Complex Phase Factors (DFT.A1)

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Discrete Fourier Transform | **Inverse Discrete Fourier Transform**

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Longleftrightarrow \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

$$W_N \triangleq e^{-j(2\pi/N)}$$

$$W_N^{nk} \triangleq e^{-j(2\pi/N)nk}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn} \quad \Longleftrightarrow \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{-kn}$$

Complex Phase Factor

$$W_N^{+kN} = 1$$

$$W_N^{-kN} = 1$$

$$W_N^{\pm N} = e^{-j(\frac{2\pi}{N})(\pm N)}$$

$$W_N^{k \pm N} = W_N^{k}$$

$$W_N^{-k \pm N} = W_N^{-k}$$

$$W_N^{\pm k \pm N} = W_N^{\pm k} \cdot W_N^{\pm N}$$

Modular N System

$$W_N^{nk} \triangleq e^{-j(2\pi/N)nk}$$

DFT Index (1)

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \qquad X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$

	n=0	n=1	n=2	• • •	n=N-1
k=0	$0 \cdot 0$	$0 \cdot 1$	$0 \cdot 2$	• • •	$0 \cdot (N-1)$
k=1	$1 \cdot 0$	$1 \cdot 1$	$1 \cdot 2$	• • •	$1 \cdot (N-1)$
k=2	$2 \cdot 0$	$2 \cdot 1$	$2 \cdot 2$	• • •	$2 \cdot (N-1)$
⋮	⋮	⋮	⋮		⋮
k=N-1	$(N-1) \cdot 0$	$(N-1) \cdot 1$	$(N-1) \cdot 2$	• • •	$(N-1) \cdot (N-1)$

A Multiplication Table in Modular N System

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DFT Index (2)

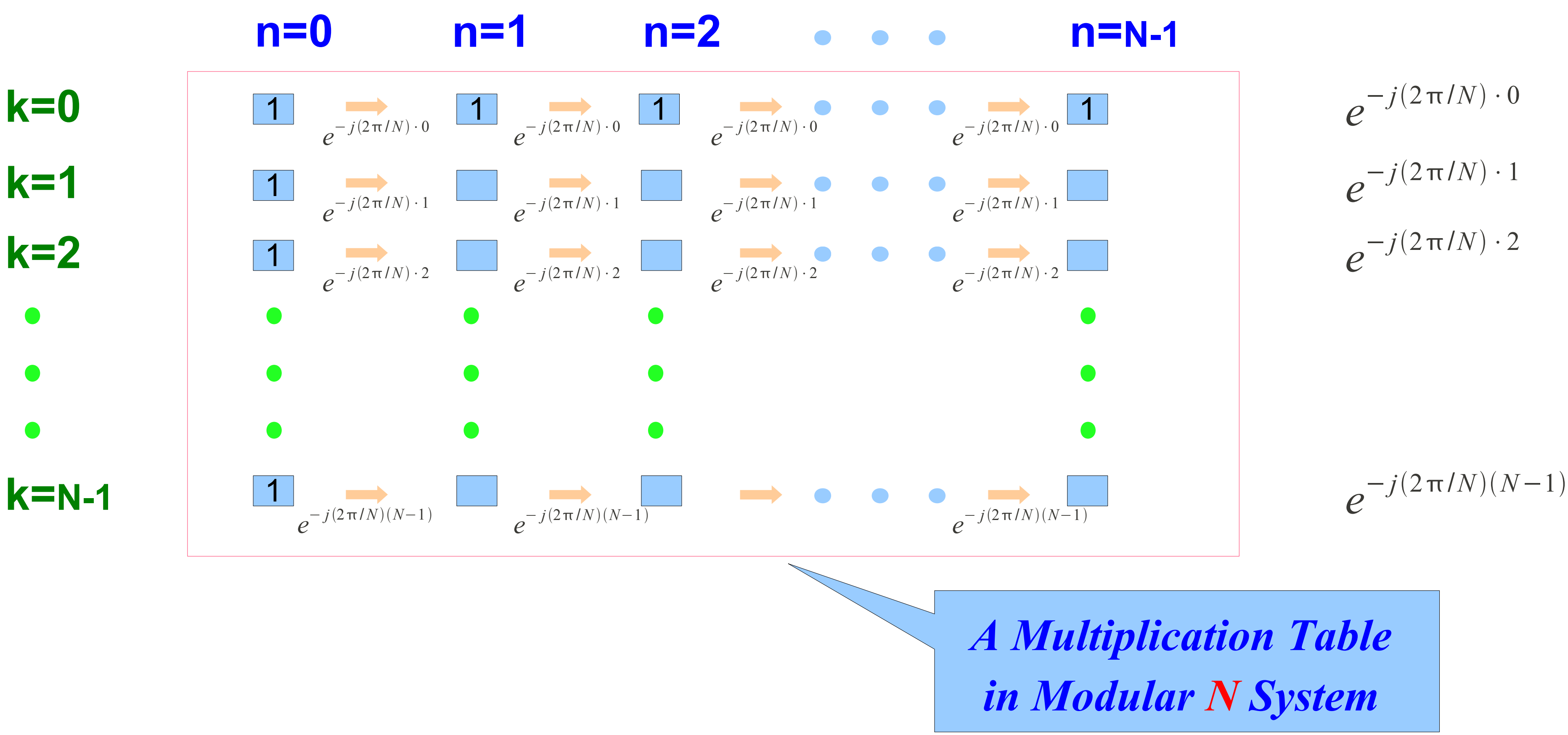
$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \qquad X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$

	n=0		n=1		n=2		···		n=N-1	
k=0	$0 \cdot 0$	→ +0	$0 \cdot 1$	→ +0	$0 \cdot 2$	→ +0	···	→ +0	$0 \cdot (N-1)$	+ 0 (mod N)
k=1	$1 \cdot 0$	→ +1	$1 \cdot 1$	→ +1	$1 \cdot 2$	→ +1	···	→ +1	$1 \cdot (N-1)$	+ 1 (mod N)
k=2	$2 \cdot 0$	→ +2	$2 \cdot 1$	→ +2	$2 \cdot 2$	→ +2	···	→ +2	$2 \cdot (N-1)$	+ 2 (mod N)
⋮	⋮		⋮		⋮				⋮	
k=N-1	$(N-1) \cdot 0$	→ +(N-1)	$(N-1) \cdot 1$	→ +(N-1)	$(N-1) \cdot 2$	→ +(N-1)	···	→ +(N-1)	$(N-1) \cdot (N-1)$	+ N-1 (mod N)

A Multiplication Table in Modular N System

DFT Matrix

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \qquad X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$



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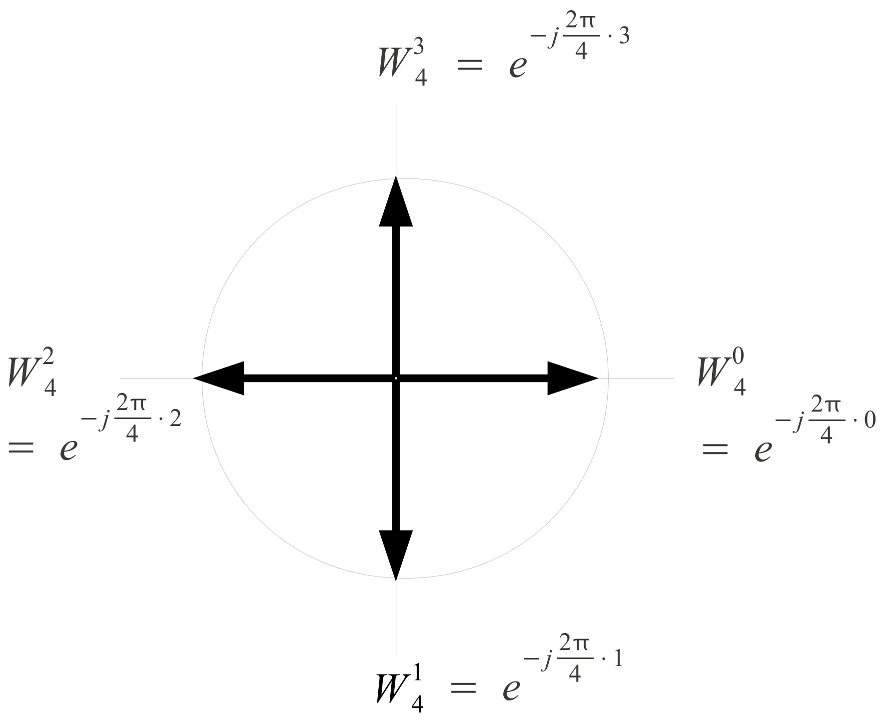
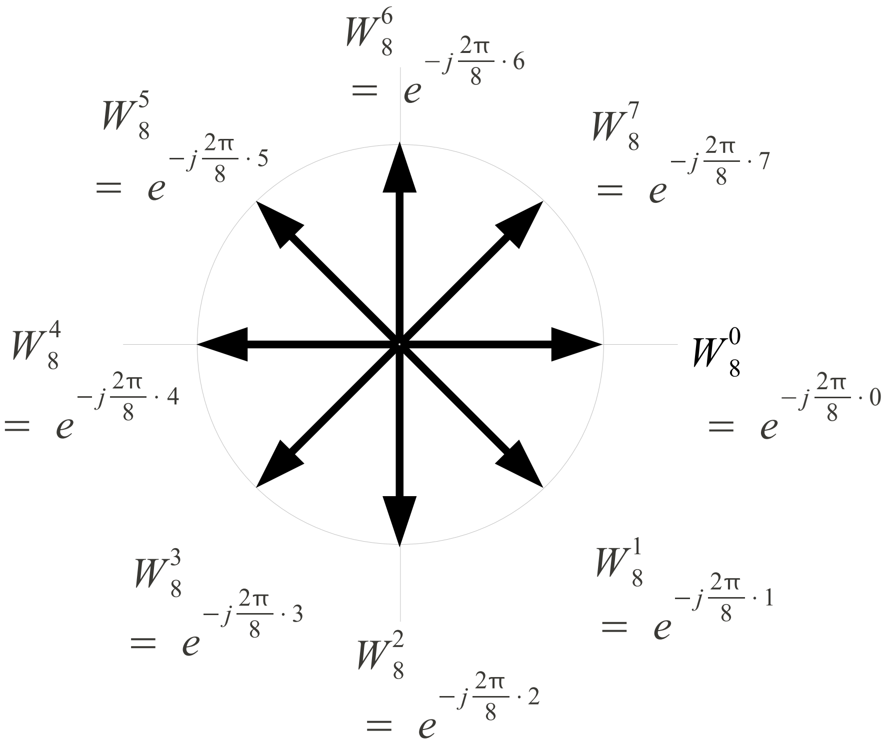
Complex Phase Factor

Modular 4 System



Modular 8 System



Complex Phase Factor Symmetry

$$W_N^{+kN} = 1$$

$$W_N^{-kN} = 1$$

$$W_N^{\pm N} = e^{-j\left(\frac{2\pi}{N}\right)(\pm N)}$$

$$W_N^{k\pm N} = W_N^{k}$$

$$W_N^{-k\pm N} = W_N^{-k}$$

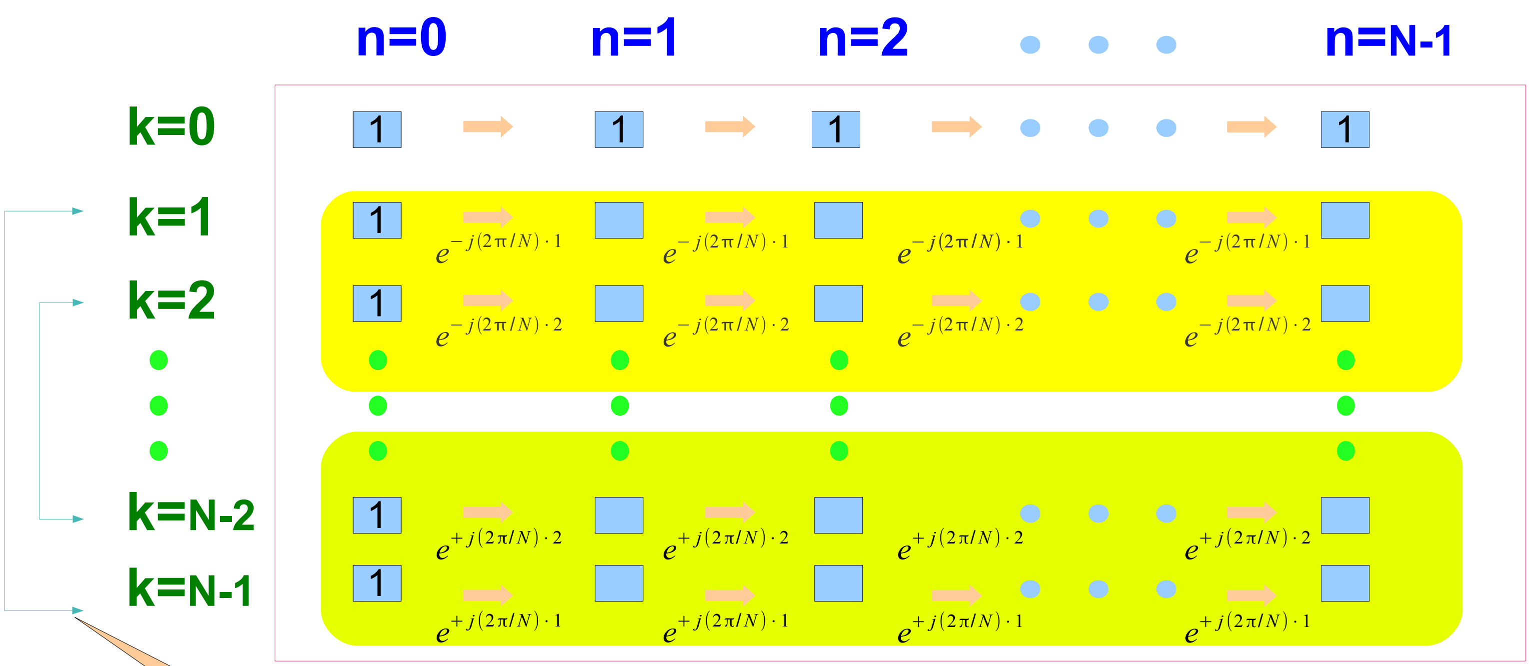
$$W_N^{\pm k \pm N} = W_N^{\pm k} \cdot W_N^{\pm N}$$

$$W_N^{N-k} = W_N^{-k} = \left\{W_N^{k}\right\}^*$$

$$e^{-j\frac{2\pi}{N}(N-k)} = e^{j\frac{2\pi}{N}k} = \left\{e^{-j\frac{2\pi}{N}k}\right\}^*$$

DFT Matrix Symmetry

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \qquad X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$



$$e^{-j(\frac{2\pi}{N})\cdot(N-2)} = e^{+j(\frac{2\pi}{N})\cdot 2}$$

$$e^{-j(\frac{2\pi}{N})\cdot(N-1)} = e^{+j(\frac{2\pi}{N})\cdot 1}$$

complex conjugate

$$e^{-j\left(\frac{2\pi}{N}\right)\cdot(N-\boldsymbol{i})} = e^{+j\left(\frac{2\pi}{N}\right)\cdot \boldsymbol{i}}$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003