Convolution (1A)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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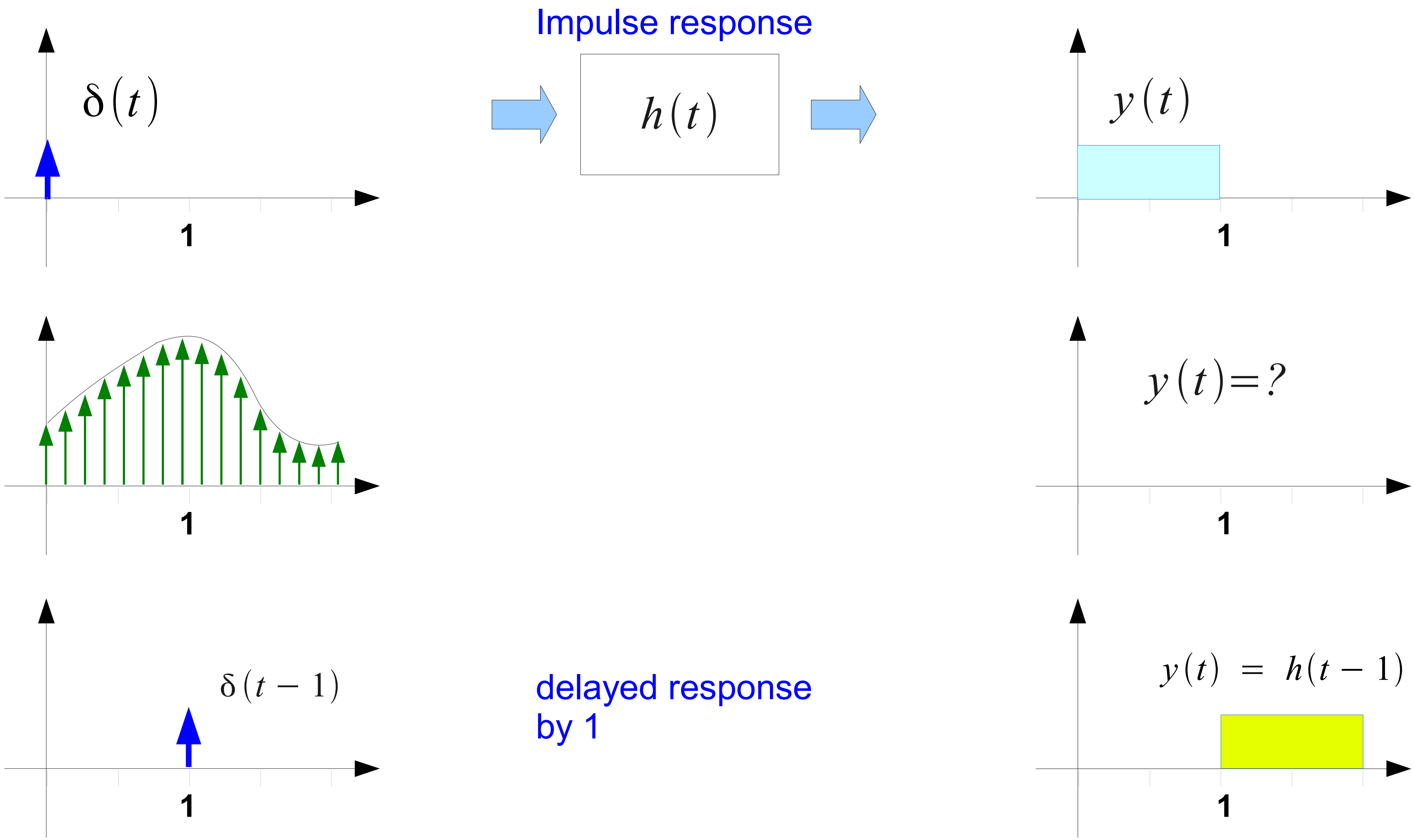
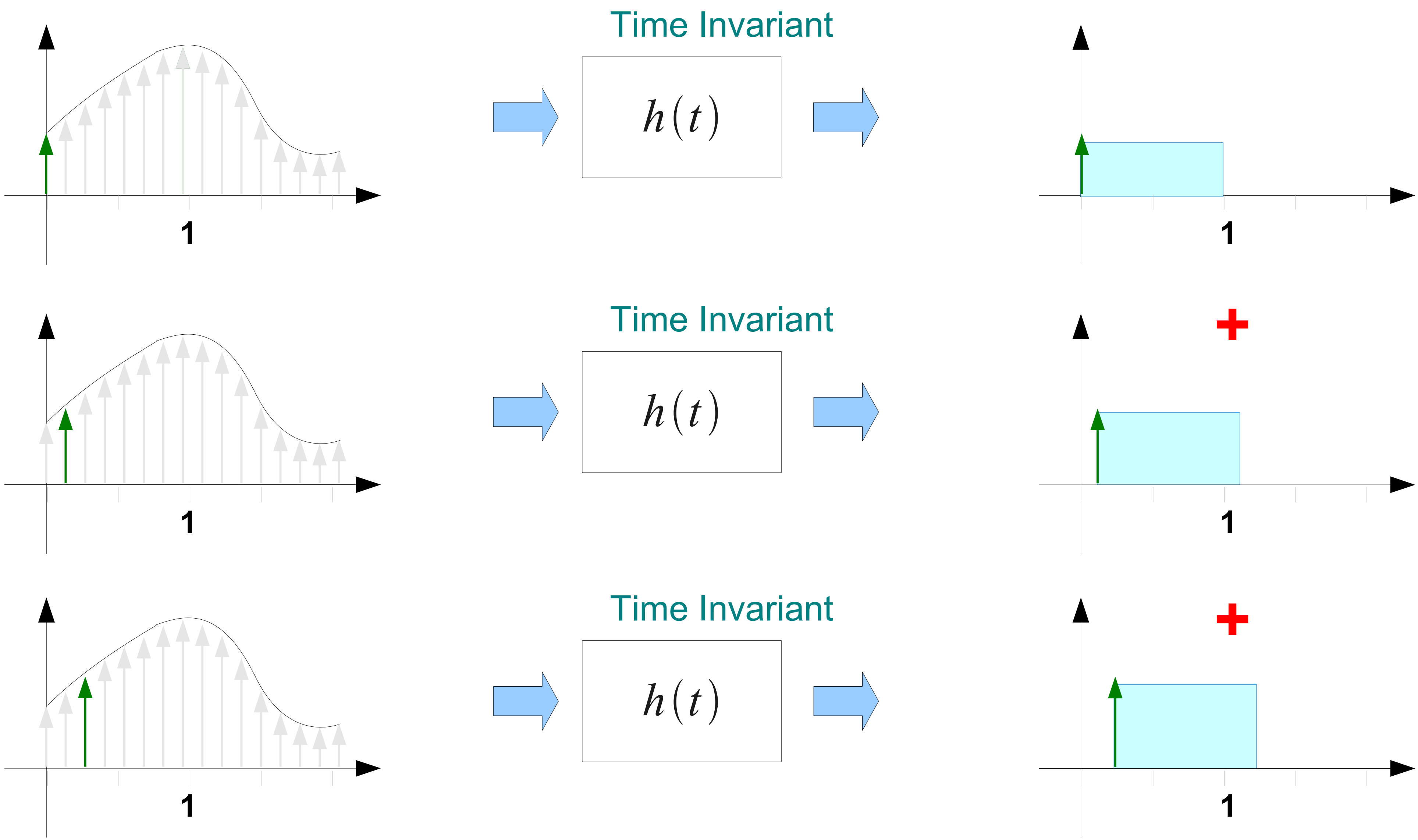
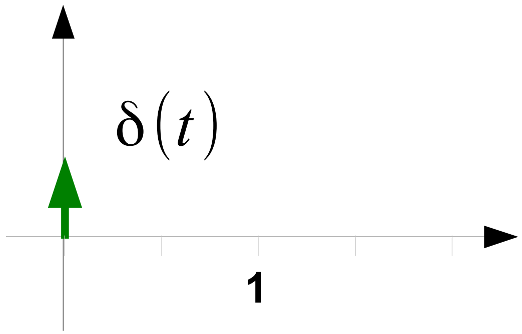
Convolution: delayed response of h(t) (1)



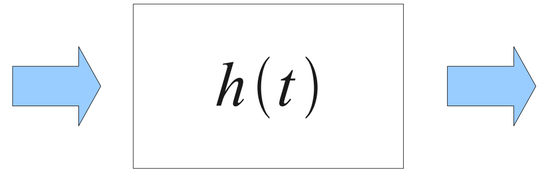
Convolution: delayed response of h(t) (2)

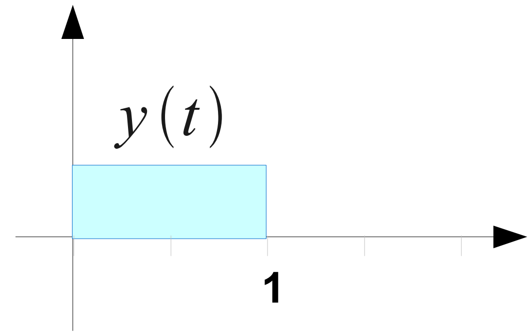


Convolution: delayed response of h(t) (3)



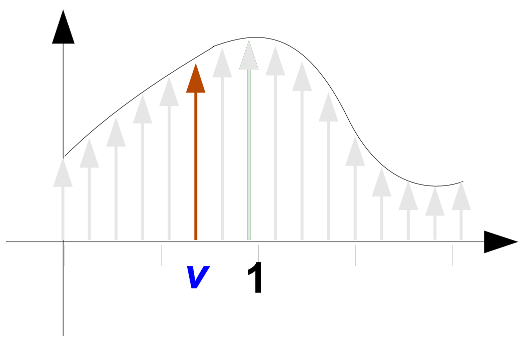
Linear System



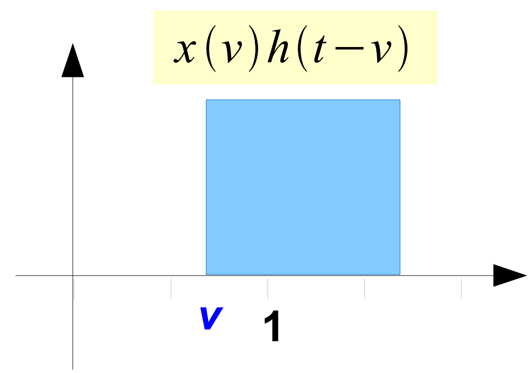


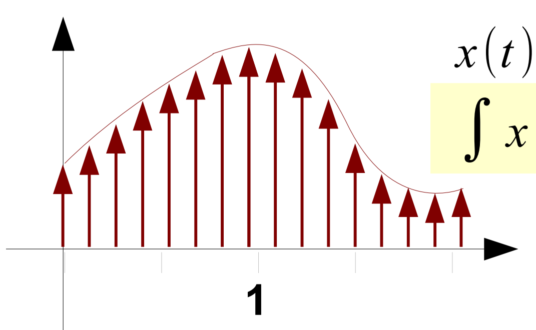
$\delta(t-v) \Rightarrow h(t-v)$

$x(v)\,\delta(t-v) \Rightarrow x(v)\,h(t-v)$



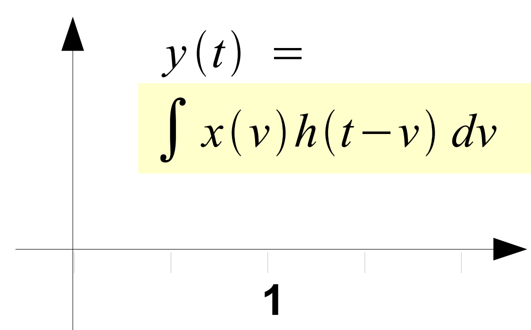
input value at time v
→ x(v)
delayed impulse response
→ x(v) h(t - v)





$x(t) = \int x(v)\delta(t-v)\,dv$

$y(t) = \int x(v)h(t-v)\,dv$



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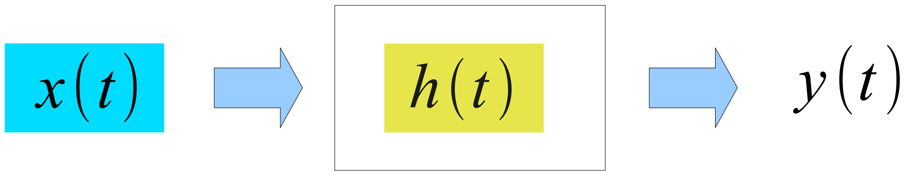
9/27/11

Convolution: Commutative Law



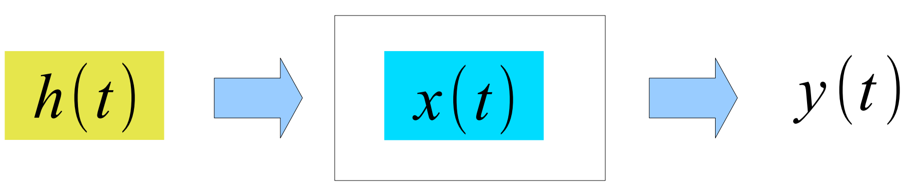
LTI

$x(v)$ $\quad$ $h(v)$ $\xrightarrow{\text{Flip}}$ $h(-v)$ $\xrightarrow{\text{Shift}}$ $h(t-v)$

$$\int x(v)\,h(t-v)\,dv = y(t)$$

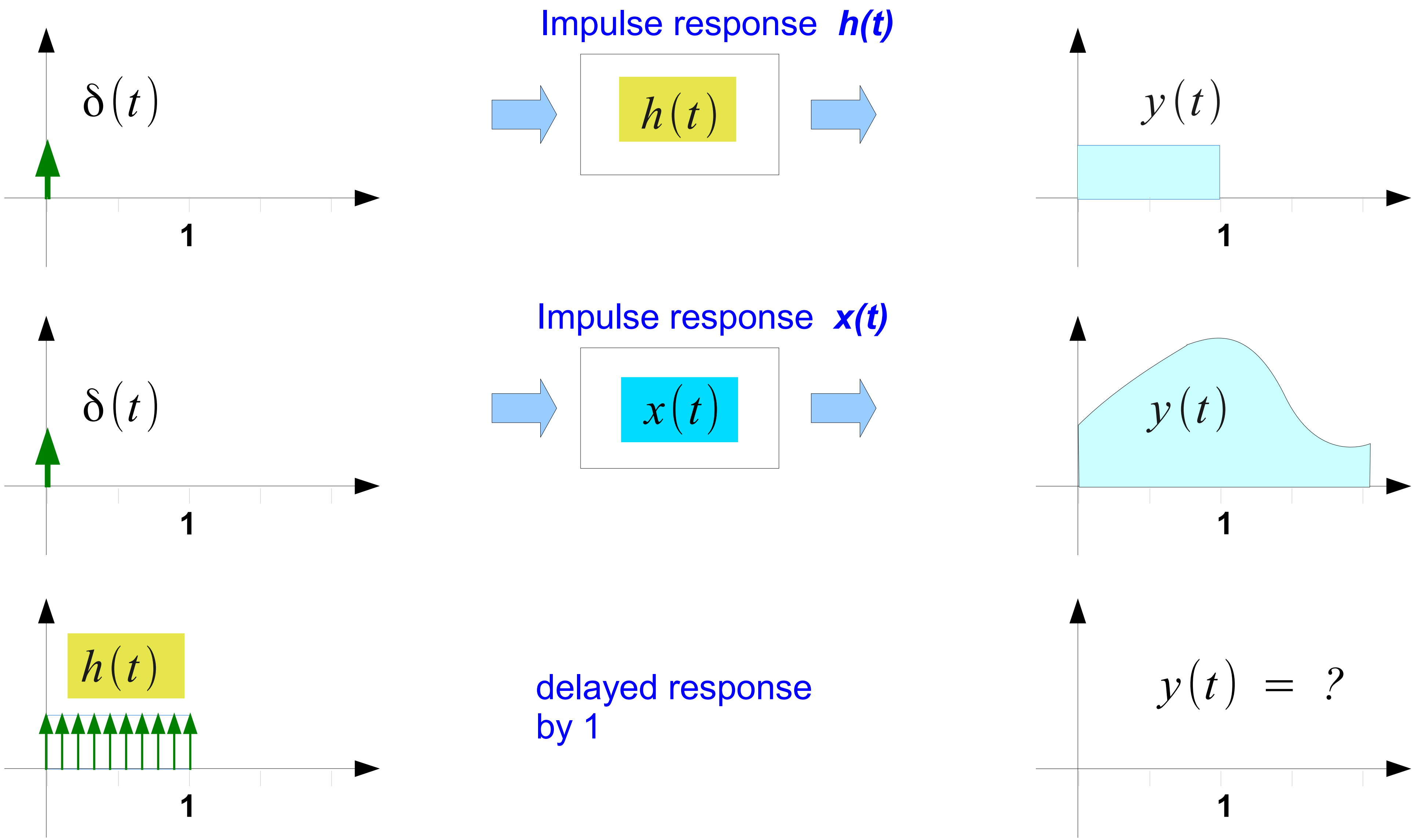
$$\int h(v)\,x(t-v)\,dv = y(t)$$

$h(v)$ $\quad$ $x(v)$ $\xrightarrow{\text{Flip}}$ $x(-v)$ $\xrightarrow{\text{Shift}}$ $x(t-v)$

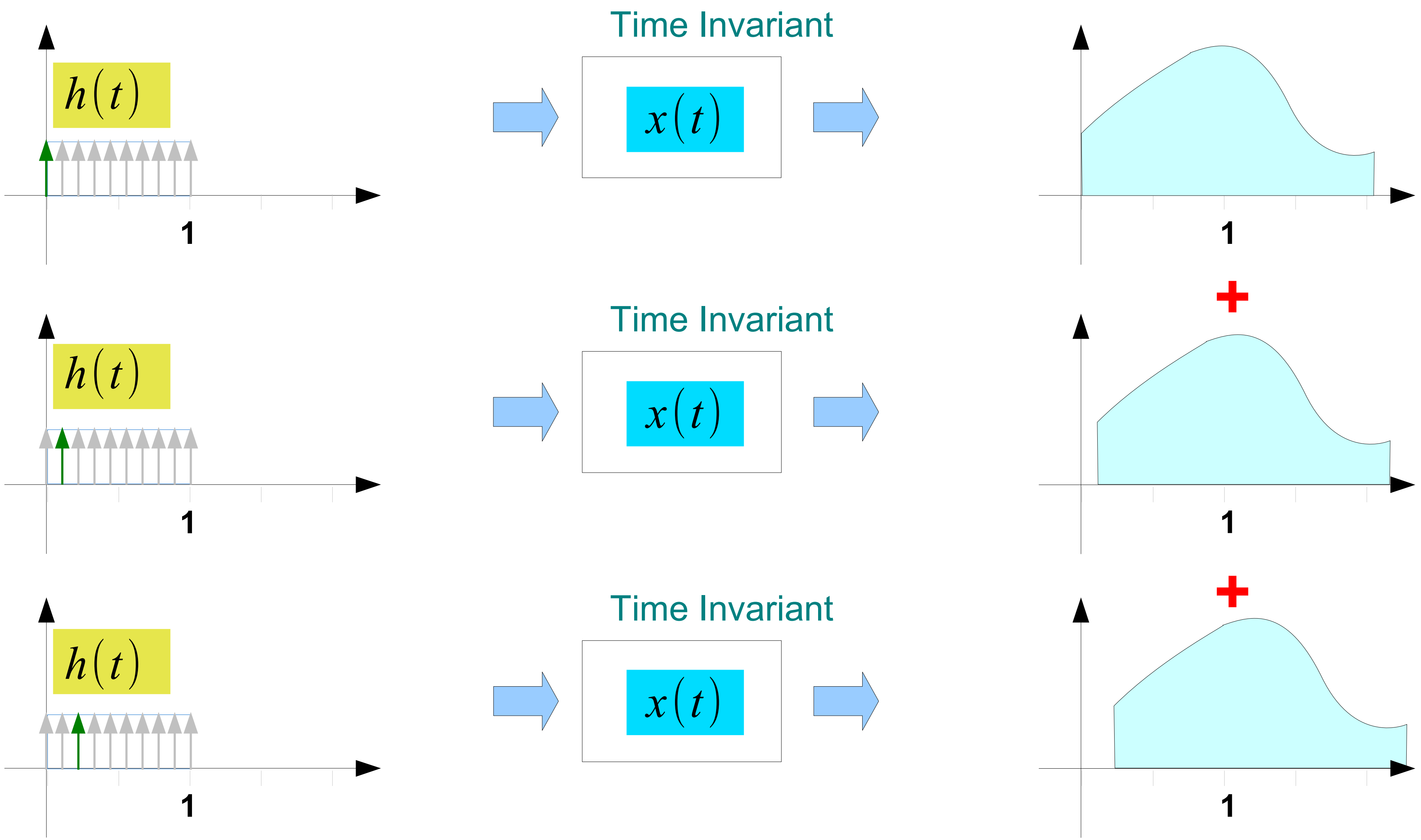


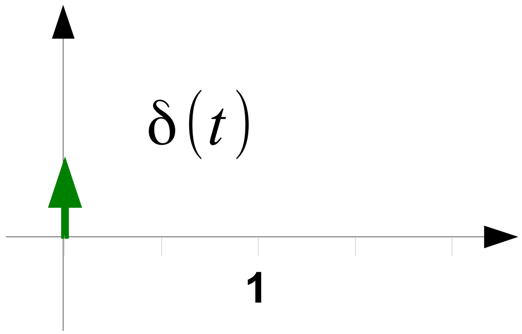
LTI

Convolution: delayed response of x(t) (1)

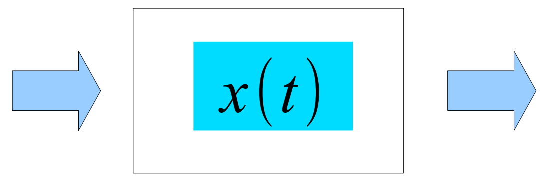


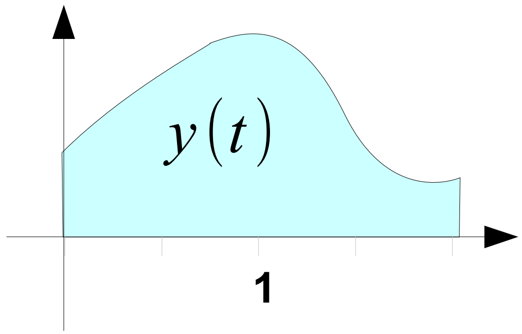
Convolution: delayed response of x(t) (2)





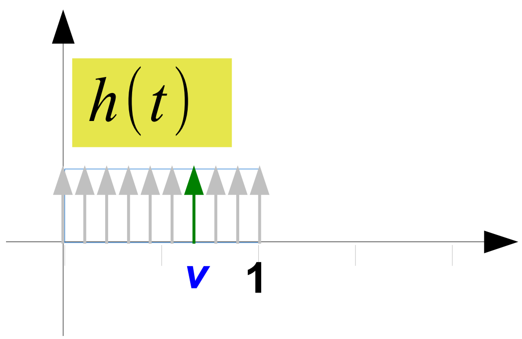
Linear System



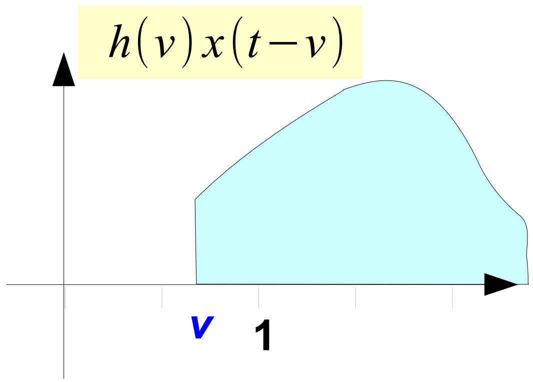


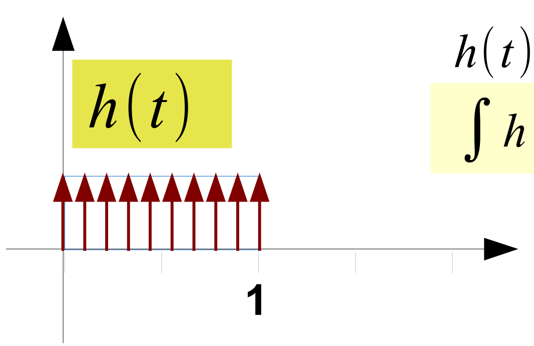
$\delta(t-v) \rightarrow x(t-v)$

$h(v)\,\delta(t-v) \rightarrow h(v)\,x(t-v)$



input value at time v
→ h(v)
delayed impulse response
→ h(v) x(t - v)

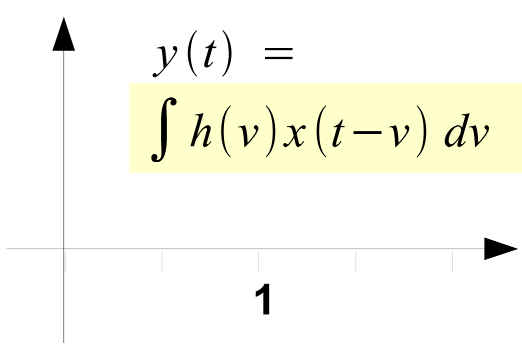




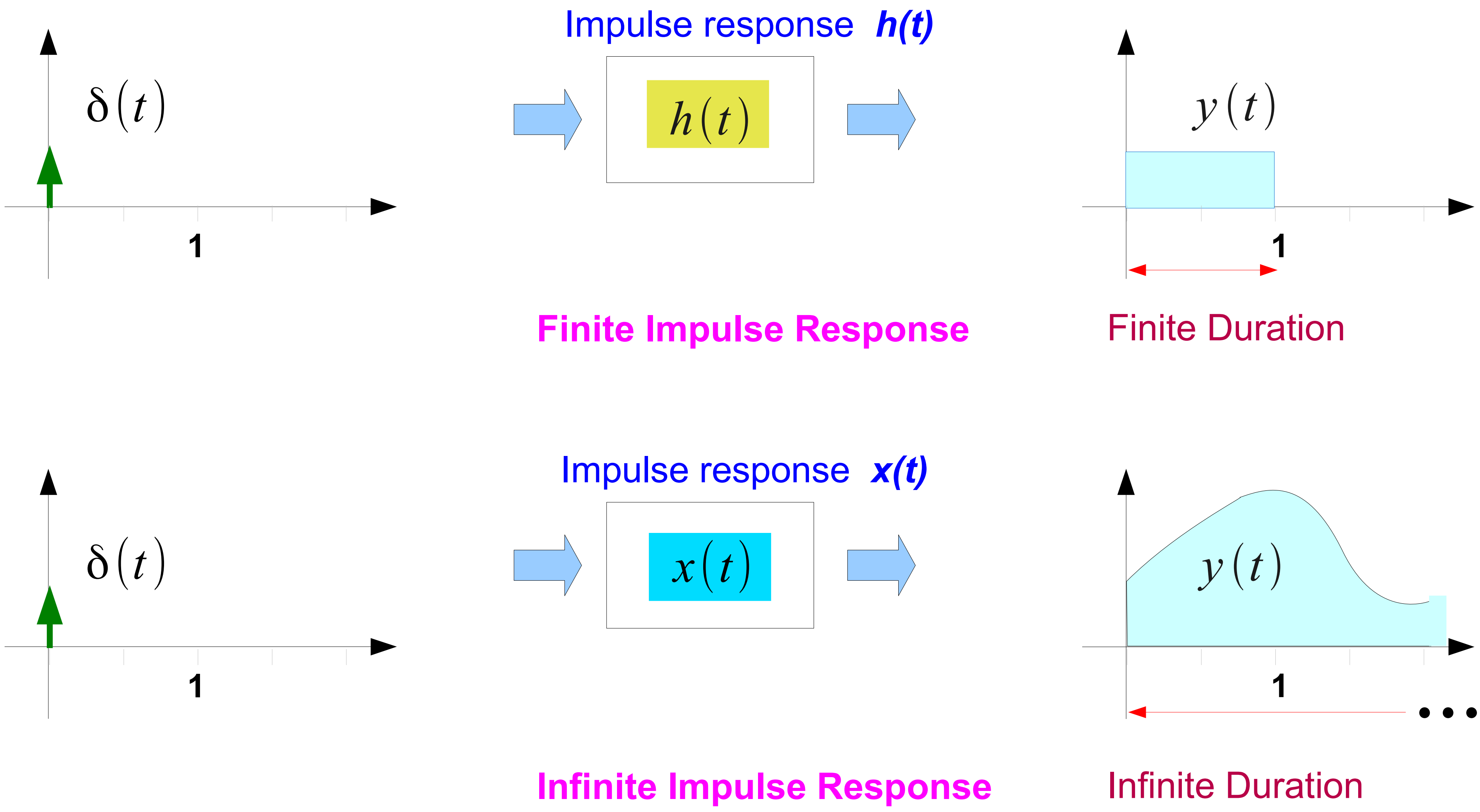
$$h(t) = \int h(v)\,\delta(t-v)\,dv$$

$$\Rightarrow$$

$$y(t) = \int h(v)\,x(t-v)\,dv$$



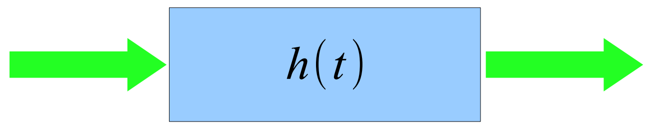
FIR and IIR


Impulse response h(t)
δ(t)
1
h(t)
y(t)
1
Finite Impulse Response
Finite Duration
Impulse response x(t)
δ(t)
1
x(t)
y(t)
1
Infinite Impulse Response
Infinite Duration

Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$x(t) = A\, e^{j\Phi}\, e^{j\omega t}$$



$$y(t) = H(jw) \cdot A\, e^{j\Phi}\, e^{j\omega t}$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, A\, e^{j\Phi}\, e^{jw(t-\tau)}\, d\tau$$

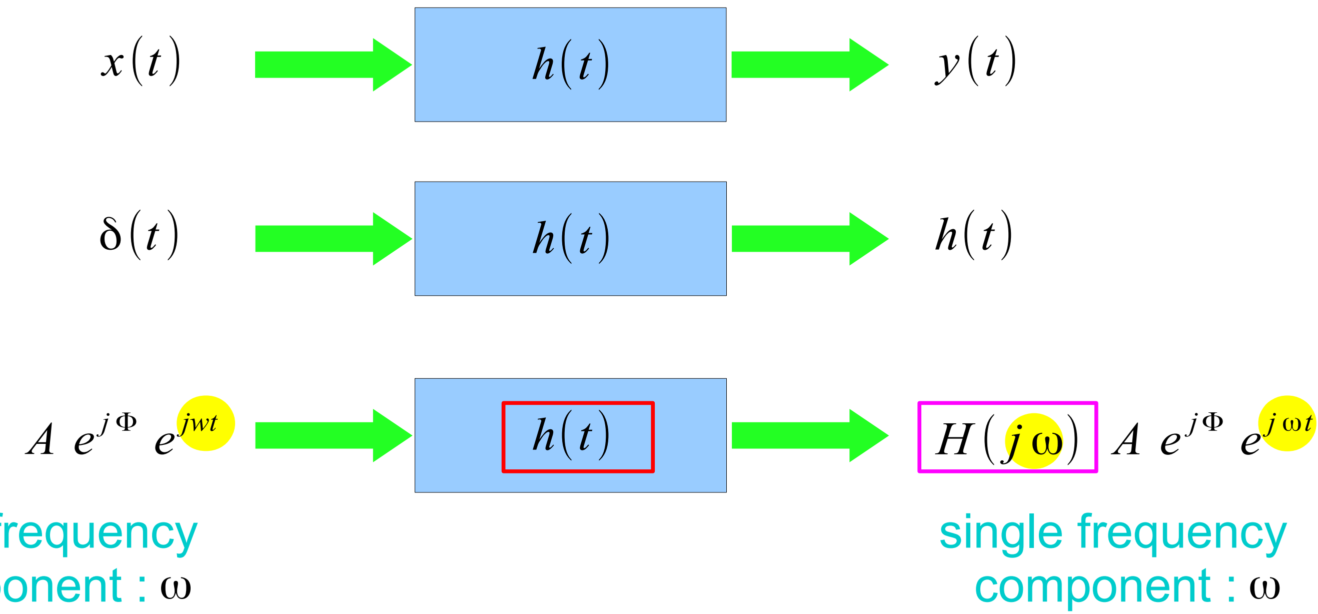
$$= \int_{-\infty}^{+\infty} h(\tau)\, A\, e^{j\Phi}\, e^{jwt}\, e^{-j\omega\tau}\, d\tau$$

$$= \underline{A\, e^{j\Phi}\, e^{jwt}} \cdot \underline{\int_{-\infty}^{+\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau}$$

$$= \underline{x(t)} \cdot \underline{H(jw)}$$

Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$



$$H(j\omega) = \int_{-\infty}^{+\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003