EigenSpaces (5A)

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EigenValues and EigenVectors

n x n

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

eigenvalue

eigenvector

$$A\,x = \lambda\,x$$

$$\begin{pmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$(A - \lambda I)\,x = 0$$

characteristic Equation

$$det(A - \lambda I) = 0$$

A nxn Matrix A (1)

1. **A** is invertible
2. **Ax** = **0** has only the trivial solution
3. The RREF(**A**) = $\mathbf{I}_n$
4. **A** can be written as a product of elementary matrix
5. **Ax** = **b** is consistent for every n x 1 **b**
6. **Ax** = **b** has exactly one solution for every n x 1 **b**
7. det(**A**) ≠ 0
8. The column vectors are linearly independent
9. The row vectors are linearly independent
10. The column vectors span R^n
11. The row vectors span R^n
12. The column vectors form a basis for R^n
13. The row vectors form a basis for R^n
14. rank(**A**) = n
15. nullity(**A**) = 0
16. The orthogonal complement of the null space is R^n
17. The orthogonal complement of the row space is {**0**}

A nxn Matrix A (2)

18. The range of T_A is R^n
19. T_A is one-to-one
20. $\lambda=0$ is not the eigenvalue of **A**

References

[1] http://en.wikipedia.org/
[2] Anton, et al., Elementary Linear Algebra, 10th ed, Wiley, 2011
[3] Anton, et al., Contemporary Linear Algebra,