GROUP T
LEUVEN ENGINEERING COLLEGE

Team Apollo

Case SSV: part 1

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Gear ratio calculation

We are going to use these five formulas:

$$U \cdot I \cdot \eta = F_{wheel} \cdot V_{AB}$$

$$T_{wheel} = C_T \cdot \eta \cdot I \cdot 10^{-3} \cdot i$$

$$T_{wheel} = F_{wheel} \cdot R_{wheel}$$

$$S = \frac{1}{2} \cdot V_{AB} \cdot t_{AB}$$

$$(F_{wheel} - F_{rolling}) \cdot t_{AB} = m \cdot V_{AB}$$

The properties of the car:

The mass of the car: m = 1kg

The radius of the wheels: R_{wheel}=0.04m

The efficiency: η=0.7

The rolling resistance: C_{rr}=0,015

De C_T van de motor: C_T = 8,55 mNm/A

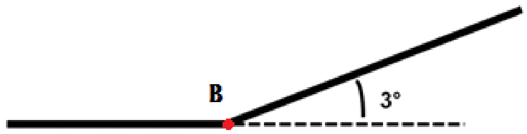
For A/B: U=7.33V , I_{graph}= 0.3A , $I_{scgraph}$=0.34A

The real short circuit current of the solar panel:

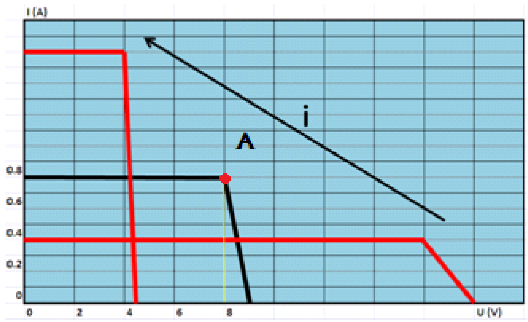
I_{sc}=0.88A → $I = \frac{0.88}{0.34} \cdot 0,3 = 0,78A$

Reaching point A/B

First we are going to calculate the velocity, gear ratio and time when reaching point A/B. Before we start our calculations we define these point.



Sketch of track. B is the point where the slope starts.



Point A is the maximum power point

Calculating the gear ratio and the velocity at point A/B:

Calculating the velocity:

$$t_{AB} \cdot (F_{wheel} - F_{rolling}) = m \cdot V_{AB}$$

$F_{rolling} = C_{rr} \cdot N \qquad = C_{rr} \cdot m \cdot g$

$\rightarrow F_{rolling} = 0{,}147$ N

$$F_{wheel} = \frac{U \cdot I \cdot \eta}{V_{AB}}$$

$$\rightarrow F_{wheel} = \frac{7{,}33V \cdot 0{,}78A \cdot 0{,}7}{V_{AB}} = \frac{4{,}00218w}{V_{AB}}$$

$S = \frac{1}{2} \cdot V_{AB} \cdot t_{AB}$ S = 6m, at point A/B

$$\rightarrow t_{AB} = \frac{2 \cdot 6m}{V_{AB}} = \frac{12m}{V_{AB}}$$

Then we get:

$$t_{AB} \cdot (F_{wheel} - F_{rolling}) = m \cdot V_{AB}$$

$$\rightarrow \frac{12m}{V_{AB}} \cdot \left(\frac{4{,}00218w}{V_{AB}} - 0{,}147\text{N}\right) = 1kg \cdot V_{AB}$$

$$\rightarrow -V_{AB}^{\ 3} + 1{,}764V_{AB} - 48{,}02616 = 0$$

$\rightarrow$ **V_{AB} =3.473 m/s**

Calculating the gear ratio:

$$\begin{cases} T_{AB} = F_{wheel} \cdot R_{wheel} \\ F_{wheel} = \dfrac{U \cdot I \cdot \eta}{V_{AB}} \end{cases}$$

$$\rightarrow T_{AB} = \frac{U \cdot I \cdot \eta}{V_{AB}} \cdot R_{wheel} \quad (*)$$

$$T_{AB} = C_T \cdot I \cdot \eta \cdot 10^{-3} \cdot i$$

$$\rightarrow C_T \cdot I \cdot \eta \cdot 10^{-3} \cdot i = \frac{U \cdot I \cdot \eta}{V_{AB}} \cdot R_{wheel} \quad (*)$$

$$\rightarrow i = \frac{U \cdot R_{wheel}}{V_{AB} \cdot C_T} \cdot 10^3$$

$$\rightarrow \boldsymbol{i = 9,87}$$

Calculating the velocity and time on the slope:

The rolling resistance on the flat part of the track:

$$F_{rolling} = 0{,}147N$$

On the slope will be an extra resistance of the weight. The air resistance will be neglected. So the resistance on the slope will be:

$$F_r = F_{rolling} + m \cdot g \cdot sin3°$$

$\rightarrow F_r = 0{,}66$N

To get equilibrium:

$F_{wheel}= F_r =0.66$ N

$$\begin{cases} T_{wheel} = C_T \cdot \eta \cdot I \cdot 10^{-3} \cdot i \\ T_{wheel} = F_{wheel} \cdot R_{wheel} \end{cases}$$

$\rightarrow F_{wheel} \cdot R_{wheel} = C_T \cdot \eta \cdot I \cdot 10^{-3} \cdot i$

$\rightarrow I = \frac{F_{wheel} \cdot R_{wheel}}{C_T \cdot \eta \cdot 10^{-3} \cdot i}$

$\rightarrow I = \frac{0{,}66N \cdot 0{,}04m}{8{,}55 \cdot 10^{-3} Nm/A \cdot 0{,}7 \cdot 9{,}87}$

$\rightarrow I=0.447A$

$$I_{graph} = \frac{I}{I_{sc}} \cdot \mathrm{I_{scgraph}}$$

$\rightarrow I_{graph} = \frac{0.447A}{0.88A} \cdot 0.34\mathrm{A}$

$\rightarrow I_{graph} = 0.173$ A

$$\mathrm{V} = \frac{\mathrm{U} \cdot \mathrm{I} \cdot \eta}{\mathrm{F_r}}$$

$\rightarrow \mathrm{V} = \frac{7.96\mathrm{V} \cdot 0.446\mathrm{A} \cdot 0.7}{0.66\mathrm{N}}$

$\rightarrow \mathrm{V} = 3.765$ m/s

We can see that: $V \approx V_{AB}$

With the velocity we can calculate the time that our SSV needs to finish the track:

$t_{slope} = \frac{8}{V_{AB}} = 2.303$ s

$t_{tot}= t_{AB}+t_{slope}$

$\rightarrow t_{tot} = \frac{6}{V_{AB}} + \frac{8}{V_{AB}}$

$\rightarrow$ **t_{tot} = 4,03 s**

Calculating the time with another gear ratio:

For i=10:

On the flat part:.

$$T_{wheel} = C_T \cdot \eta \cdot \mathrm{I} \cdot 10^{-3} = 8{,}55 \cdot 10^{-3}\mathrm{Nm/A} \cdot 0{,}7 \cdot 0{,}78\mathrm{A} \cdot 10 = 0{,}0467\mathrm{Nm}$$

$$F_{wheel} = \frac{T_{wheel}}{R_{wheel}} = \frac{0{,}0467Nm}{0{,}04m} = 1{,}17N$$

$$a = \frac{(F_{wheel} - F_{rolling})}{m} = \frac{(1{,}17N - 0{,}147\ \mathrm{N})}{1kg} = 1{,}02m/s^2$$

At point A (somewhere on the flat part of the track):

$$U \cdot I \cdot \eta = F_{wheel} \cdot V_A$$

$$\rightarrow V_A = \frac{U \cdot I \cdot \eta}{F_{wheel}} = \frac{7{,}33V \cdot 0{,}78A \cdot 0{,}7}{1{,}17N} = 3{,}42m/s$$

$$t_A = \frac{V_A}{a} = \frac{3{,}42m/s}{1{,}02m/s^2} = 3{,}35s$$

$$S_A = \frac{a \cdot t^2}{2} = \frac{1{,}02m/s^2 \cdot (3{,}35s)^2}{2} = 5{,}72m$$

$F_{Wheel} = F_{rolling}$ for the rest of the flat path:

$$T_{wheel} = F_{wheel} \cdot R_{wheel} = 1{,}17N \cdot 0{,}04m = 0{,}0468Nm$$

$$I = \frac{T_{wheel}}{C_T \cdot \eta \cdot 10^{-3} \cdot i} = \frac{0{,}0468Nm}{8{,}55Nm/A \cdot 0{,}7 \cdot 10^{-3} \cdot 10} = 0{,}782A$$

$$\mathrm{I_{graph}} = \frac{0.782\mathrm{A}}{0.89\mathrm{A}} \cdot 0.34\mathrm{A} = 0.30\ \mathrm{A}$$

→ U = 7,33

$$V_{AB} = \frac{U \cdot I \cdot \eta}{F_{wheel}} = \frac{7{,}33V \cdot 0{,}782A \cdot 0{,}7}{1{,}17N} = 3{,}43m/s$$

On the slope,

F_{wheel} = 0.66 N

→ I = $\frac{0.66\times0.04}{8.55\times0.7\times10^{-3}\times10}$ = 0.441 A

→I_{graph} = $\frac{0.441}{0.88}\times 0.34 = 0.170\ A$

→ U = 8.00 V

→$V_{eq,slope}$ = $\frac{8.00\times0.441\times0.7}{0.660}$ = 3.742 m/s

→t_{tot} = t_A + $\frac{6-S_A}{V_{eq,flat}}$ + $\frac{8}{V_{eq,slope}}$

=5.561 s < 5.758 s

→ i = 10 is better than i = 9.874

Consider i = 9:

On the ground,

T_{wheel}= 8.55·0.7·0.78·10^{-3}·9

=0.0420147 N·m

F_{wheel} = $\frac{T_{wheel}}{R_{wheel}}$ = 1.0503675 N

a =$\frac{(\text{Fwheel– Frolling})}{m}$ = 0.903 m/s^2

U·I·η=F_{wheel}·V_A

→ V_A = $\frac{7.33\cdot0.78\cdot0.7}{1.0503675}$ = 3.810 m/s

t_A = $\frac{V_A}{a}$ = 4.220 s

S_A = $\frac{1}{2}\cdot a\cdot t_A^2$ = 8.039 m > 6m

This result means that the car keeps accelerating after it steps on the slope.

$\rightarrow \frac{1}{2} \cdot a \cdot t_{flat}^2 = 6$ m $\rightarrow t_{flat} = 3.645$s

$V_{AB} = a \cdot t_{flat} = 3.292$ m/s

$\rightarrow t_{slope} > \frac{8}{V_{AB}} = 2.430$s

$\rightarrow t_{tot} > 3.645 + 2.430 = 6.075$ s > 5.561 (i = 10)

i = 9 is not suitable for the car

Consider i=11:

On the ground,

$\rightarrow T_{wheel} = 8.55 \cdot 0.7 \cdot 0.78 \cdot 10^{-3} \cdot 11$

$= 0.0513513$ N·m

$\rightarrow F_{wheel} = \frac{0.0513513}{0.04} = 1.283$ N·m

$U \cdot I \cdot \eta = F_{wheel} \cdot V_A$

$\rightarrow V_A = \frac{0.78 \times 7.33 \times 0.7}{1.2837825} = 3.117$ m/s

$a = \frac{F_{wheel} - F_{rolling}}{m} = 1.137$ m/s

$\rightarrow S_A = \frac{1}{2} \cdot a \cdot t_A^2 = 4.273$ m

$F_{wheel} = F_{rolling}$ for the rest of the flat path:

$\rightarrow F_{wheel} \cdot R_{wheel} = 8.55 \cdot 0.7 \cdot I \cdot 10^{-3} \cdot 11$

$\rightarrow I = 0.089$ A

$\rightarrow I_{graph} = \frac{0.089}{0.88} \cdot 0.34 = 0.0345$ A

$\rightarrow U = 8.20$ V

$\rightarrow V_{eq,flat} = \frac{8.20 \times 0.089 \times 0.7}{0.147} = 3.475$ m/s

On the slope,

$F_{wheel} = F_r = 0.66$ N

→ $I = \frac{0.66\times0.04}{8.55\times0.7\times10^{-3}\times11} = 0.401$ A

$I_{graph} = \frac{0.401}{0.88} \times 0.34 = 0.155\ A$

U = 8.04 V

→$V_{eq,slope} = \frac{8.04\times0.401\times0.7}{0.66} = 3.419$ m/s

→$t_{tot} = t_A + \frac{6-S_A}{V_{eq,flat}} + \frac{8}{V_{eq,slope}} = 5.578$ s> 5.561s

→ i=11 is not the best choice, so i=10 is better than others.

About the air resistance :

$F_W = \frac{1}{2} \cdot C_W \cdot A \cdot \rho \cdot v^2$ if i=10,

Take $v = v_{max} = v_{AB} = v_{eq,flat} = 3.827$ m/s

$\rho = 1.293\ kg/m^3$

The area of the solar panel: 21.8×28 cm

Take $C_W = 0.02$

θ = 20°(The best angle to make the solar panel absorb more solar energy)

$A = 0.218\times0.28\times\sin\theta = 0.0209\ m^2$

For $F_{wheel} = 1.17$ N >> $F_{W=}$ 0.039 N

Which means the car can always run against the air resistance, F_W can be neglected;

Consider static friction,

Take the friction coefficient μ =0.71

(between rubber and ordinary road)

→ Max static friction on the ground, $f=\mu*m*g=6.867$ N>> $F_{rolling}$ =0.147 N →The solar car will not slip while moving on flat road;

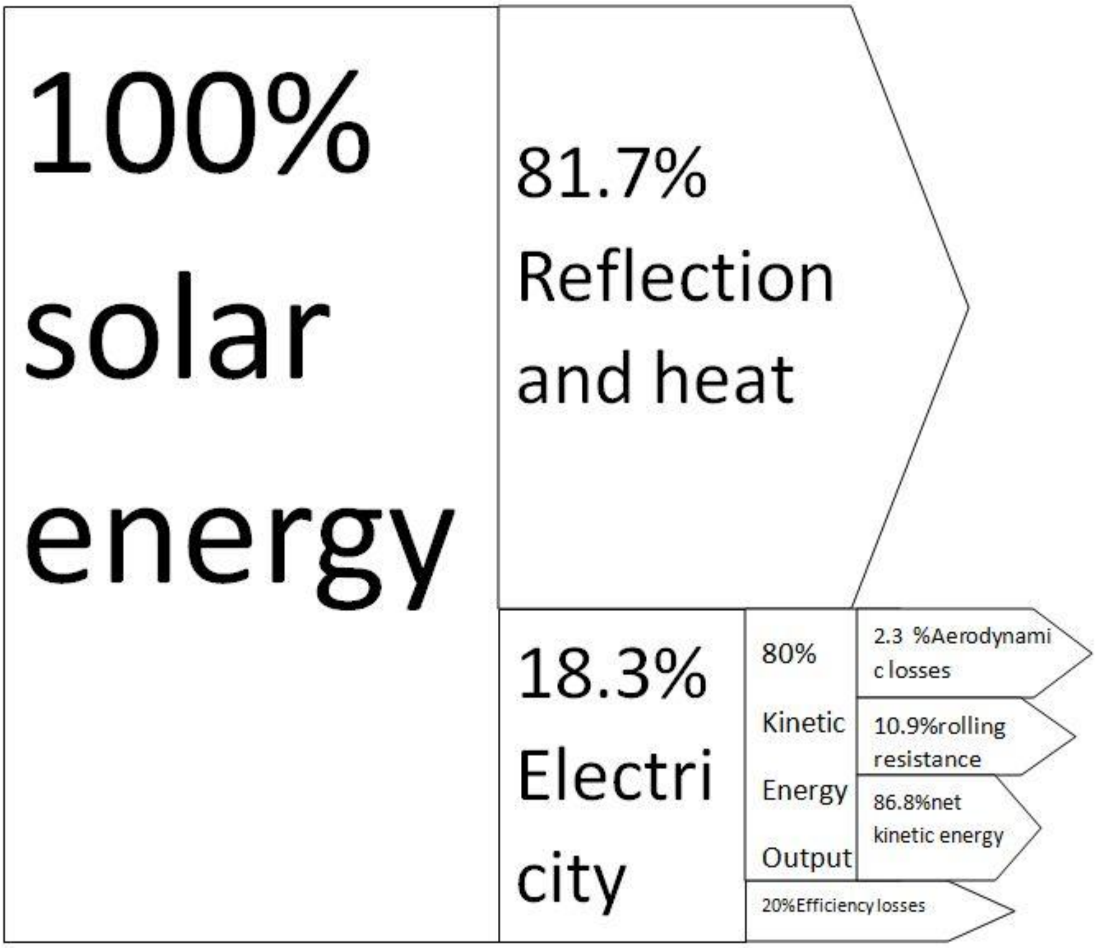
And max static friction on the slope, $f_s = \mu \cdot mg \cdot \cos 3° = 6.858$ N >>$F_r = 0.660$ N

→ The solar car will not slip on the slope.

Sankey Diagram

To have a visual idea how the energy from the sun is being used by the solar car, we use a Sankey-diagram. Here are the formulas to create this diagram.

This Sankey-diagram is for the situation where the SSV is on the flat part of the track and has reached top power.



Figuur 1: Sankey Diagram

The amount of solar energy delivered by the solar panel depends on the solar-intensity and de surface of the solar cells. We have taken 800W/m² as solar-intensity, this is the solar-intensity in countries with the same latitude as Belguim

The total area of solar cells is: $0{,}062m * 0{,}042m * 15 = 0{,}03906m^2$

So the total amount of solar energy becomes:

$$800\frac{W}{m^2} * 0{,}03906m^2 = 31.248W$$

We have measured the characteristics (the current at each voltage) of the solar panel and the maximum power that can be generated is equal to 5,72W. This is 18.3% of the total solar energy. The rest of the energy is lost in reflection and heat.

This electric energy can be converted into movement, into kinetic energy.

The kinetic energy consists of the movement, rolling resistance and aerodynamic losses. We are calculating the Sankey-diagram at top power point, where speed is equal to 3,429$\frac{m}{s}$.

- **Rolling resistance:** $M \cdot C_{rr} \cdot v$ = $0{,}505\ W$

Where M is the force due to the mass of the SSV, Crr is the rolling resistance coefficient and v is the speed. With $M = 1kg \cdot 9{,}81 \frac{m}{s^2}$, $C_{rr} = 0{,}015$, $v = 3{,}429 \frac{m}{s}$ the rolling resistance takes 0,505 W of the kinetic energy.

- **Aerodynamic:** $\frac{1}{2} \cdot \rho \cdot v^3 \cdot A \cdot C_d = 0{,}108\ W$

Where ρ is the density of the fluid, v the speed, A is the frontal surface and Cd is the drag coefficient. With $\rho = 1{,}29 \frac{kg}{m^3}$, $v = 3{,}429 \frac{m}{s}$, $C_d = 0{,}2$ and $A = 0{,}02087\ m^2$ the aerodynamic loss is 0,108 W.

- **Net kinetic power :** $F \cdot v = 4{,}002W$

Kinetic: $1.167\mathrm{N} \cdot 3.429\mathrm{m/s} = 4.002\mathrm{W}$

Where F is the driving force, v is the speed at top power point. This is the net kinetic power the car can output.

So the total of kinetic energy output becomes 4,615 W. This is equal to 80% of the electricity. The other 20% are lost due to efficiency losses.

This leads to the Sankey-diagram shown above, when the SSV is on the flat part of the track and has achieved top power point.