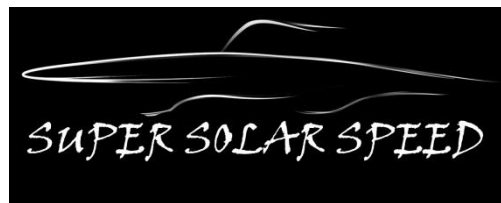




GroupT International Engineering College Feb-Jun 2012 BAC 2
Engineering Experience 4: "Make things work" Small Solar Vehicle
Group 17A/B

Super Solar Speed Entertainment Articles
Engineering Office



Small Solar Vehicle

SSV Part 1

Version 1

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0. Contents

1. Introduction.....	5
2. Design elements.....	5
2.1 Motor.....	5
2.2Solar panel.....	5
2.3 Wheels and bearings.....	5
2.4 Calculation of the rolling resistance.....	5
2.5Fixation system for the solar panel.....	5
2.6 Frame shape and material.....	5
2.7 Drive configuration.....	5
2.8 Guiding system.....	5
3. Gearbox calculations and simulations.....	7
3.1Gear ratio calculation examples.....	5
3.2Gear ratio simulations Matlab.....	5
3.3 Conclusion.....	5
3.4 Sankey diagrams.....	5
4. Case Matlab-Simulink.....	5
4.1 Simulation of the solar panel behavior.....	5
4.2 Total roll distance simulation.....	5

1. Introduction

In this document our team will discuss the different matters that belong to the SSV Case 1. We will start with a discussion on our design parameters; thereafter we will do manual and Matlab calculations on the gear ratio and energy efficiency. We finish with an overall simulation of the SSV using Matlab-Simulink.

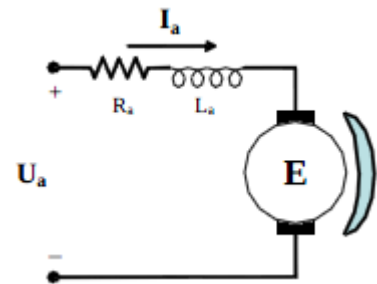
2. Design elements

2.1 Motor

The DC motor we will use to drive our SSV can simply be modeled using a series resistor and coil. The series resistor, R , and inductor, L , represent the terminal resistance and inductance of the rotor's winding, through which we apply the supply voltage.

$$V_R = I_a * R_a$$

$$V_L = L_a * \frac{di}{dt}$$



Let's first have a look at the many parameters on our motor's specifications table:

Motor Data:		Order Number	
with CLL		110117	110119
1	Assigned power rating	W	5.0 5.0
2	Nominal voltage	Volt	6.0 9.0
3	No load speed	rpm	9630 9970
4	Stall torque	mNm	20.7 23.2
5	Speed/torque gradient	rpm/mNm	469 433
6	No load current	mA	30 21
7	Starting current	mA	3510 2710
8	Terminal resistance	Ohm	1.71 3.32
9	Max. permissible speed	rpm	10600 10600
10	Max. continuous current	mA	840 751
11	Max. continuous torque	mNm	4.96 6.43
12	Max. power output at nominal voltage	mW	5210 6030
13	Max. efficiency	%	83 84
14	Torque constant	mNm/A	5.90 8.55
15	Speed constant	rpm/V	1620 1120
16	Mechanical time constant	ms	19 19
17	Rotor inertia	gcm ²	3.88 4.10
18	Terminal inductance	mH	0.11 0.22
19	Thermal resistance housing-ambient	K/W	20 20
20	Thermal resistance rotor-housing	K/W	6.0 6.0
21	Thermal time constant winding	s	9 10

→ R_a

→ $K_t/1000$

→ $1/K_e * 60/(2\pi)$

As the motor turns, it generates a back EMF (Faraday's law) and output torque. The output torque is proportional to the winding's current:

$$T_m = K_t * I_a$$

Where $K_t = 8.55 \text{ mNm/A}$ is the torque constant

And the back EMF is proportional to the rotor's speed:

$$E = K_e * \omega$$

Where $K_e = 8.52\text{mV}/(\text{rad}/\text{sec})$ is the Back-EMF Constant
 At zero speed, the back EMF is zero. Hence the current is:

$$I = \frac{U}{R} = \frac{9}{3.32} = 2.7 \text{ A}$$

Therefore, the output torque is:

$$\text{Stall torque} = 8.55 * 2.7 = 23.2 \text{ mNm}$$

At no-load condition, the back EMF is:

$$U = E + I_0 * R$$

$$E = U - I_0 * R = 9 - 0.021 * 3.32 = 8.93 \text{ V}$$

$$\text{No load speed } \omega = \frac{E}{K_e} = \frac{8.93}{8.52 * 0.001} = 1048 \frac{\text{rad}}{\text{s}} = 10\text{k rpm}$$

This normally should be 9970 rpm if we look in the specs.

The no-load current, I_0 , is required to overcome the motor's internal friction. The output torque at the motor shaft is essentially zero.

At other speeds, the torque is:

$$EMF = K_e * \omega$$

$$I = \frac{U - EMF}{R}$$

$$\begin{aligned} \text{Output Torque} &= K_m * I \\ &= K_m * \frac{U - K_e * \omega}{R} \\ &= \frac{K_m * U}{R} - \frac{K_m * K_e * \omega}{R} \\ &= \text{Torque (stalled)} - \frac{K_m * K_e * \omega}{R} \end{aligned}$$

$\frac{K_m * K_e}{R}$ is a constant. Hence the torque vs. speed curve is a straight line with a negative slope.

Other parameters:

Output power is the maximum output mechanical power from the motor.

Efficiency is the maximum power efficiency of the motor.

Friction torque is the torque required to start rotating the rotor. The friction is due to brushes, bearings etc. It is equal to the no-load current * Torque constant ($I_0 * K_m$).

$$\text{Mechanical power output (W)} = \text{Torque (N * m)} * \text{speed } \left(\frac{\text{rad}}{\text{s}}\right)$$

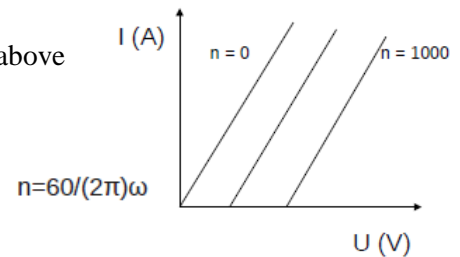
Power output is maximum at half stall torque and half no-load speed

$$\begin{aligned} \text{Power input} &= V * I \\ &= \text{Power (mechanical)} + \text{heat}(I_a * R_a) \\ &= \text{speed } \omega * \text{torque} + I_a * R_a \\ &= U_a * I_a \end{aligned}$$

Efficiency of a motor is the ratio of mechanical output power over electrical input power. Maximum efficiency occurs at motor speed above half no-load speed.

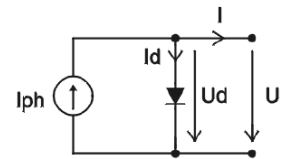
We can conclude that the relation Current –Voltage is:

$$U_a = K_t * \omega + R_a * I_a$$



2.2 Solar panel

Our silicon solar panel can simply be modeled using a current source and a diode connected in parallel. The current coming from the current source is directly proportional to the solar radiation. The diode represents the PN junction of a solar cell. The current coming from an ideal solar cell using the ideal solar cell model, is given by:



$$I(t) = I_{sc} - I_s \left(e^{\frac{U}{mN U_r}} - 1 \right)$$

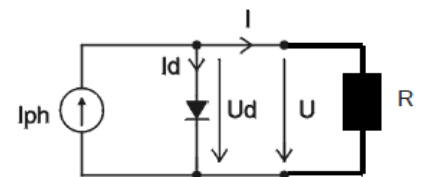
Where is: I_{sc} —short circuit current or photocurrent (A), I_s - reverse saturation current (approximately range 10^{-8} A/m²), U - diode voltage (V), U_r - thermal voltage (25.7 mV at 25 °C), m - diode ideality factor = $1 \dots 5 \times VT$ (-) ($m = 1$ for ideal diode), N – number of solar cells in series.

The thermal voltage U_r was calculated using the following equation:

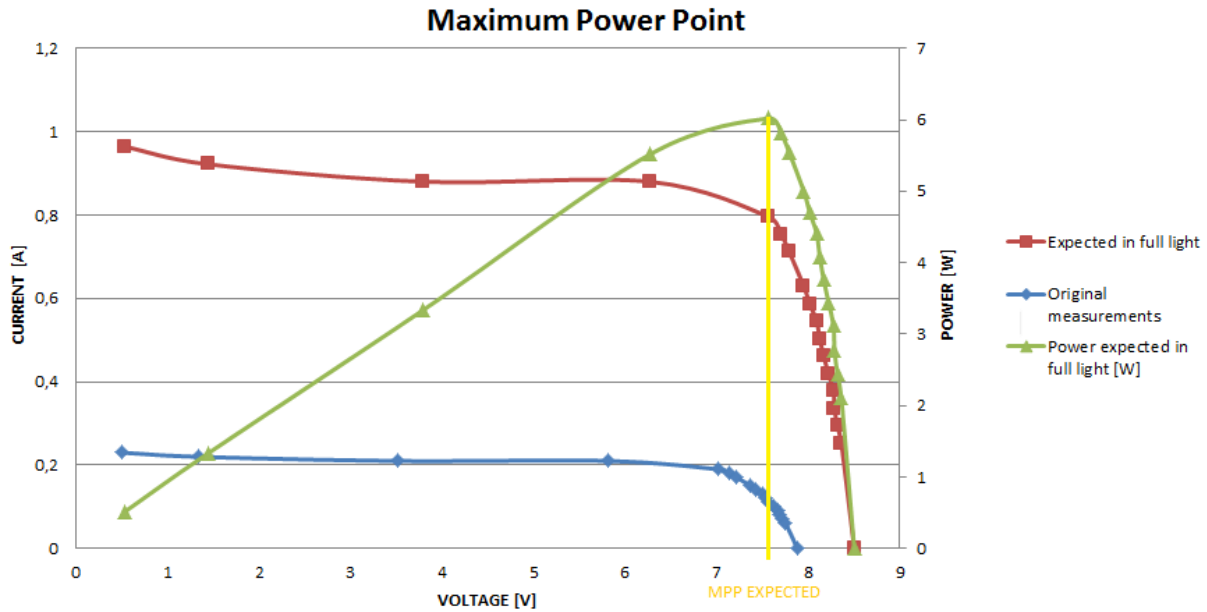
$$U_r = \frac{k * T}{q}$$

Where is: k - Boltzmann constant = 1.38×10^{-23} J/K, T - temperature (K), q - charge of electron = 1.6×10^{-19} A*s

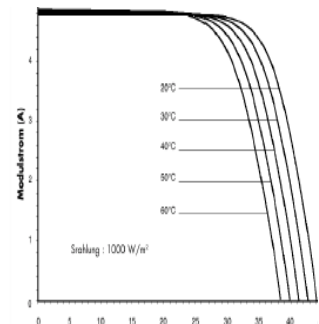
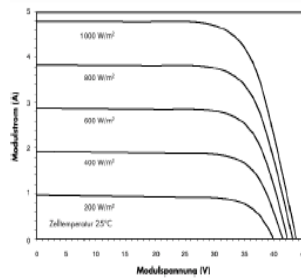
To calculate the approximate value of the diode factor m , we did a small test in the lab. First we connected the solar panel to various resistance values from 0 to 100 Ohms. For every resistor value we measured the current I and voltage U . From the solar panel current equation we calculated the value m that fitted the measurements.



The resulting I-U characteristic of our solar panel is plotted in this graph:



The working point of the solar panel depends on the load and on the solar insolation. The above graph shows the importance of operating solar-photovoltaic arrays at their maximum power point. Since temperature and sunlight density can have a significant influence on the U-I characteristic of our solar panel, we will examine the feasibility of building a maximum power point tracking circuit. Sunlight density effect that automatically draws maximum power from the solar panel.



Temperature effect

2.3 Wheels and bearings

We are convinced that the wheels are an extremely important part of the SSV. Therefore we decided not to make them by ourselves and order them on a highly professional and specialized online store in Australia. The wheels are made of Delrin® acetalhomopolymer resin. This lightweight material is stiff and has a high tensile strength and a low coefficient of friction. One of the main reasons why we opted for these wheels is because they are balanced. They are also flawlessly connectable with the bearings discussed in the next paragraph. The wheels are 54mm in diameter, have an internal diameter of 7mm, 6mm width at the center and are 3mm wide at the perimeter.

The bearings we will use are stainless steel 3x7x3mm flanged ball bearings. Each bearing has 2 metal shields to protect the bearing from dust or any possible contamination. These bearings are also pre-lubricated with grease. The wheels are designed to take two of these bearings on each side as shown.



2.4 Calculation of the rolling resistance

The rolling resistance of our SSV is mainly dependent on the vehicle's weight and the rolling resistance coefficient, f_r . Because our wheels are stiff and do not significantly deform plastically, we can use the following general relation for maximum rolling resistance on a flat surface:

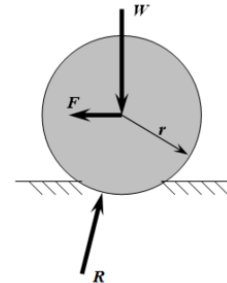
$$F_R = C_{rr} * N$$

Where:

F_R is the SSV's total rolling resistance in N

C_{rr} is the rolling resistance coefficient (dimensionless)

N is the normal force (m*g)



Notes:

- This formula is only valid when the wheels don't slip on the surface.
- C_{rr} is hard to obtain, it is in the range of 0.012

$$F_R = 0.012 * 1kg * 9.81 \frac{N}{kg} \approx 0,118 N$$

2.5 Fixation system for the solar panel

The solar panel will be attached to the frame by an old GPS stand. This is equipped with a suction pad on one side, to which the solar panel will be attached. The other side will be connected by screws to a plate that will be connected to the frame (see below).

The stand itself is adjustable in two directions (originally three, but we shortened it to improve stability, to bring the solar panel down somewhat and because two directions is sufficient for the purpose we are giving it). This way, even though the average angle of inclination of the sunlight is known, we can still adjust the aim of our solar panel at the race itself, so we will not depend on things like the direction of the track or the time of day.

2.6 Frame shape and material

Our vehicle will consist out of a triangular chassis, on which the solar panel, motor, gear box and Maximum Power Point Tracker and (MPPT) will fit. A vertical bar will protrude through this triangle, and even stick out in the front, to connect the front wheel to. The rear axle will be extended as well, and the wheels will be put at the end. These two measures make sure that both weight and air resistance are of the frame are limited to the minimum, but without making it unable to carry the components. The extensions also serve as an extra means to stabilize the vehicle while driving, since its center of gravity will be fairly high.

The bars are U-shaped aluminum bars of two different types: two-centimeter sided ones for the container for the rear axle,

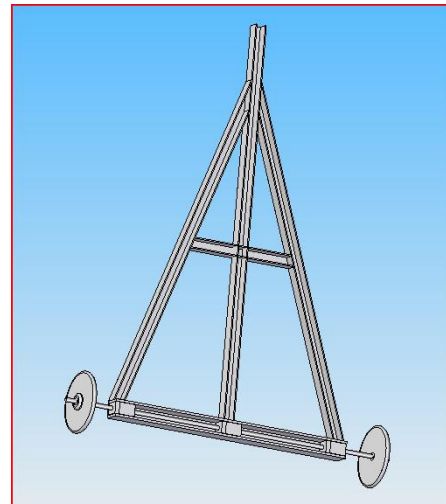


Figure 1: The vehicle as it is modeled at this moment

and one-centimeter sided ones for the rest of the frame. We took these small and hence light bars once again to reduce the weight to a minimum. But to have a firm connection and container for the bearings and axle, this was not possible for the rear bar, which we decided to make wider after testing.

The rear bar contains bearing holders, which are printed by the 3D-printer in FabLab. This was necessary because these are hard to build parts, and because their volume is limited the cost will not be too high. The holders consist out of a block, with a hole cut out in it. The hole is wider in the mid, so the bar can fit completely through it, but it is impossible for the bearings to slip out.

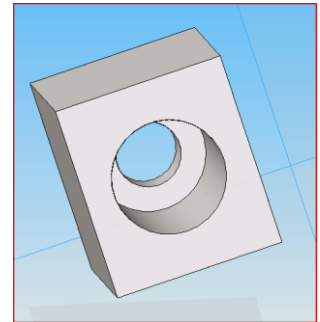


Figure 2: The bearing holder

A bar in the middle has been added to make the attachment of the plate to connect the solar panel stand to easier. More bars through the middle might be added in the future for stability, or to attach other components to, if this turns out to be necessary.

The frame will be welded together, since the limited surface area at the connections will not allow the use of glue. This can be done in FabLab as well. Since aluminum welding is rather difficult, it might take some time for this to be done, since courses will have to be taken by some of our members.

In the middle of the frame, the stand for the solar panel will be located. Since the motor will be placed near the rear axle to make the gear configuration easier, the MPPT will have to be placed in between.

2.7 Drive configuration

We have chosen for a three-wheel drive configuration (single wheel in front) to reduce the rolling resistance and body weight. The two rear wheels will be driven, by connecting the motor to the rear axle. We chose to drive two wheels to make the deviations from the track as small as possible. Since the entire axle can be driven this way, it will also be an easy system to implement. The rear wheel drive is more of an arbitrary decision, although one could argue that the aerodynamics of the frame would have been worse if we would turn it around (flat vs. triangular shape).

2.8 Guiding system

For the guiding system, we decided to keep things as simple as possible. The system as it is right now will consist out of two pairs of horizontal bearings, which are attached to the beam that runs from front to back in our vehicle. These will loosely fit around the guiding bar, allowing them to gently steer the vehicle in the right direction in case it deviates too much from its main path. Since the bearings can roll on the guiding bar, this will not cause a too large amount of friction. By connecting them near the position of the wheels, we can also prevent them from slipping off the guiding bar when the angle of the track changes.

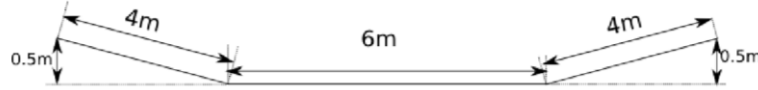
We chose this system over systems that could steer the front wheel because with those, steering the car gently enough would have become a major issue. The simplicity of this design will also allow us to keep the weight of the car down.

3. Gearbox calculations and simulations

3.1 Gear ratio calculation examples

To develop a good understanding of the physics behind the energy equation, we will do some calculations by hand. First let's assume we are on the first part of the track ($s \leq 4m$).

Racetrack



Using some basic physics we can now put on the stage the displacement $s(t)$, velocity $v(t)$, back emf $E(t)$ and terminal current $I(t)$, all in function of t . Before we can start putting together the energy equation, we do a little refresher on several kinds of energies that are involved in our SSV system:

$$\text{Kinetic energy: } E_k = \frac{M \times v(t)^2}{2}$$

$$\text{Change of Potential energy: } E_p = M \times g \times \sin(\alpha) \times s(t)$$

$$\text{Dissipated power: } P = F \times v(t) = M \times g \times C_{rr} \times \cos(\alpha) \times v(t); C_w \times A \times \rho \times \frac{v(t)^3}{2}$$

In the above formula we defined

C_{rr} as the rolling resistance [dimensionless]

C_w as the air drag coefficient [dimensionless]

A as the frontal area of the SSV [m^2] and ρ as the air density [kg/m^3]

$$\text{Input work: } W = E(t) \times I(t)$$

Because of the imbalance between dissipated and input energies, we assume that our SSV system does not conserve energy. We can now apply the Lagrange equation on our system:

$$\frac{d}{dt} \left[\frac{\delta(E_k - E_p)}{\delta v} \right] - \left[\frac{\delta(E_k - E_p)}{\delta s} \right] = \left[\frac{-\delta P}{\delta v} \right] + \left[\frac{\delta W}{\delta s} \right]$$

When we fill in the corresponding items, calculate the derivatives and put everything but the acceleration on the right side, we can rewrite this second order differential energy equation as follows:

$$a(t) = g \times (\sin(\alpha) - \cos(\alpha) \times C_{rr}) + \frac{I(t) \times E(t)}{M \times v(t)} - 3 \times C_w \times A \times \rho \times \frac{v(t)^2}{2 \times M}$$

With

$$a(t) = \frac{d^2 s(t)}{dt^2}; v(t) = \frac{ds(t)}{dt}$$

We can approximate the motor voltage as follows:

$$E(t) = K_e \times \omega = C_E \varphi \times 60 \times v(t) \times \frac{\text{gear ratio}}{2\pi r}$$

Here C_E is defined as the motor's inverse speed constant [V/(rad/s)]

The current coming from the solar panel can be approached like this:

$$I(t) = I_{sc} - I_s \left(e^{\frac{U(t)}{m n U_r}} - 1 \right)$$

With $U(t)$ equaling $E(t) + I(t) \times R$

In the prior formula, I_{sc} is the short circuit current [A]

Unfortunately, it's impossible to solve this second order differential energy equation analytically. But luckily we can easily solve this mathematical problem by using a numerical approximation method. We will go through this method step by step and at the end we will present a recording of some calculated data points relating to our specific SSV system.

Step one: in this step we appoint the initial conditions.

For these approximative calculations, we rate the rolling resistance coefficient to be around $C_{rr} = 0.012$, the total mass some 1kg, the air drag coefficient C_w close to 0.5 and the frontal area A just about 0.02 m^2 . The track angle α equals 0.1253 rad ; $C_E \cdot \Phi = 8.9285e^{-4} \text{ V/rpm}$; the radius of our wheels is 0.027 m and $\rho = 1.293 \text{ kg/m}^3$

Concerning the solar panel current we use the formula discussed before:

$$I(t) = I_{sc} - I_s \left(e^{\frac{E(t)+I(t) \times R}{mNUr}} - 1 \right)$$

$I_{sc} = 0.88 \text{ A}$; $I_s = 1e^{-8} \text{ A/m}^2$; $m = 1.1$; the number of solar cells in series $N = 15$; thermal voltage $U_r = 0.0257 \text{ V}$; $R_a = 3.32 \Omega$

We take the time interval $T = 0.1 \text{ s}$. The gear ratio we will test here is 10.

Initial conditions: $s(0) = 0$; $v(0) = 0$; $I(0) = 0.88 \text{ A}$

Step two: we solve the energy equation for $t=0$

$$a(0) = g(\sin(\alpha) - \cos(\alpha) \times C_{rr}) - 3C_w A \rho \times \frac{v^2(0)}{2M}$$

Step three: we calculate the initial condition for $t=0.1 \text{ s}$

$$v(0.1) = a(0) \times T$$

$$s(0.1) = v(0) \times T + a(0) \times \frac{T^2}{2}$$

$$E(0.1) = C_E \cdot \Phi \times 60 \times v(0.1) \times \frac{\text{gear ratio}}{2\pi}$$

$$I(0.1) = I_{sc} - I_s \left(e^{\frac{E(0.1)+I(0.1)R}{mNUr}} - 1 \right)$$

Step four: we solve the energy equation for $t=0.1 \text{ s}$

$$a(0.1) = g(\sin(\alpha) - \cos(\alpha) C_{rr}) + \frac{I(0.1)E(0.1)}{M v(0.1)} - 3C_w A \rho \frac{v^2(0.1)}{2M}$$

Step five: we calculate the initial condition for $t=0.2 \text{ s}$

$$v(0.2) = v(0.1) + a(0.1) \times T$$

$$s(0.2) = s(0.1) + v(0.1) \times T + a(0.1) \times \frac{T^2}{2}$$

$$E(0.2) = C_E \cdot \Phi \times 60 \times v(0.2) \times \frac{\text{gear ratio}}{2\pi}$$

$$I(0.2) = I_{sc} - I_s \left(e^{\frac{E(0.2)+I(0.2)R}{mNUr}} - 1 \right)$$

Step six: we solve the energy equation for $t=0.2 \text{ s}$:

$$a(0.2) = g(\sin(\alpha) - \cos(\alpha) C_{rr}) + \frac{I(0.2)E(0.2)}{M v(0.2)} - 3C_w A \rho \frac{v^2(0.2)}{2M}$$

Step 7: etc...

If we calculated all values till a distance of 4m, we can calculate the values for the flat part ($s = 4..10 \text{ m}$) of the track. We can consider here the initial values equal to the last values of the sloping part of the track at $\approx 4 \text{ m}$. The energy equation now simplifies since α becomes zero degrees:

$$a(t) = -C_{rr} * g + \frac{E * I}{M * v(t)} - 3C_w A \rho \frac{v^2(t)}{2M}$$

Again we apply the numerical approximation method.

To calculate the uphill part (s = 10..14m) of the track we apply the following adapted energy equation:

$$a(t) = -g(\sin(\alpha) + \cos(\alpha) Crr) + \frac{I(t)E(t)}{M v(t)} - 3CwA\rho \frac{v^2(t)}{2M}$$

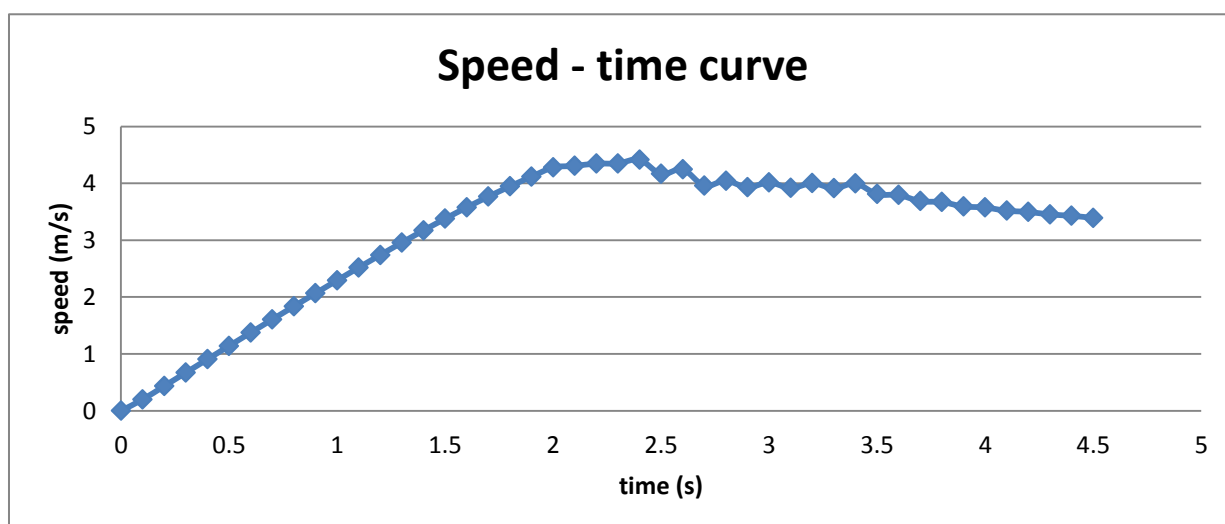
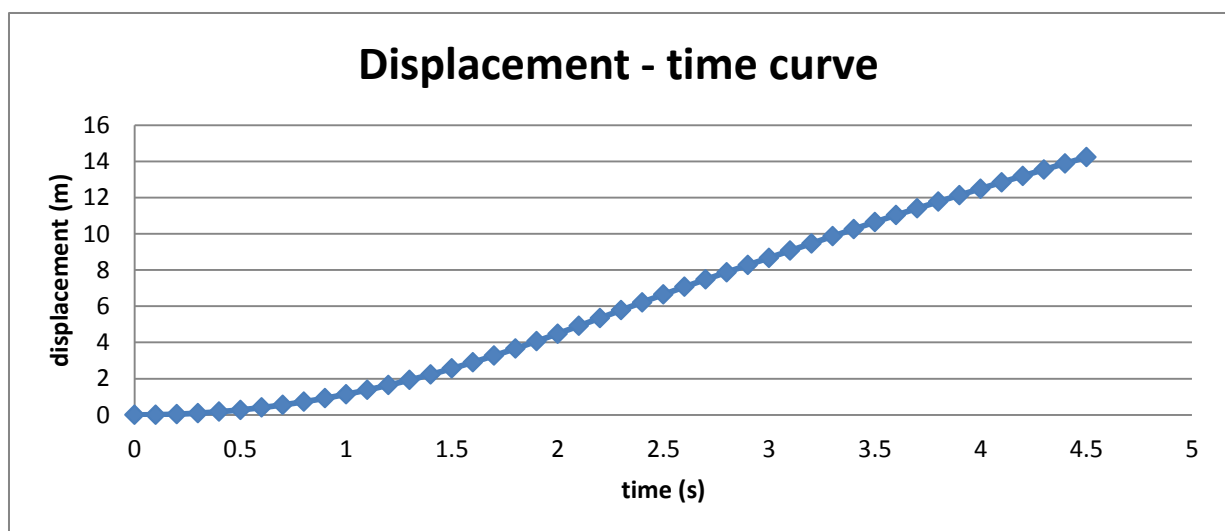
If we enter these lines into excel, we get the following values:

dt [s]	a(t) [m/s^2]	v(t) [m/s]	s(t) [m]	I(t) [A]	E(t) [V]
0	1,98918	0	0	0,88	0
0,1	2,35889	0,198918	0,009946	0,87999	0,282666
0,2	2,35598	0,434808	0,041632	0,879979	0,617868
0,3	2,3509	0,670405	0,096893	0,879957	0,952656
0,4	2,34365	0,905495	0,175688	0,879911	1,286722
0,5	2,33422	1,13986	0,277956	0,879817	1,619758
0,6	2,32256	1,373281	0,403613	0,879625	1,951453
0,7	2,30859	1,605538	0,552554	0,879235	2,281493
0,8	2,29207	1,836397	0,72465	0,87845	2,609547
0,9	2,27249	2,065603	0,91975	0,876884	2,935253
1	2,24891	2,292853	1,137673	0,873806	3,258178
1,1	2,21955	2,517744	1,378203	0,867912	3,577752
1,2	2,18154	2,739699	1,641075	0,857086	3,893153
1,3	2,13111	2,957853	1,925953	0,838563	4,203153
1,4	2,06545	3,170964	2,232394	0,810184	4,505987
1,5	1,98579	3,377508	2,559817	0,77259	4,79949
1,6	1,89716	3,576088	2,907497	0,729066	5,081674
1,7	1,80355	3,765804	3,274592	0,682201	5,351264
1,8	1,70822	3,94616	3,66019	0,634099	5,607552
1,9	1,6128	4,116982	4,063347	0,585746	5,850293
2	0,29274	4,278262	4,483109	0,538672	6,079474
2,1	0,39437	4,307537	4,912399	0,61362	6,121074
2,2	0,03462	4,346974	5,345125	0,36511	6,177114
2,3	0,64143	4,350435	5,779995	0,792549	6,182033
2,4	-2,49757	4,414578	6,218246	-1,40876	6,273181
2,5	0,79635	4,164821	6,647216	0,88	5,918273
2,6	-2,82921	4,244456	7,067679	-1,66225	6,031436
2,7	0,82839	3,961536	7,477979	0,88	5,629401
2,8	-1,13834	4,044375	7,878275	-0,49498	5,747117
2,9	0,83307	3,930541	8,27702	0,87995	5,585357
3	-0,95803	4,013848	8,67424	-0,37145	5,703737
3,1	0,83487	3,918044	9,070834	0,879883	5,567599
3,2	-0,88927	4,001532	9,466813	-0,32441	5,686236
3,3	0,83564	3,912604	9,86252	0,879838	5,559869
3,4	-1,85277	3,996168	10,25796	-0,30435	5,678614
3,5	-0,16639	3,810891	10,64831	0,879863	5,415332

3,6	-1,07116	3,794252	11,02857	0,242926	5,391688
3,7	-0,17053	3,687135	11,40264	0,875257	5,239474
3,8	-0,76157	3,670082	11,7705	0,459097	5,215241
3,9	-0,18587	3,593924	12,1337	0,863194	5,10702
4	-0,57163	3,575337	12,49216	0,591473	5,080608
4,1	-0,20945	3,518175	12,84684	0,845564	4,999379
4,2	-0,44432	3,497229	13,19761	0,679997	4,969615
4,3	-0,23481	3,452798	13,54511	0,826824	4,906477
4,4	-0,36046	3,429316	13,88922	0,738086	4,873109
4,5	-0,25358	3,39327	14,23034	0,812802	4,821888
	Avg speed.	11,26957			

Table 1: Excel calculations of the acceleration, speed and displacement for a gear ratio of 9.

Graph 1 and 2 plot the displacement and speed in function of time for gear ratio 9 for the full 14 m of the track



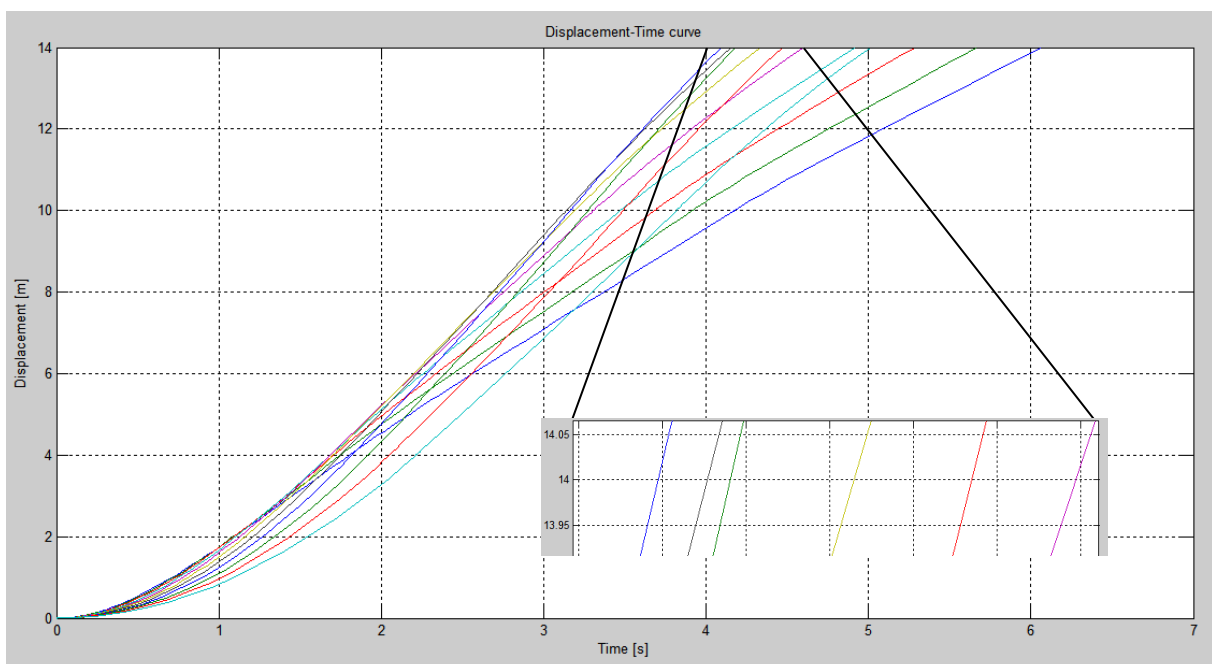
But for now, let's make things more comprehensive and recalculate it using Matlab:

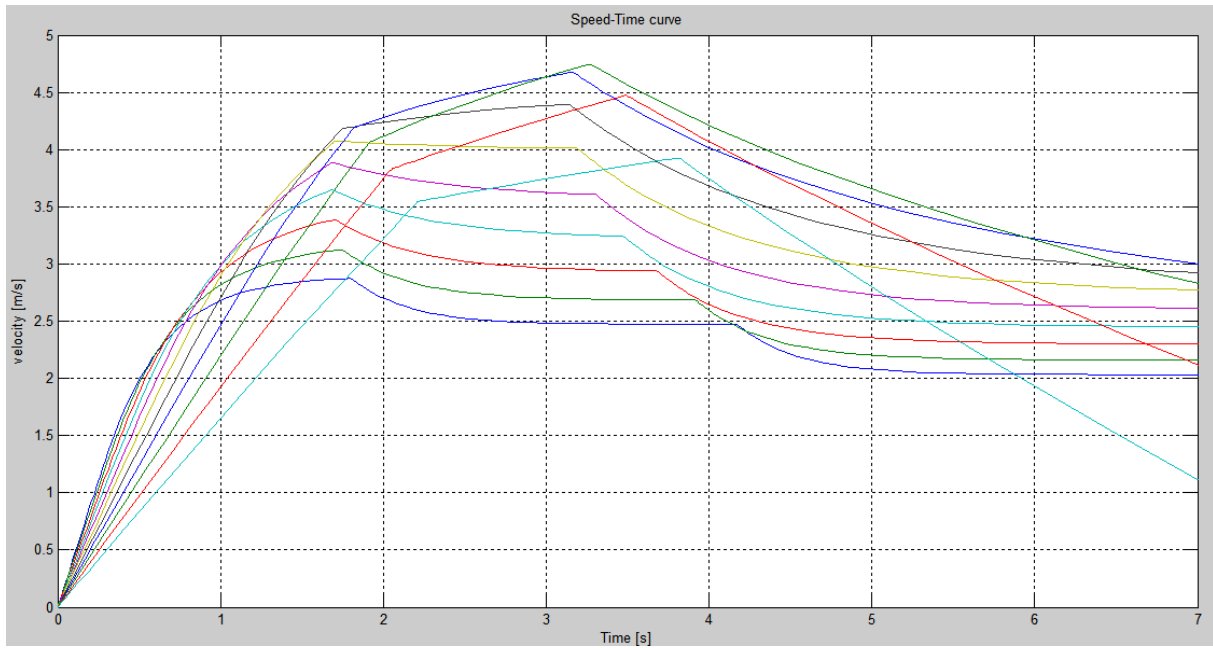
3.2 Gear ratio simulations Matlab

The Matlab program is developed to do exactly the same as we were doing by hand, only more precise (without the deviations that come with the numerical approximation method). First, we need to set the variables of our solar car:

```
a=0.1253; % angle [radians]
M=1; % mass [kg]
g=9.81; % acceleration [m/s^2]
Ce=8.9285e-4; % inverse of the speed constant [V/(rad/sec)]
r=0.027; % radius of the wheel [m]
Crr=0.012; % rolling resistance coefficient [-]
ratio=9; % we pick [-]
Cw=0.5; % air drag coefficient [-]
A=0.0148; % get value from car [m^2]
p=1.293; % air density [kg/m^3]
Isc=0.88; % short circuit current [A]
Is=1e-8; % saturation current [A/m^2]
Ur=0.0257; % thermal voltage [V] (25.7 mV at 25 C)
m=1.22; % diode factor[-]; original 1.1
N=15; % number of solar cells in series [-]
R=3.32; % motor internal resistance [ohms]
```

We will not go into detail on the exact programming of the code because it is doing exactly the same as we were doing in Excel. So the next thing we had to do was to run it. With some tweaking, we were able to automatically generate the displacement and speed vs. time plots for different gear ratios on one graph:





Color	Gear Ratio	Color	Gear Ratio	Color	Gear Ratio
Blue:	12	Purple:	8	Clear green	4
Green:	11	Beige:	7	Clear red	3
Red:	10	Black:	6	Clear cyan	2
Cyan:	9	Clear blue	5		

3.3 Conclusion

From the displacement diagram we can see that the best gear ratios, for our values, are between 4 and 6 because after examining the diagram closely, by zooming in, we can see that the fastest finish-time that we could achieve would be by using a gear ratio of 5.

Now we look at the time it would take to reach 14 meters with the gear ratio of 5 and find that it is at approximately 4 seconds. If we look at that point on the speed curve, for a gear ratio of 5, we find that the velocity at that point will be approximately 4 m/s.

3.4 Investigation of first second at optimal gear ratio

Solar vehicle parameters:

$\alpha = 7.18^\circ$; $g = 9.81 \text{ N/kg}$; $C_{rr} = 0.012$; $C_E \cdot \Phi = 8.9285 \cdot 10^{-4} \text{ V/rpm}$;
 $r = 0.027 \text{ m}$; $\pi = 3.14$; $M = 1 \text{ kg}$; $C_w = 0.5$; $A = 0.0148$; $\rho = 1.293 \text{ kg/m}^3$

Solar panel parameters: $I(t) = I_{sc} - I_s (e^{(E(t)+I(t)R)/(mNUr)} - 1)$

$I_{sc} = 0.88 \text{ A}$; $I_s = 1e^{-8} \text{ A/m}^2$; $m = 1.22$; $N = 15$; $U_r = 0.0257 \text{ V}$; $R_a = 3.32 \Omega$

testing gear ratio: 9

$$\text{Motor: } E(t) = K_e \times \omega = C_E \cdot \Phi \times 60 \times v(t) \times \text{gear ratio} / (2\pi r)$$

Situation 1: Time interval (T) 0.1s

Initial condition (at t = 0s)

$$s(0) = 0; v(0) = 0; I(0) = 0.88A$$

[During each time interval (T, T+0.1), the acceleration remains constant.]

(0,0.1)

$$a(0) = g(\sin(\alpha) - \cos(\alpha)) \times C_{\pi} + I(0) \times E(0) / (M \times v(0)) - 3C_w A \rho v^2(0) / 2M \approx 2.399 \text{ m/s}^2$$

$$v(0.1) = a(0) \times T = 0.2399 \text{ m/s}$$

$$s(0.1) = v(0) \times T + a(0) \times T^2 / 2 = 0.0120 \text{ m}$$

$$E(0.1) = C_E \cdot \Phi \times 60 \times v(0.1) \times \text{gear ratio} / (2\pi r) = 0.6818 \text{ V}$$

$$I(0.1) = I_{sc} - I_s (e^{(E(0.1)+I(0)R)/(mNUr)} - 1) \approx 0.87 \text{ A}$$

(0.1,0.2)

$$a(0.1) = g(\sin(\alpha) - \cos(\alpha)) C_{\pi} + I(0.1)E(0.1) / (M v(0.1)) - 3C_w A \rho v^2(0.1) / 2M \approx 2.370 \text{ m/s}^2$$

$$v(0.2) = v(0.1) + a(0.1) \times T = 0.4769 \text{ m/s}$$

$$s(0.2) = s(0.1) + v(0.1) \times T + a(0.1) \times T^2 / 2 = 0.0478 \text{ m}$$

$$E(0.2) = C_E \cdot \Phi \times 60 \times v(0.2) \times \text{gear ratio} / (2\pi r) = 1.3557 \text{ V}$$

$$I(0.2) = I_{sc} - I_s (e^{(E(0.2)+I(0.1)R)/(mNUr)} - 1) \approx 0.87 \text{ A}$$

(0.2, 0.3)

$$a(0.2) = g(\sin(\alpha) - \cos(\alpha)) C_{\pi} + I(0.2)E(0.2) / (M v(0.2)) - 3C_w A \rho v^2(0.2) / 2M \approx 2.368 \text{ m/s}^2$$

$$v(0.3) = v(0.2) + a(0.2) \times T = 0.7137 \text{ m/s}$$

$$s(0.3) = s(0.2) + v(0.2) \times T + a(0.2) \times T^2 / 2 = 0.1073 \text{ m}$$

$$E(0.3) = C_E \cdot \Phi \times 60 \times v(0.3) \times \text{gear ratio} / (2\pi r) = 2.0289 \text{ V}$$

$$I(0.3) = I_{sc} - I_s (e^{(E(0.3)+I(0.2)R)/(mNUr)} - 1) \approx 0.87 \text{ A}$$

(0.3, 0.4)

$$a(0.3) = g(\sin(\alpha) - \cos(\alpha)) C_{\pi} + I(0.3)E(0.3) / (M v(0.3)) - 3C_w A \rho v^2(0.3) / 2M \approx 2.3639 \text{ m/s}^2$$

$$v(0.4) = v(0.3) + a(0.3) \times T = 0.9501 \text{ m/s}$$

$$s(0.4) = s(0.3) + v(0.3) \times T + a(0.3) \times T^2 / 2 = 0.1905 \text{ m}$$

$$E(0.4) = C_E \cdot \Phi \times 60 \times v(0.4) \times \text{gear ratio} / (2\pi r) = 2.7010 \text{ V}$$

$$I(0.4) = I_{sc} - I_s (e^{(E(0.4)+I(0.3)R)/(mNUr)} - 1) \approx 0.87 \text{ A}$$

(0.4, 0.5)

$$a(0.4) = g(\sin(\alpha) - \cos(\alpha)) C_{\pi} + I(0.4)E(0.4) / (M v(0.4)) - 3C_w A \rho v^2(0.4) / 2M \approx 2.3584 \text{ m/s}^2$$

$$v(0.5) = v(0.4) + a(0.4) \times T = 1.1859 \text{ m/s}$$

$$s(0.5) = s(0.4) + v(0.4) \times T + a(0.4) \times T^2/2 = 0.2973 \text{ m}$$

$$E(0.5) = C_E \cdot \Phi \times 60 \times v(0.5) \times \text{gear ratio} / (2\pi r) = 3.3713 \text{ V}$$

$$I(0.5) = I_{sc} - I_s (e^{(E(0.5)+I(0.4)R)/(mNUr)} - 1) \approx 0.87 \text{ A}$$

(0.5, 0.6)

$$a(0.5) = g(\sin(\alpha) - \cos(\alpha) C_{\pi}) + I(0.5)E(0.5)/(M v(0.5)) - 3C_w A \rho v^2(0.5)/2M \approx 2.3511 \text{ m/s}^2$$

$$v(0.6) = v(0.5) + a(0.5) \times T = 1.4210 \text{ m/s}$$

$$s(0.6) = s(0.5) + v(0.5) \times T + a(0.5) \times T^2/2 = 0.4276 \text{ m}$$

$$E(0.6) = C_E \cdot \Phi \times 60 \times v(0.6) \times \text{gear ratio} / (2\pi r) = 4.0397 \text{ V}$$

$$I(0.6) = I_{sc} - I_s (e^{(E(0.6)+I(0.5)R)/(mNUr)} - 1) \approx 0.87 \text{ A}$$

(0.6, 0.7)

$$a(0.6) = g(\sin(\alpha) - \cos(\alpha) C_{\pi}) + I(0.6)E(0.6)/(M v(0.6)) - 3C_w A \rho v^2(0.6)/2M \approx 2.3424 \text{ m/s}^2$$

$$v(0.7) = v(0.6) + a(0.6) \times T = 1.6552 \text{ m/s}$$

$$s(0.7) = s(0.6) + v(0.6) \times T + a(0.6) \times T^2/2 = 0.5814 \text{ m}$$

$$E(0.7) = C_E \cdot \Phi \times 60 \times v(0.7) \times \text{gear ratio} / (2\pi r) = 4.7054 \text{ V}$$

$$I(0.7) = I_{sc} - I_s (e^{(E(0.7)+I(0.6)R)/(mNUr)} - 1) \approx 0.87 \text{ A}$$

(0.7, 0.8)

$$a(0.7) = g(\sin(\alpha) - \cos(\alpha) C_{\pi}) + I(0.7)E(0.7)/(M v(0.7)) - 3C_w A \rho v^2(0.7)/2M \approx 2.3321 \text{ m/s}^2$$

$$v(0.8) = v(0.7) + a(0.7) \times T = 1.8884 \text{ m/s}$$

$$s(0.8) = s(0.7) + v(0.7) \times T + a(0.7) \times T^2/2 = 0.7586 \text{ m}$$

$$E(0.8) = C_E \cdot \Phi \times 60 \times v(0.8) \times \text{gear ratio} / (2\pi r) = 5.3683 \text{ V}$$

$$I(0.8) = I_{sc} - I_s (e^{(E(0.8)+I(0.7)R)/(mNUr)} - 1) \approx 0.87 \text{ A}$$

(0.8, 0.9)

$$a(0.8) = g(\sin(\alpha) - \cos(\alpha) C_{\pi}) + I(0.8)E(0.8)/(M v(0.8)) - 3C_w A \rho v^2(0.8)/2M \approx 2.3202 \text{ m/s}^2$$

$$v(0.9) = v(0.8) + a(0.8) \times T = 2.1204 \text{ m/s}$$

$$s(0.9) = s(0.8) + v(0.8) \times T + a(0.8) \times T^2/2 = 0.9744 \text{ m}$$

$$E(0.9) = C_E \cdot \Phi \times 60 \times v(0.9) \times \text{gear ratio} / (2\pi r) = 6.0279 \text{ V}$$

$$I(0.9) = I_{sc} - I_s (e^{(E(0.9)+I(0.8)R)/(mNUr)} - 1) \approx 0.87 \text{ A}$$

(0.9, 1.0)

$$a(0.9) = g(\sin(\alpha) - \cos(\alpha) C_{\pi}) + I(0.9)E(0.9)/(M v(0.9)) - 3C_w A \rho v^2(0.9)/2M \approx 2.3069 \text{ m/s}^2$$

$$v(1.0) = v(0.9) + a(0.9) \times T = 2.3511 \text{ m/s}$$

$$s(1.0) = s(0.9) + v(0.9) \times T + a(0.9) \times T^2/2 = 1.1962 \text{ m}$$

Situation 2: Time interval (T) 0.2s

Initial condition (at t = 0s)

$$s(0) = 0; v(0) = 0; I(0) = 0.88 \text{ A}$$

[During each time interval (T, T+0.2), the acceleration remains constant.]

(0,0.2)

$$a(0) = g(\sin(\alpha) - \cos(\alpha) \times C_{rr}) + I(0) \times E(0)/(M \times v(0)) - 3C_w A \rho \times v^2(0)/2M \approx 2.399 \text{ m/s}^2$$

$$v(0.2) = a(0) \times T = 0.4798 \text{ m/s}$$

$$s(0.2) = v(0) \times T + a(0) \times T^2/2 = 0.0480 \text{ m}$$

$$E(0.2) = C_E \cdot \Phi \times 60 \times v(0.2) \times \text{gear ratio} / (2\pi r) = 1.3640 \text{ V}$$

$$I(0.2) = I_{sc} - I_s (e^{(E(0.2)+I(0)R)/(mNUR)} - 1) \approx 0.87 \text{ A}$$

(0.2,0.4)

$$a(0.2) = g(\sin(\alpha) - \cos(\alpha) \times C_{rr}) + I(0.2) \times E(0.2)/(M \times v(0.2)) - 3C_w A \rho \times v^2(0.2)/2M \approx 2.3680 \text{ m/s}^2$$

$$v(0.4) = v(0.2) + a(0.2) \times T = 0.9534 \text{ m/s}$$

$$s(0.4) = s(0.2) + v(0.2) \times T + a(0.2) \times T^2/2 = 0.1913 \text{ m}$$

$$E(0.4) = C_E \cdot \Phi \times 60 \times v(0.4) \times \text{gear ratio} / (2\pi r) = 2.7103 \text{ V}$$

$$I(0.4) = I_{sc} - I_s (e^{(E(0.4)+I(0.2)R)/(mNUR)} - 1) \approx 0.87 \text{ A}$$

(0.4,0.6)

$$a(0.4) = g(\sin(\alpha) - \cos(\alpha) \times C_{rr}) + I(0.4) \times E(0.4)/(M \times v(0.4)) - 3C_w A \rho \times v^2(0.4)/2M \approx 2.3582 \text{ m/s}^2$$

$$v(0.6) = v(0.4) + a(0.4) \times T = 1.4250 \text{ m/s}$$

$$s(0.6) = s(0.4) + v(0.4) \times T + a(0.4) \times T^2/2 = 0.4291 \text{ m}$$

$$E(0.6) = C_E \cdot \Phi \times 60 \times v(0.6) \times \text{gear ratio} / (2\pi r) = 4.0510 \text{ V}$$

$$I(0.6) = I_{sc} - I_s (e^{(E(0.6)+I(0.4)R)/(mNUR)} - 1) \approx 0.87 \text{ A}$$

(0.6,0.8)

$$a(0.6) = g(\sin(\alpha) - \cos(\alpha) \times C_{rr}) + I(0.6) \times E(0.6)/(M \times v(0.6)) - 3C_w A \rho \times v^2(0.6)/2M \approx 2.3416 \text{ m/s}^2$$

$$v(0.8) = v(0.6) + a(0.6) \times T = 1.8933 \text{ m/s}$$

$$s(0.8) = s(0.6) + v(0.6) \times T + a(0.6) \times T^2/2 = 0.7609 \text{ m}$$

$$E(0.8) = C_E \cdot \Phi \times 60 \times v(0.8) \times \text{gear ratio} / (2\pi r) = 5.3823 \text{ V}$$

$$I(0.8) = I_{sc} - I_s (e^{(E(0.8)+I(0.6)R)/(mNUR)} - 1) \approx 0.87 \text{ A}$$

(0.8,1.0)

$$a(0.8) = g(\sin(\alpha) - \cos(\alpha) \times C_{rr}) + I(0.8) \times E(0.8)/(M \times v(0.8)) - 3C_w A \rho \times v^2(0.8)/2M \approx 2.3200 \text{ m/s}^2$$

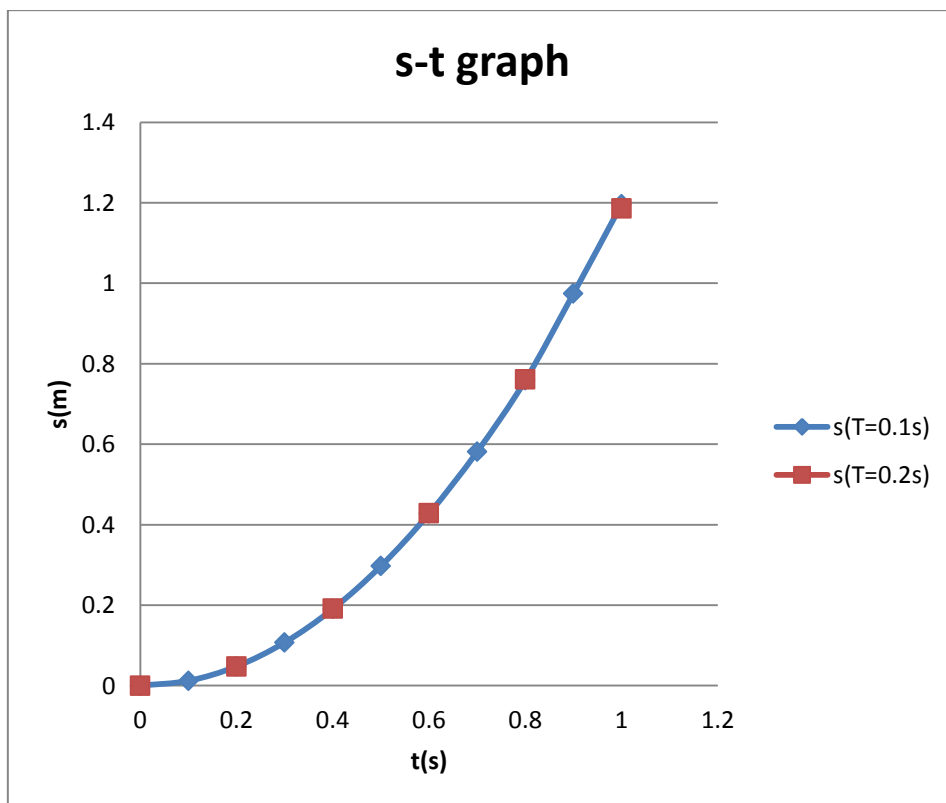
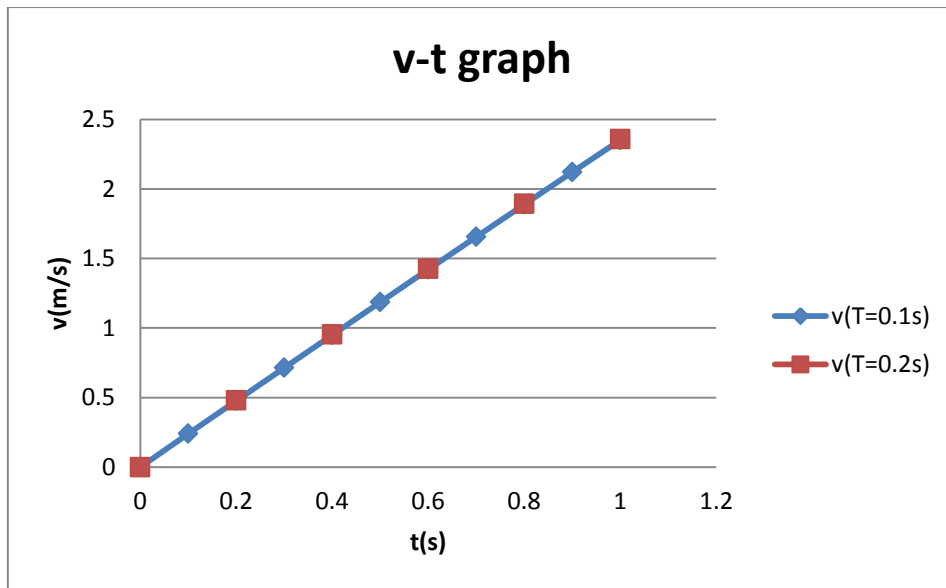
$$v(1.0) = v(0.8) + a(0.8) \times T = 2.3573 \text{ m/s}$$

$$s(1.0) = s(0.8) + v(0.8) \times T + a(0.8) \times T^2/2 = 1.1860 \text{ m}$$

Conclusion:

t(s)	v(T=0.1s)	v(T=0.2s)	s(T=0.1s)	s(T=0.2s)
0	0	0	0	0
0.1	0.2399		0.012	
0.2	0.4769	0.4798	0.0478	0.048
0.3	0.7137		0.1073	

0.4	0.9501	0.9534	0.1905	0.1913
0.5	1.1859		0.2973	
0.6	1.421	1.425	0.4276	0.4291
0.7	1.6552		0.5814	
0.8	1.8884	1.8933	0.7586	0.7609
0.9	2.1204		0.9744	
1	2.3511	2.3573	1.1962	1.186



Based on the graph above, we can find that two curves are nearly overlapped, there is no obvious difference between these two curves, because we can find that the accelerations are nearly constant (which can be shown on the data chart above), thus the the vehicle is taking constant acceleration motion, the displacement-time curve or velocity-time curve does not change very much.

3.4 Sankey diagrams

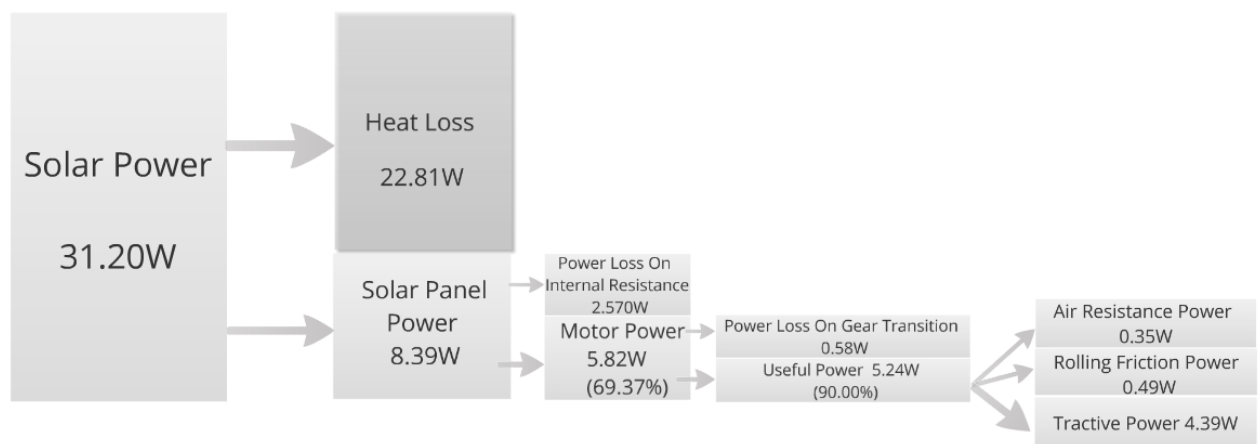
Sankey diagram is used to indicate the power flow instantaneously. With the help of Sankey diagram, we can get an intuitive view of how the power flows from the solar power (initial) to the kinetic power (terminal).

Here we choose two conditions to discuss. One is $s=4m$, which means that the car is at the bottom of the first slope. The other is when the velocity is the maximum value. All the data are simulated based on the Matlab-Simulink except the solar power ($800 W/m^2$) and the efficiency of the gear transition (90%).

Sankey diagrams are followed by detailed interpretations of power calculation.

Condition 1 (at $s = 4m$)

According to the graph drawn by Matlab, at $s = 4m$, $t = 1.818s$, $v = 4.19m/s$
The radius of wheels $r = 0.027m$



$$w_{\text{wheel}} = v/r = 155.18 \text{ rad/s}$$

$$w_{\text{motor}} = \text{gear ratio} * w_{\text{wheel}} = 775.92 \text{ rad/s}$$

$$E(\text{back emf}) = K_e * w_{\text{motor}} = \frac{1}{1120 * \frac{2\pi}{60}} * 1640.3 = 6.62 \text{ V}$$

$$U_a = R_a I_a + E \quad (R_a = 3.32 \Omega)$$

$$I_a = I_{sc} - I_s \left(e^{\frac{U_a}{mN\Phi r}} - 1 \right)$$

Applying Maple we can calculate that $I_a = 0.88 \text{ A}$

$$U_a = R_a I_a + E = 9.54 \text{ V}$$

The solar power is about $800 W/m^2$ at summer in Belgium.

The area of the solar panel is $22.7 * 17.2 = 390 \text{ cm}^2$

$$P_{\text{solar}} = 800 * 390 * 10^{-4} \text{ W} = 31.2 \text{ W}$$

$$P_{\text{solar panel}} = U_a * I_a = 8.39 \text{ W}$$

The heat loss on the solar panel is $\Delta P_1 = P_{\text{solar}} - P_{\text{solar panel}} = 22.81 \text{ W}$

The efficiency of the solar panel $\eta_1 = P_{\text{solar panel}} / P_{\text{solar}} = 26.90\%$

The power loss on the internal resistance of motor is $\Delta P_2 = I_a^2 * R_a = 2.57 \text{ W}$

The real power of the motor $P_{\text{motor}} = P_{\text{solar panel}} - \Delta P_2 = 5.82 \text{ W}$

The efficiency of the motor $\eta_2 = P_{\text{motor}} / P_{\text{solar panel}} = 69.37\%$

The power loss on the gear transition between the motor and the wheel $\Delta P_3 = 10\% * P_{\text{motor}} = 0.58 \text{ W}$.

The wheels get the real power $P_{\text{wheel}} = P_{\text{motor}} * 90\% = 5.24 \text{ W}$

Consider that there are air resistance and rolling friction,

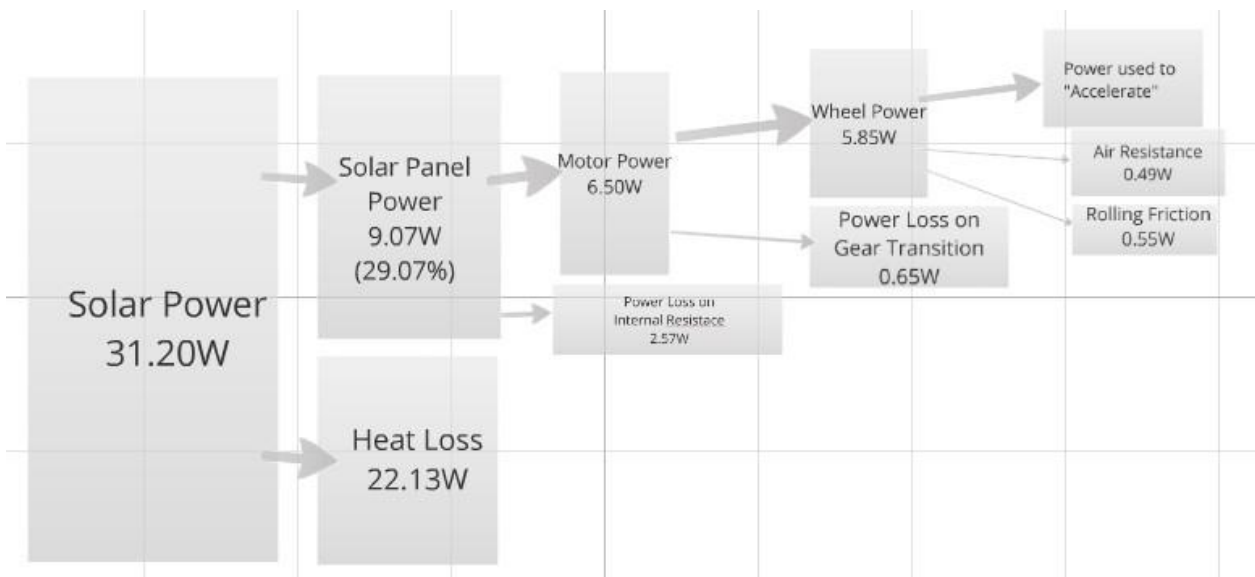
$$\Delta P_{\text{air resistance}} = F_w * v = \frac{1}{2} * C_w * A * \rho * v^3 = 0.5 * 0.5 * 0.014797 * 1.293 * 4.19^3 = 0.35 \text{ W}$$

$$\Delta P_{\text{rolling friction}} = F_r * v = C_{rr} * mg * v = 0.012 * 1 * 9.81 * 4.19 = 0.49 \text{ W}$$

Finally, the power which is used to accelerate the car

$$P = 5.24 - 0.35 - 0.49 = 4.39 \text{ W}$$

Condition 2 (at the maximum speed)



From the graph we can get the maximum speed $v_{\text{max}} = 4.68 \text{ m/s}$.

At this moment, $\omega_{\text{motor}} = \text{gear ratio} * v_{\text{max}} / r = 866.67 \text{ rad/s}$

$$E(\text{back emf}) = K_e * \omega_{\text{motor}} = 7.39 \text{ V}$$

Use the same method as before we can get $I_a = 0.88 \text{ A}$, just the same as the previous one. That is not strange since I_s is too small to effect the value of I_a .

$$U_a = I_a R_a + E = 10.31 \text{ V}$$

$P_{\text{solar}} = 31.2 \text{ W}$, does not change.

$$P_{\text{solar panel}} = U_a * I_a = 10.31 * 0.88 = 9.07 \text{ W}$$

$$P_{\text{motor}} = P_{\text{solar panel}} - I_a^2 * R_a = 6.50W$$

$$P_{\text{wheel}} = P_{\text{motor}} * 90\% = 5.85W$$

$$\Delta P_{\text{air resistance}} = F_w * v_{\text{max}} = \frac{1}{2} * C_w * A * \rho * v_{\text{max}}^3$$

$$= 0.5 * 0.5 * 0.014797 * 1.293 * 4.68^3 = 0.49W$$

$$\Delta P_{\text{rolling friction}} = F_r * v_{\text{max}} = C_{rr} * mg * v_{\text{max}} = 0.012 * 1 * 9.81 * 4.68 = 0.55W$$

In theory when the car is at the maximum speed, which means it does not accelerate anymore, that is, it does not have any power to increase its speed. Thus the input power P_{wheel} should be equal to the total loss of power $\Delta P_{\text{air resistance}} + \Delta P_{\text{rolling friction}}$.

But actually $P_{\text{wheel}} > \Delta P_{\text{air resistance}} + \Delta P_{\text{rolling friction}}$, which means there is still some power to accelerate the car. But why the car does not accelerate in the speed-time graph?

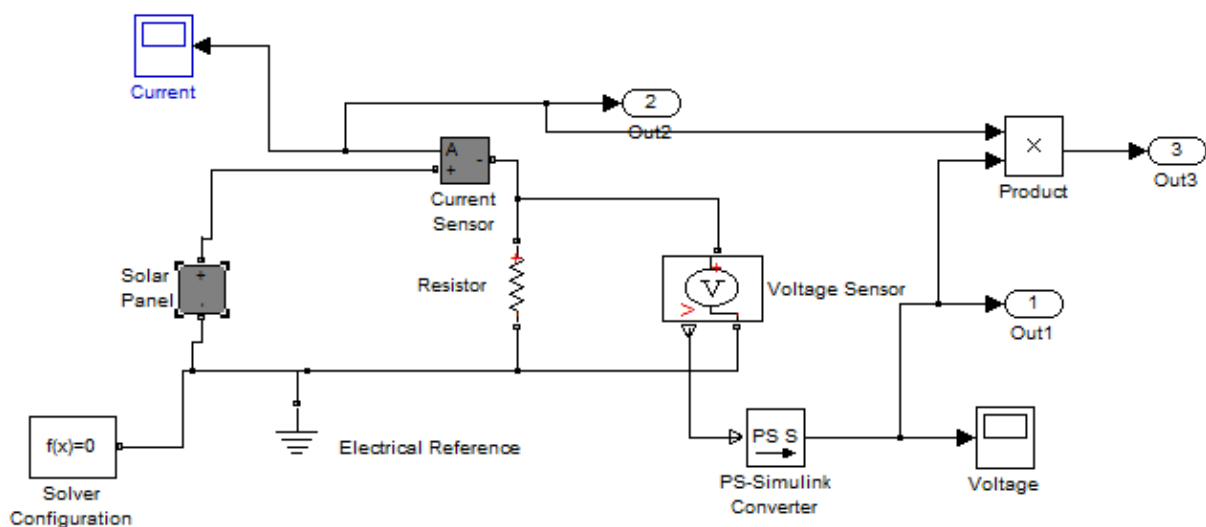
According to the graph, when the speed is $v_{\text{max}} = 4.68\text{m/s}$. At this moment, the position of the car $s \approx 10\text{m}$, which means that the car is going uphill at this moment. In this instant, the acceleration changes abruptly. After that, due to the gravity when the car goes uphill, it will decelerate. So at this moment, the velocity is the maximum value, but there is still “accelerating” power meanwhile.

What’s more, some values are estimated during the calculation, such as the solar power (800W/m^2) and the efficiency of the gear transition system (90%). They are not precise values. Thus this calculation can just give us a sketchy view of how the power flows. It cannot give accurate data.

4. Case Matlab-Simulink

4.1 Simulation of the solar panel behavior

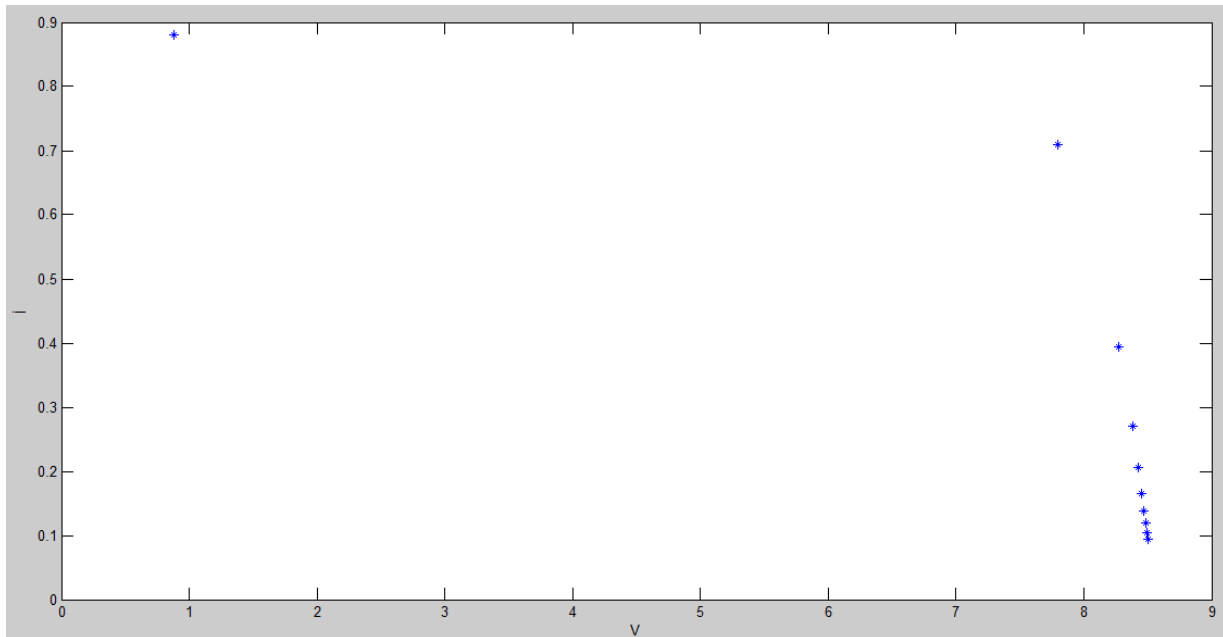
Here we will simulate the U-I and power diagram of our solar panel connected to some resistors ranging from 10 to 100 Ohms. First we built the circuit in Simulink:



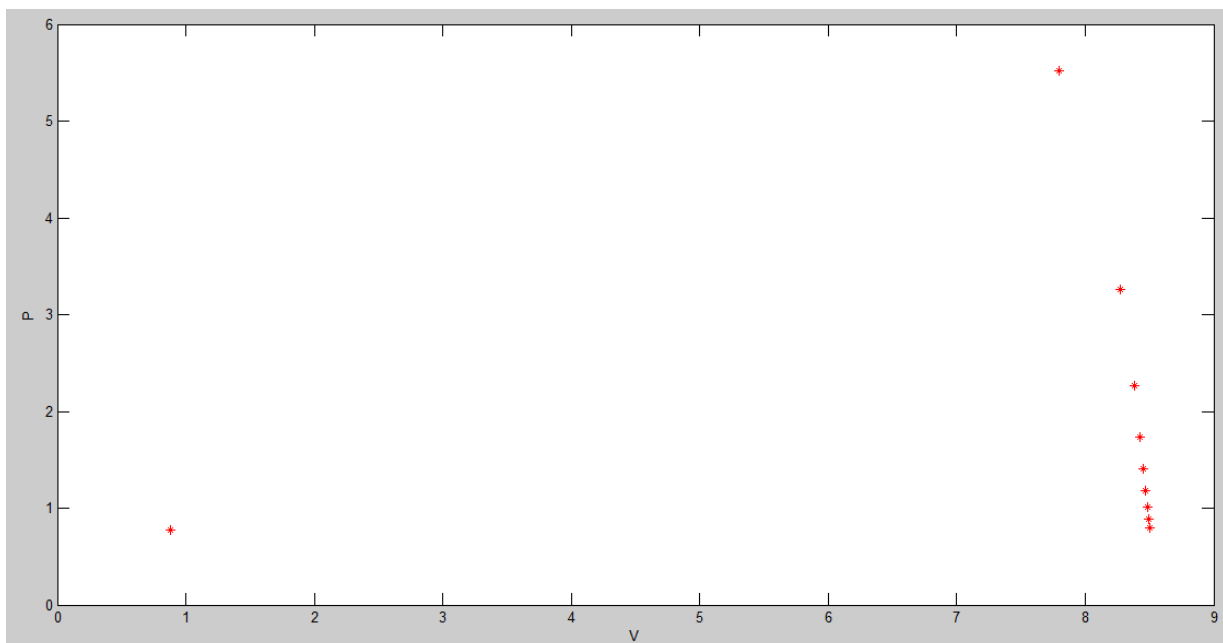
We set the variables in the Matlab program:

```
%%% Solar Power
Ir = 800; % solar irradiance [W/m^2]
Is= 1e-8;% saturation current [ A/m^2]
Isc= 0.22;%short circuit current [A]
Voc = 0.52;%Open circuit voltage [V]
Ir0=500;%irradiqance used for measurements [W/m^2]
N = 1.2;%diode quality factor
```

And we plotted the UI an power graphs:



Graph 3: Solar panel U-I graph simulation in Matlab-Simulink



Graph 4: Solar panel U-P graph simulation in Matlab-Simulink

The power will be maximal when we connect a resistor with a value around 20 Ohms.

