R3.1) Consider the same system as in the example p. 7-3, i.e., with the same L2-ODE-CC(4) p. 5-5 and initial conditions (2) p. 3-4, but with an excitation of $r(x) = 7e^{5x} - 2x^2$. Find the solution, and plot it with the solution to the example.

$$y'' - 10y' + 25y = r(x)$$

I.C.

$y(0) = 4$

$y'(0) = -5$

Roots (2) p. 5-5

$\lambda = 5$ (double)

Excitation

$r(x) = 7e^{5x} - 2x^2$

$$y_h = Ae^{5x} + Bxe^{5x}$$

$$y_p = Cx^2e^{5x} + Dx^2 + Ex + F$$

$$y_p'' - 10y_p' + 25y = 7e^{5x} - 2x^2$$

$$y_p' = C(5x^2e^{5x} + 2xe^{5x}) + 2Dx + E$$

$$y_p'' = C(10xe^{5x} + 25x^2e^{5x} + 10xe^{5x} + 2e^{5x}) + 2D$$

$$y_p'' = Ce^{5x}(25x^2 + 20x + 2) + 2D$$

$$Ce^{5x}(25x^2 + 20x + 2) + 2D - 10Ce^{5x}(5x^2 + 2x) - 20Dx - 10E + 25Cx^2e^{5x} + 25Dx^2 + 25Ex + 25F = 7e^{5x} - 2x^2$$

$$Ce^{5x}(\cancel{25x^2} + \cancel{20x} + 2 - \cancel{50x^2} - \cancel{20x} + \cancel{25x^2}) + D(25x^2 - 20x + 2) - E(25x - 10) + 25F = 7e^{5x} - 2x^2$$

$$\underline{2Ce^{5x}} + \underline{25(Dx^2 + Ex + F)} - 10(2Dx + E) + 2D = \underline{7e^{5x}} \ \underline{-2x^2}$$

$2C = 7$

$C = \frac{7}{2}$

$25D = -2$

$D = \frac{-2}{25}$

$25E - 20D = 0$

$E = \frac{4}{5}\left(\frac{-2}{25}\right) = \frac{-8}{125}$

$25F - 10E + 2D = 0$

$F = \frac{1}{25}\left(10\left(\frac{-8}{125}\right) - 2\left(\frac{-2}{25}\right)\right)$

$F = \frac{1}{25}\left(-\frac{80}{125} + \frac{20}{125}\right)$

$F = \frac{1}{25}\left(\frac{-60}{125}\right) = \frac{-12}{625}$

$$\boxed{y_p = \frac{7}{2}x^2e^{5x} - \frac{2}{25}x^2 - \frac{8}{125}x - \frac{12}{625}}$$

$$y = Ae^{5x} + Bxe^{5x} + \frac{7}{2}x^2e^{5x} - \frac{2}{25}x^2 - \frac{8}{125}x - \frac{12}{625}$$

$y(0)=4$ $\qquad 4 = A + 0 + 0 + 0 + 0 - \frac{12}{625}$

$$A = 3.9808 = \frac{2488}{625}$$

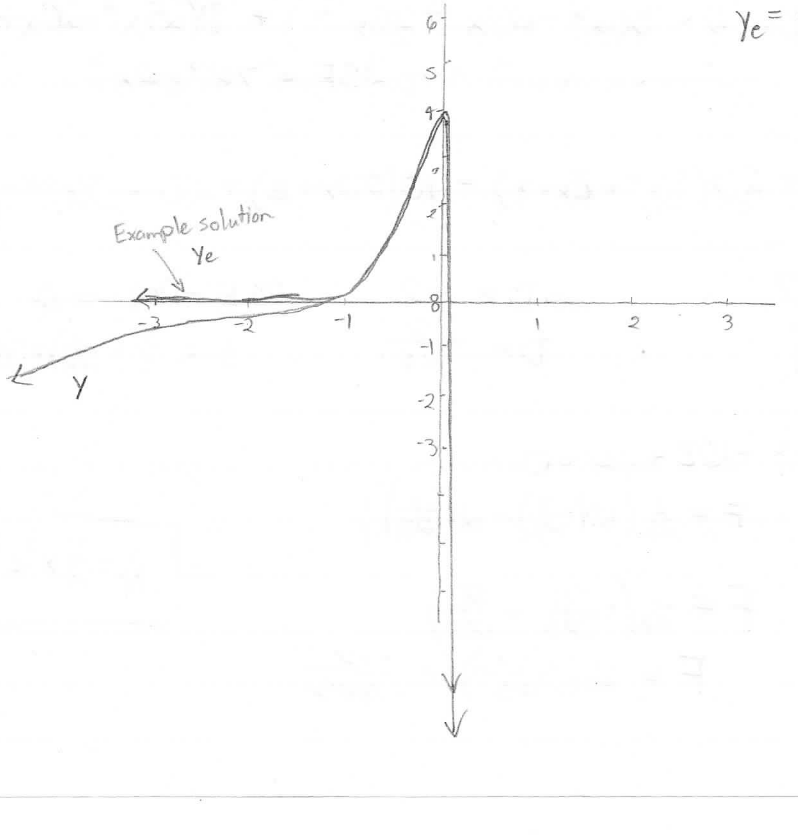
$$y' = 5Ae^{5x} + B(e^{5x} + 5xe^{5x}) + \frac{7}{2}(2xe^{5x} + 5x^2e^{5x}) - \frac{4}{25}x - \frac{8}{125}$$

$$-5 = 5A + B - \frac{8}{125}$$

$$B = -24.84 = -\frac{621}{25}$$

$$y = \frac{2488}{625}e^{5x} - \frac{621}{25}xe^{5x} + \frac{7}{2}x^2e^{5x} - \frac{2}{25}x^2 - \frac{8}{125}x - \frac{12}{625}$$

$$\boxed{y = \frac{1}{625}\left(2488e^{5x} - 15525xe^{5x} + 2187.5x^2e^{5x} - 50x^2 - 40x - 12\right)}$$



$$y_e = 4e^{5x} - 25xe^{5x} + \frac{7}{2}x^2e^{5x}$$

R3.7) Expand the series on both sides of (1)-(2) p.7-12b to verify these equalities.

(1) p.7-12b

$$\sum_{j=2}^{5} c_j j(j-1)x^{j-2} \overset{k:=j+2}{=} \sum_{k=0}^{3} c_{k+2}(k+2)(k+1)x^k = \sum_{j=0}^{3} c_{j+2}(j+2)(j+1)x^j$$

$$c_2(2)(1)x^0 + c_3(3)(2)x^1 + c_4(4)(3)x^2 + c_5(5)(4)x^3 = c_{0+2}(0+2)(0+1)x^0 + c_{1+2}(1+2)(1+1)x^1 + c_{2+2}(2+2)(2+1)x^2 + c_{3+2}(3+2)(3+1)x^3$$

$$2c_2 + 6c_3x + 12c_4x^2 + 20c_5x^3 = 2c_2 + 6c_3x + 12c_4x^2 + 20c_5x^3 \checkmark$$

(2) p.7-12b

$$\sum_{j=1}^{5} c_j j x^{j-1} \overset{k:=j-1}{=} \sum_{k=0}^{4} c_{k+1}(k+1)x^k = \sum_{j=0}^{4} c_{j+1}(j+1)x^j$$

$$c_1(1)x^{1-1} + c_2(2)x^{2-1} + c_3(3)x^{3-1} + c_4(4)x^{4-1} + c_5(5)x^{5-1} = c_{0+1}(0+1)x^0 + c_{1+1}(1+1)x^1 + c_{2+1}(2+1)x^2 + c_{3+1}(3+1)x^3 + c_{4+1}(4+1)x^4$$

$$c_1 + 2c_2x + 3c_3x^2 + 4c_4x^3 + 5c_5x^4 = c_1 + 2c_2x + 3c_3x^2 + 4c_4x^3 + 5c_5x^4 \checkmark$$