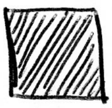
Probability - combining areas & Ratios

Definition: a(1): the ratio of the number of outcomes that produce a given event to the total number of possible outcomes.
(2): the chance that a given event will occur. - merriam-webster

Main question -
Could someone build a machine that would take random numbers and from them produce a closer & closer approximation to π?
Let's see how we can build our way to this conclusion?

What's the probability that my jelly beans will land in the shaded region?



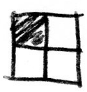
my jelly beans

100% chance since the entire region is shaded.

What's the probability that my jelly beans will land in the shaded region?

$\frac{1}{2}$ chance or 50% since ~~ont~~ excctly half of the area is shaded the probability is 1/2.

What's the probability that my jelly beans will land in the shaded region?

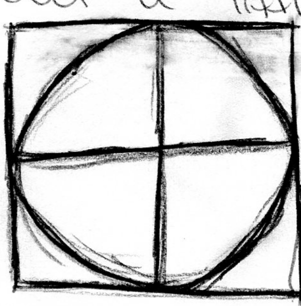


1/4 or 25% chance. since only a quarter is shaded the probability is 1/4.

What's the probability that my jelly beans will land in the shaded region?

The area of the square ~~will~~ is $\underline{1}$.

Let's think of pie in order to figure this problem out a little easier.



The entire area of the shape is 4.
The area of the circle is π.
The radius of the circle is $\underline{1}$.
So the probability that one of my jelly beans lands in the large

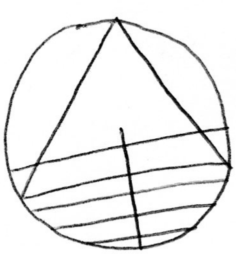
shaded circle is $\frac{\pi}{4}$. The probability of my jelly bean landing in the little square and the little quarter-circle would be only $\frac{1}{4}$ of this each. The probability of the jelly bean landing in the shaded region in the little square is

$$\frac{\pi/4}{4/4} = \frac{\pi}{4}$$

* what does random mean?
  - Unpredictability, random distribution

Betrand's Paradox

Given a circle what is the probability that a chord chosen at random is longer than the side on an inscibed equilateral triangle?

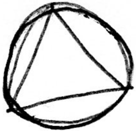


Choose a radius of the circle, choose a point on the radius & then construct the chord through this point and the perpendicular to the radius. To calculate the probability in think of the triangle rotated so a side is perpendicular to the radius. The chord is longer than a side of the triangle if the chosen point is nearer the center of the circle than the point where the side of the triangle intersects the radius. The side of the triangle bisects the radius therefore the probability a random chord is longer than a side of the inscribed triangle is onl half.

- wikipedia

ner way of viewing this.....

There are 60° in a triangle & there



are 180° in a straight line.
A probability that the random chord can be longer than the edge of the triang is $\frac{60}{180} = \frac{1}{3}$

The previous example gives a hint as to ~~wh~~ now we can take numbers & turn them into an approximation for π. Here is a helpful algorithm.

1. Take a random set of numbers between 0 and 1.
   Ex: 0.2, 0.3, 0.4
2. Take pairs of these numbers (a,b). Let n be the total number of pairs the we have.
   Ex: (0.2, 0.3)
   (0.2, 0.4)
   (0.3, 0.4) total pairs $n=3$
3. To see if a pair (a,b) lands inside the circle use the Pythagorean Theorem, that is if
   $$a^2+b^2 \le 1$$ then the point is inside the circle, otherwise the point is outside the circle.

   $0.2^2 + 0.3^2$
   $0.21 \le 1$
   (Inside circle)

   $0.2^2 + 0.4$
   $.2 \le 1$
   (Inside circle)

   $0.3^2 + .4^2$

   $.25$

   Inside the circle

4. Count how many pairs land inside the circle, divide by the total number of pairs

$$\frac{\text{3 pairs land inside}}{\text{3 pairs is the total of pairs}}$$

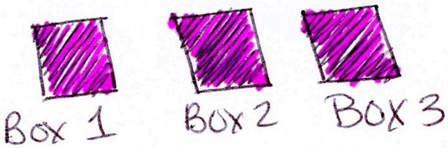
1 answer

5. This ratio approximates $\frac{\pi}{4}$. To get an approximation for π, mult your answer by 4.

$1 \times 4 = 4$

– Let's play a game to better understand probability ....

We have 3 boxes in front of us!



Behind 1 box. is a prize & behind the remaing 2 you will find nothing.

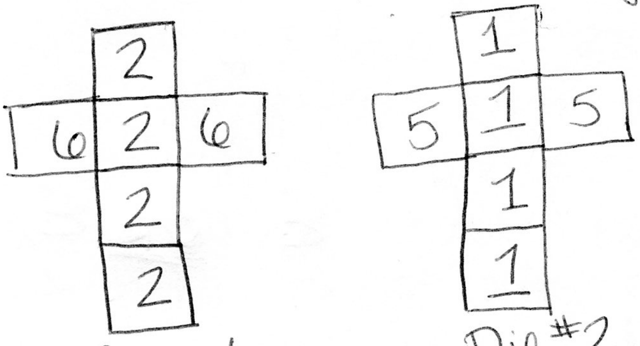
Rules to the game:

First you will choose whatever box you like. Say you pick box # 3. I will then uncover another box which you did not choose. Let's say in this case I uncover box # 1 in this situation which reveals an empty box. I then give you the option to either swith your choosen box or stay. If you decide to switch your box choice then I will open the other remaining door to reveal either a prize or no prize at all.

Try and Figure out on your own if it would be better to switch your choice or stay?

Dice Game

what is the probability of die 1 beating die 2



Die#1 Die#2.

Let's begin by thinking. what is the probability that the number 2 on die 1 will beat the number 1 on die 2.

Die 1 Die 2 $\frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6}$ or $\frac{1}{3}$

$\frac{4}{6}$ or $\frac{2}{3}$ $\frac{3}{6}$ or $\frac{1}{2}$

1/3 probability that the number 2 of die 1 will beat the number 1 in die 2.

In order to get all the possible outcomes we must finish the problem.

what is the probability that the number 6 on die will beat any of the numbers on die 2.

Die 1 Die 2.

$\frac{2}{6}$ or $\frac{1}{3}$

Since the number 6 will beat every number on die 2 every time. Die 2 will always be beat by the number 2, 100% of the time

So with die 1 beating die 2 $\frac{1}{3}$ of the with ~~die~~ number 2

and die 1 also beats die 2 $\frac{1}{3}$ of the time with number 6 $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

Die 1 will beat die 2; $\frac{2}{3}$ of the time.

Lemma 19: If two events $\mathcal{E}_1$ and $\mathcal{E}_2$ are disjoint, meaning that they can not both occur together, then the probability that both $\mathcal{E}_1$ **OR** $\mathcal{E}_2$ will happen is the sun of the probability that $\mathcal{E}_1$ happens with the probabilithy that $\mathcal{E}_2$ happens. Symbolic if $P(\mathcal{E}_i)$ is the probability that event $\mathcal{E}_i$ happens, then

$$P(\mathcal{E}_1 \vee \mathcal{E}_2) = P(\mathcal{E}_1) + P(\mathcal{E}_2).$$

Lemma 20: If two events $\mathcal{E}_1$ and $\mathcal{E}_2$ are independent, meaning that the occurence of one has no effec on the occurence of the other then the probability that $\mathcal{E}_1$ a $\mathcal{E}_2$ will happen is the product of the probability that $\mathcal{E}_1$ happens with the probabilit that $\mathcal{E}_2$ happens. Symbolicall if $P(\mathcal{E}_i)$ is the probability th event $\mathcal{E}_i$ happens, then

$$P(\mathcal{E}_1 \wedge \mathcal{E}_2) = P(\mathcal{E}_1) \cdot P(\mathcal{E}_2).$$