

Double Integrals (5A)

- Double Integral
- Double Integrals in Polar Coordinates
- Green's Theorem

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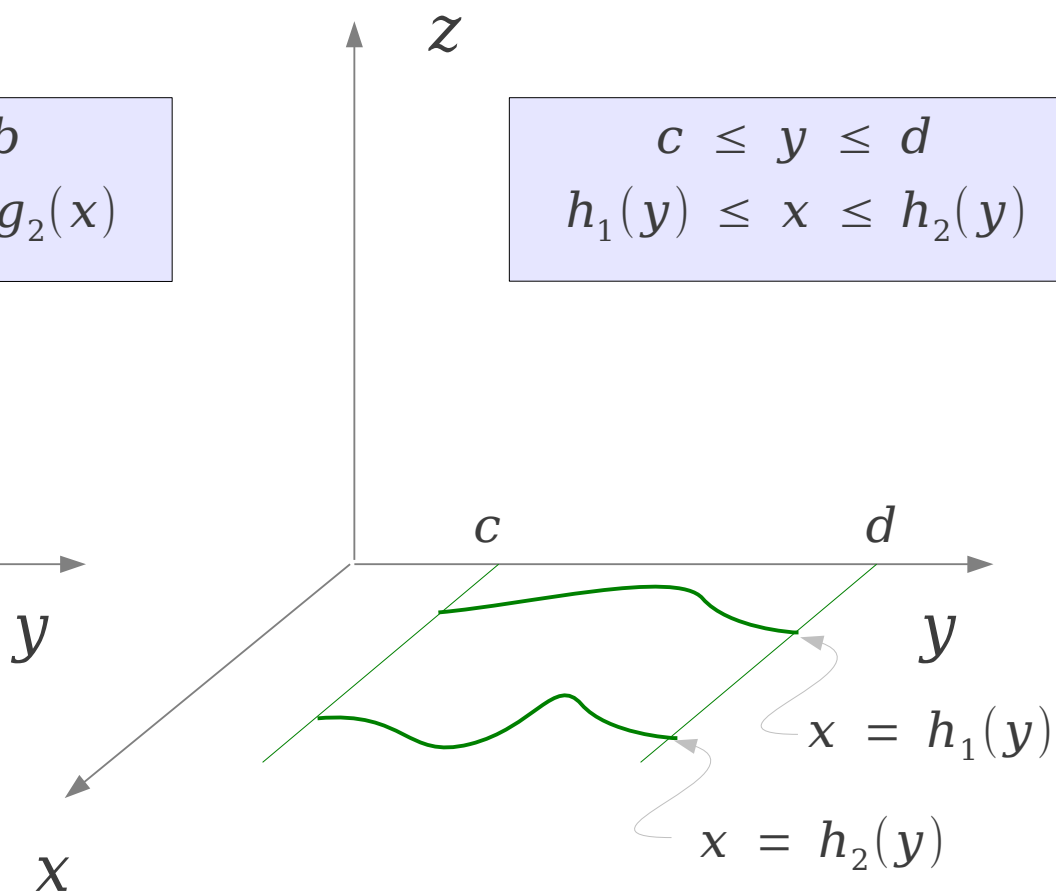
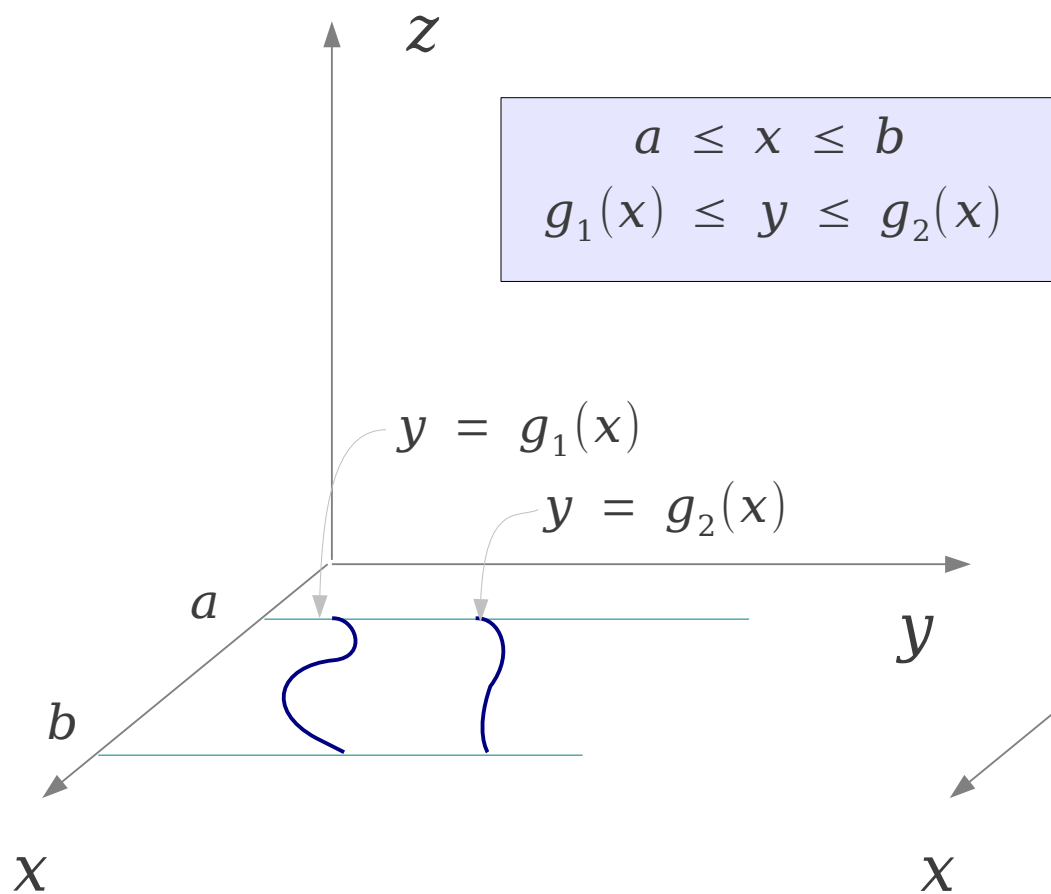
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Area and Volume

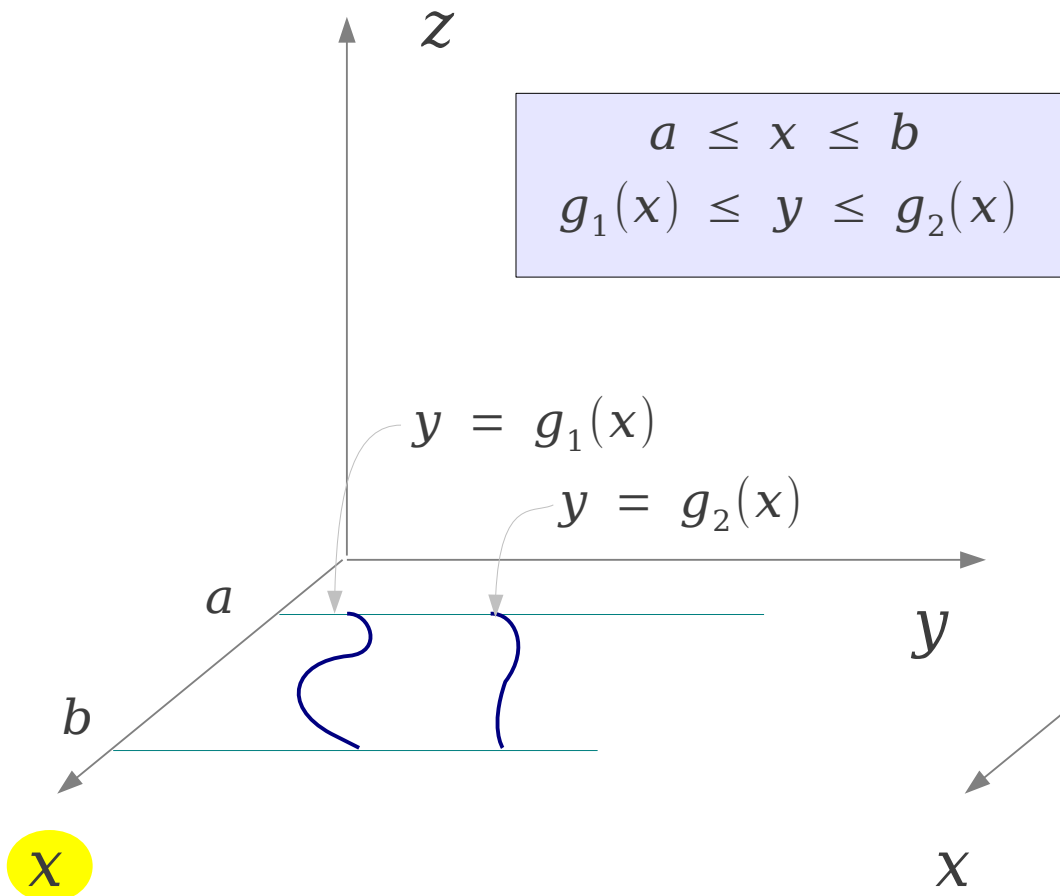
$$A = \iint_R dA$$

$$V = \iint_R f(x, y) dA$$

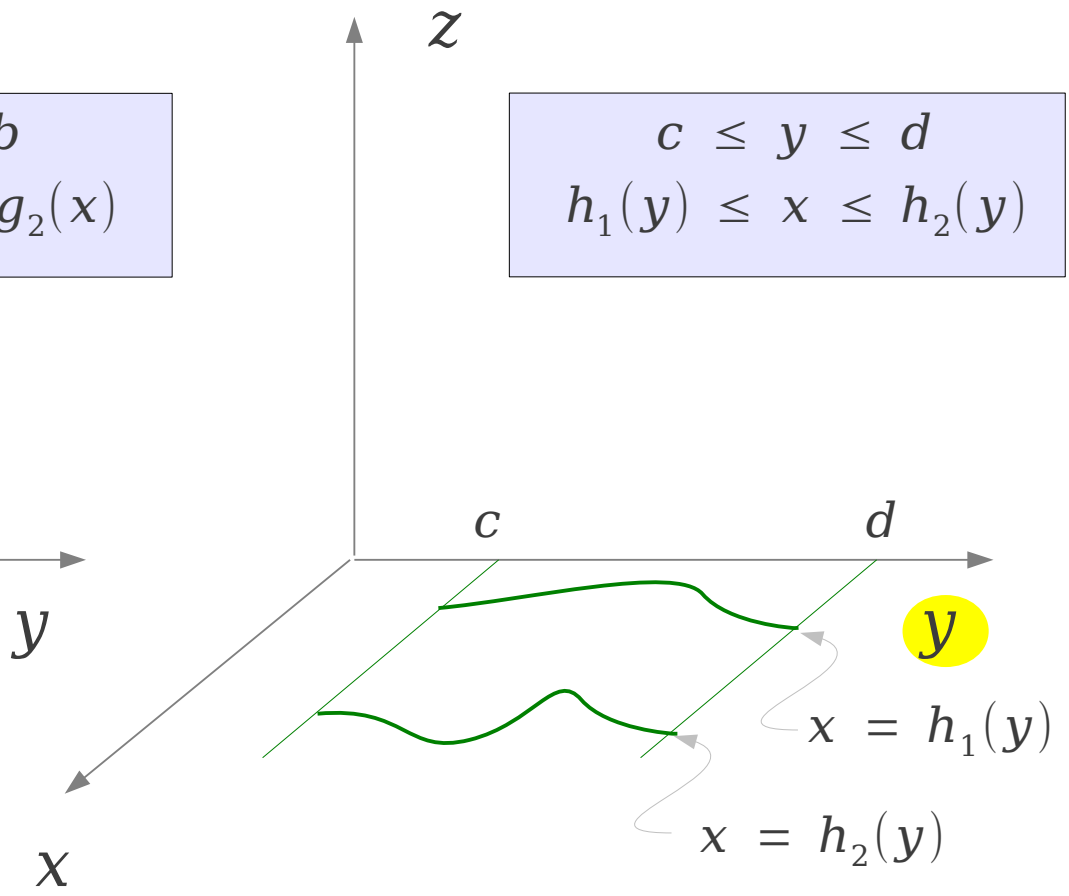
Type I and Type II



Fubini's Theorem

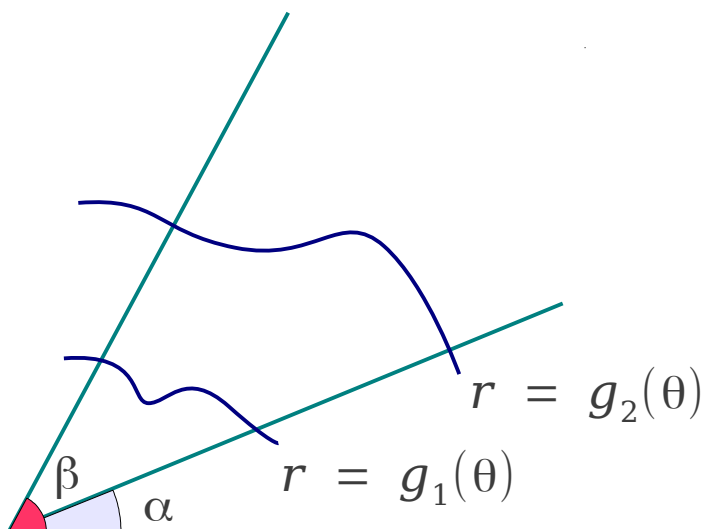


$$\begin{aligned}
 & \iint_R f(x, y) \, dA \\
 &= \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, dy \, dx
 \end{aligned}$$

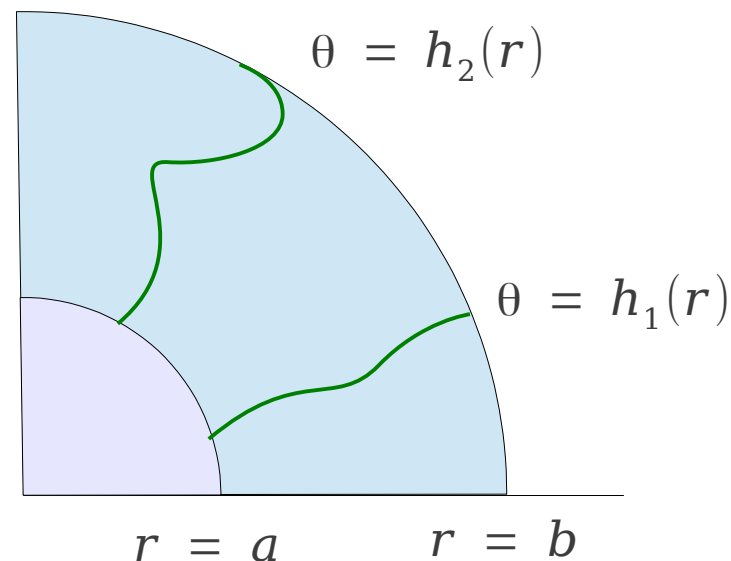


$$\begin{aligned}
 & \iint_R f(x, y) \, dA \\
 &= \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) \, dx \, dy
 \end{aligned}$$

Type A and Type B



$$\iint_R f(r, \theta) dA \\ = \int_{\alpha}^{\beta} \int_{g_1(\theta)}^{g_2(\theta)} f(r, \theta) r dr d\theta$$



$$\iint_R f(r, \theta) dA \\ = \int_a^b \int_{h_1(r)}^{h_2(r)} f(r, \theta) d\theta r dr$$

Equivalence in 3-D

In an open connected region

Path Independence $\int_C \mathbf{F} \cdot d\mathbf{r}$ \longleftrightarrow

Conservative \mathbf{F} \longleftrightarrow

Closed path C $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ \longleftrightarrow

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \quad \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x} \quad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}$$

$$\text{curl } \mathbf{F} = \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right) \mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \mathbf{j} + \left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y} \right) \mathbf{k}$$

$$\mathbf{F} = P \mathbf{i} + Q \mathbf{j} + R \mathbf{k} \quad \mathbf{F} = \nabla \Phi = \frac{\partial \Phi}{\partial x} \mathbf{i} + \frac{\partial \Phi}{\partial y} \mathbf{j} + \frac{\partial \Phi}{\partial z} \mathbf{k}$$

2-Divergence

Flux across rectangle boundary

$$\approx \left(\frac{\partial M}{\partial x} \Delta x \right) \Delta y + \left(\frac{\partial N}{\partial y} \Delta y \right) \Delta x = \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) \Delta x \Delta y$$

Flux density

$$= \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right)$$

Divergence of \mathbf{F}

Flux Density

References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
- [4] D.G. Zill, “Advanced Engineering Mathematics”