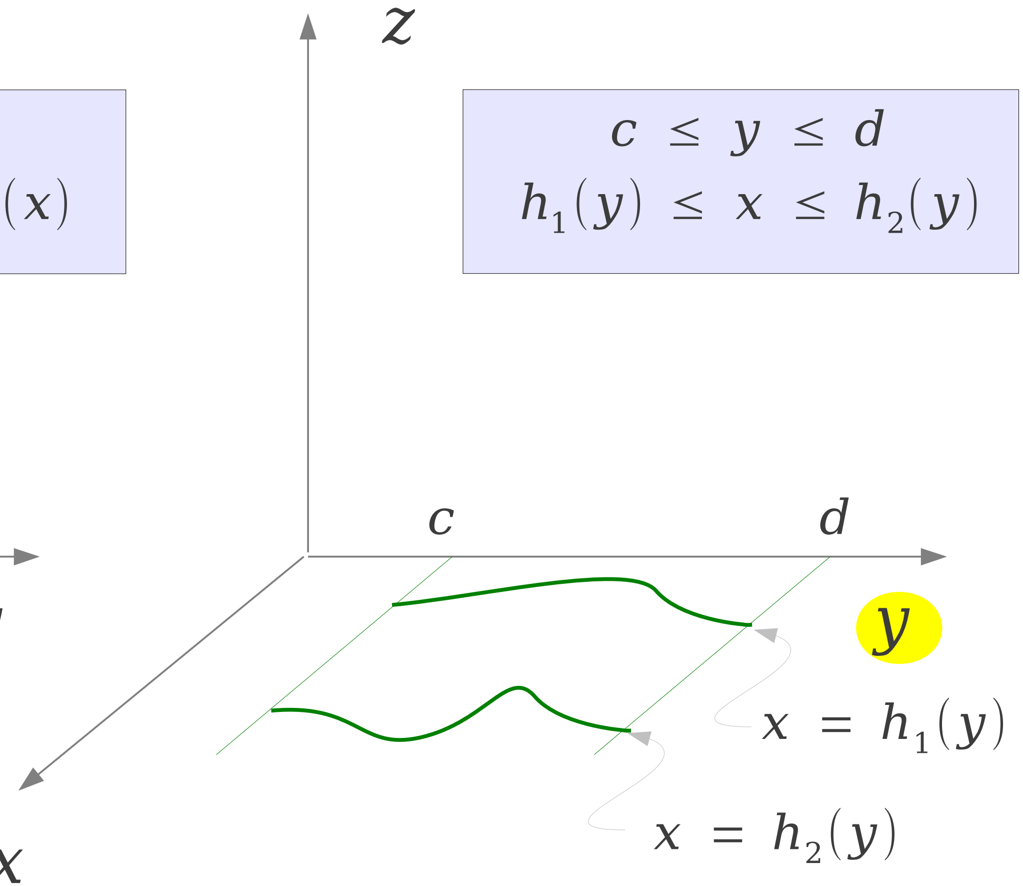
Fubini's Theorem

$$a \le x \le b$$
$$g_1(x) \le y \le g_2(x)$$



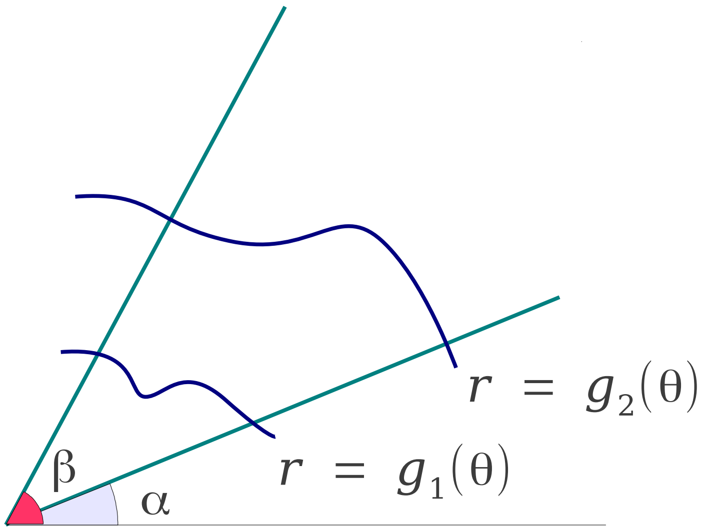
$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

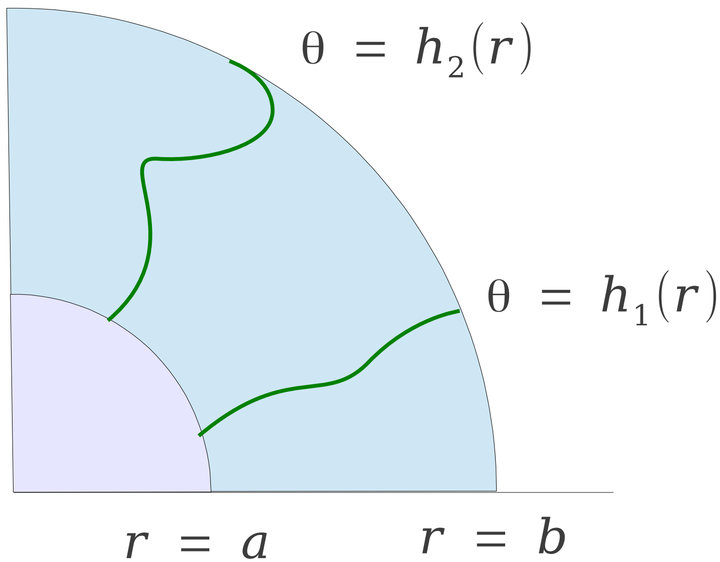
$$c \le y \le d$$
$$h_1(y) \le x \le h_2(y)$$



$$\iint_R f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

Type A and Type B





$$\iint_R f(r,\theta)\, dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(r,\theta)\, r\, dr\, d\theta$$

$$\iint_R f(r,\theta)\, dA = \int_a^b \int_{h_1(r)}^{h_2(r)} f(r,\theta)\, d\theta\, r\, dr$$

Equivalence in 3-D

In an open connected region

Path Independence $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ ⟺

Conservative $\boldsymbol{F}$ ⟺

Closed path C $\oint_C \boldsymbol{F} \cdot d\boldsymbol{r} = 0$ ⟺

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \qquad \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x} \qquad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}$$

$$curl\,\boldsymbol{F} = \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) + \left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right)$$

$$\boldsymbol{F} = P\,\boldsymbol{i} + Q\,\boldsymbol{j} + R\,\boldsymbol{k} \qquad \boldsymbol{F} = \nabla\Phi = \frac{\partial\Phi}{\partial x}\boldsymbol{i} + \frac{\partial\Phi}{\partial y}\boldsymbol{j} + \frac{\partial\Phi}{\partial z}\boldsymbol{k}$$

2-Divergence

Flux across rectangle boundary

$$\approx \left(\frac{\partial M}{\partial x}\Delta x\right)\Delta y + \left(\frac{\partial N}{\partial y}\Delta y\right)\Delta x \qquad = \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)\Delta x \Delta y$$

Flux density

$$= \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)$$

Divergence of **F**

Flux Density

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[4] D.G. Zill, “Advanced Engineering Mathematics”