DLTI z-Transform

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Finding ZIR & ZSR Using Laplace Transform (1)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition $y[1]=3, \quad y[-2]=2$

input $x[n] = u[n]$

$y[n]$	$\Leftrightarrow$	$Y[z]$
$y[n-1]$	$\Leftrightarrow$	$y[-1] + z^{-1}Y[z]$ $= z^{-1}Y[z] + 3$
$y[n-2]$	$\Leftrightarrow$	$y[-2] + z^{-1}y[-1] + z^{-2}Y[z]$ $= z^{-2}Y[z] + 3z^{-1} + 2$

$x[n]$	$\Leftrightarrow$	$X[z] = \frac{z}{z-1}$
$x[n-1]$	$\Leftrightarrow$	$\frac{1}{z-1}$
$x[n-2]$	$\Leftrightarrow$	$\frac{1}{z(z-1)}$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}(z^{-1}Y[z] + 3) - \frac{1}{20}(z^{-2}Y[z] + 3z^{-1} + 2)$$

initial condition terms

Finding ZIR & ZSR Using Laplace Transform (2)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition $y[1]=3, \quad y[-2]=2$

input $x[n] = u[n]$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}\left(z^{-1}Y[z] + 3\right) - \frac{1}{20}\left(z^{-2}Y[z] + 3z^{-1} + 2\right)$$

$$\frac{Y[z]}{z} = \frac{2z^2 - z + 3}{(z-1)(z-\frac{1}{3})(z-\frac{1}{4})} + \frac{\left(\frac{5}{4}z - \frac{3}{20}\right)}{(z-\frac{1}{5})(z-\frac{1}{4})}$$

init cond terms

input terms

$$\frac{Y[z]}{z} = \frac{\frac{20}{3}}{(z-1)} + \frac{72}{(z-\frac{1}{5})} + \frac{\frac{230}{3}}{(z-\frac{1}{4})} - \frac{2}{(z-\frac{1}{5})} + \frac{\frac{13}{4}}{(z-\frac{1}{4})}$$

$$y[n] = \frac{30}{3} + 72\left(\frac{1}{5}\right)^n - \frac{230}{3}\left(\frac{1}{4}\right)^n - 2\left(\frac{1}{5}\right)^n + \frac{13}{4}\left(\frac{1}{4}\right)^n \qquad = \frac{20}{3} + 70\left(\frac{1}{5}\right)^n - \frac{881}{12}\left(\frac{1}{4}\right)^n$$

Zero State Resp ***Zero Input Resp***

Region Of Convergence (1)

$$X(z) = \frac{1}{1-0.5z^{-1}} \quad \Longleftrightarrow \quad X(z) = \frac{(0.5)^{-1}z}{0.5z^{-1}-1} = -\frac{(0.5)^{-1}z}{1-0.5z^{-1}}$$

$\frac{a}{1-r}$ Infinite Geometric Series $\frac{a}{1-r}$

$a = 1$	Initial Term	$a = -(0.5)^{-1}z$
$r = 0.5z^{-1}$	Common Ratio	$r = (0.5)^{-1}z$
$\lvert (0.5)z^{-1} \rvert < 1$	ROC	$\lvert (0.5)^{-1}z \rvert < 1$
$1 + r + r^2 + r^3 + \cdots$		$a + ar + ar^2 + ar^3 + \cdots$
$1 + 0.5z^{-1} + 0.5^2z^{-2} + 0.5^3z^{-3} + \cdots$		$-0.5^{-1}z - 0.5^{-2}z^2 - 0.5^{-3}z^3 + \cdots$
$x[0] + x[1]z^{-1} + x[2]z^{-2} + x[3]z^{-3} + \cdots$		$x[-1]z^1 + x[-2]z^2 + x[-3]z^3 + \cdots$
		$-2z - 2^2z^2 - 2^3z^3 - \cdots$

Region Of Convergence (2)

$$X(z) = \frac{1}{1-0.5z^{-1}} \quad \Longleftrightarrow \quad X(z) = \frac{(0.5)^{-1}z}{(0.5)^{-1}z-1} = -\frac{(0.5)^{-1}z}{1-(0.5)^{-1}z}$$

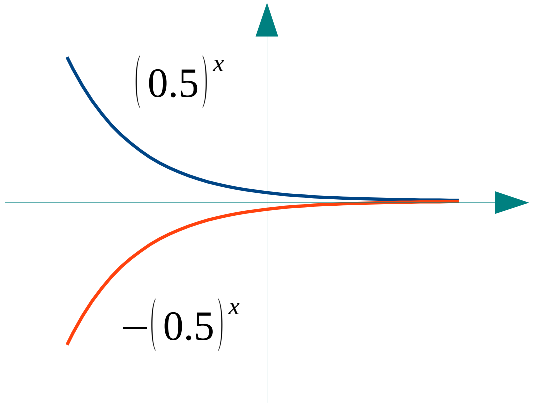
ROC

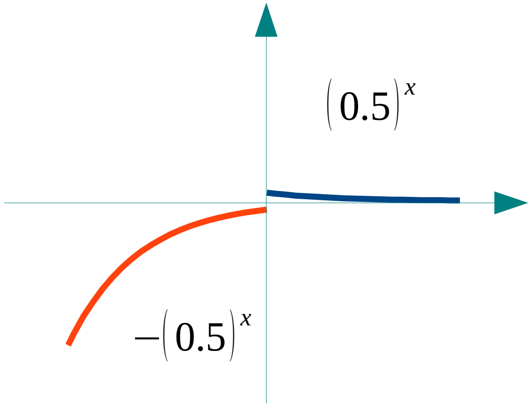
$$|(0.5)z^{-1}| < 1 \qquad\qquad |(0.5)^{-1}z| < 1$$

$$1 + r + r^2 + r^3 + \cdots \qquad\qquad a + ar + ar^2 + ar^3 + \cdots$$

$$1 + 0.5z^{-1} + 0.5^2z^{-2} + 0.5^3z^{-3} + \cdots \qquad\qquad -2z - 2^2z^2 - 2^3z^3 - \cdots$$

$$x[0] + x[1]z^{-1} + x[2]z^{-2} + x[3]z^{-3} + \cdots \qquad\qquad x[-1]z^1 + x[-2]z^2 + x[-3]z^3 + \cdots$$





Region Of Convergence (3)

$$X(z) = \frac{1}{1-0.5z^{-1}} \quad \Longleftrightarrow \quad X(z) = \frac{(0.5)^{-1}z}{(0.5)^{-1}z-1} = -\frac{(0.5)^{-1}z}{1-(0.5)^{-1}z}$$

ROC

$\left|(0.5)z^{-1}\right| < 1$ $\quad |z| > 0.5$

$1 + 0.5z^{-1} + 0.5^2z^{-2} + 0.5^3z^{-3} + \cdots$

$x[0] + x[1]z^{-1} + x[2]z^{-2} + x[3]z^{-3} + \cdots$

$\left|(0.5)^{-1}z\right| < 1$ $\quad |z| < 0.5$

$-2z - 2^2z^2 - 2^3z^3 - \cdots$

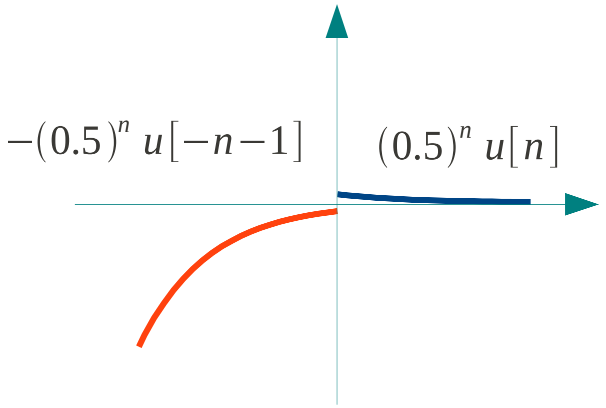
$x[-1]z^1 + x[-2]z^2 + x[-3]z^3 + \cdots$

$$x[0] = 1$$
$$x[1] = 0.5$$
$$x[2] = (0.5)^2$$
$$x[3] = (0.5)^3$$
$$\cdots$$

$$x[n] = (0.5)^n u[n]$$

$$x[-1] = -(0.5)^{-1} = -2$$
$$x[-2] = -(0.5)^{-2} = -2^2$$
$$x[-3] = -(0.5)^{-3} = -2^3$$
$$x[-4] = -(0.5)^{-4} = -2^4$$
$$\cdots$$

$$x[n] = -(0.5)^n u[-n-1]$$



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] B.P. Lathi, Linear Systems and Signals (2^{nd} Ed)
[4] D. Sundararajan, A Practical Approach to Signals and Systems