Anti-aliasing Prefilter (6B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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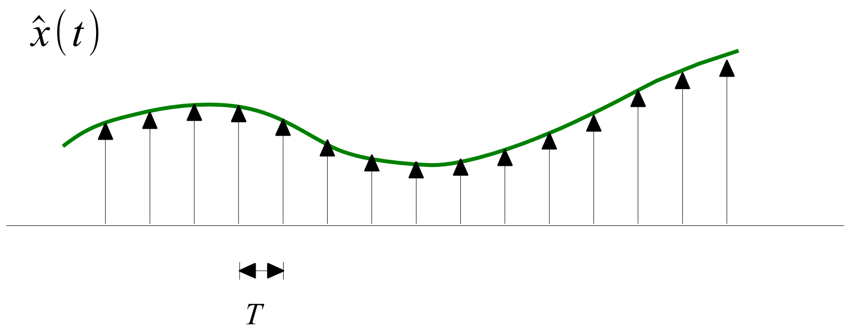
Sampler

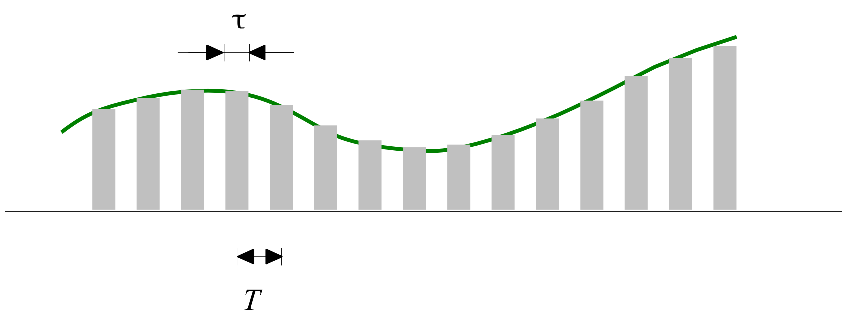
Ideal Sampling



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

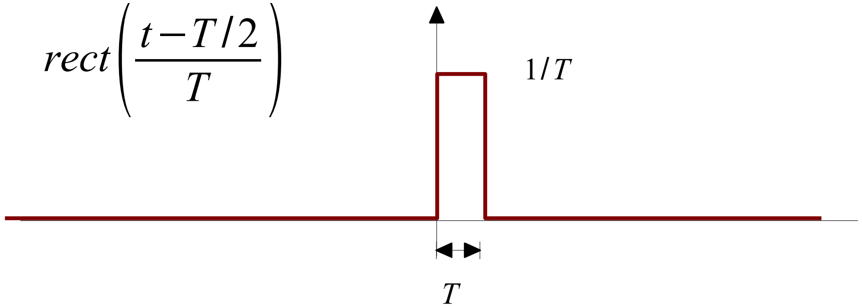
$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t)\, e^{-j2\pi f t}\, dt$$

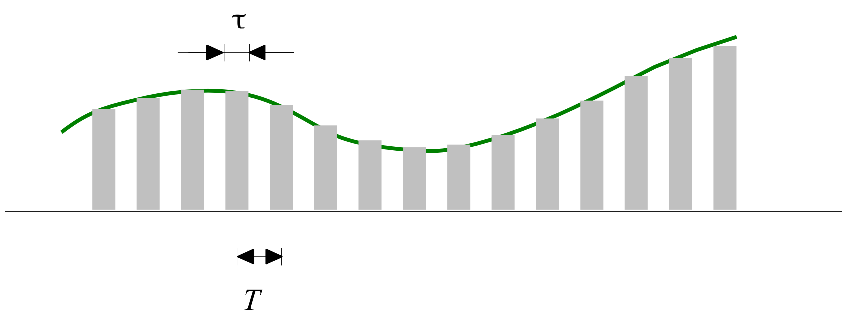
Practical Sampling



$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT)\,p(t-nT)$$

Zero Order Hold (ZOH)





$$x_{ZOH}(t) = \sum_{n=-\infty}^{+\infty} x[n] \cdot rect\left(\frac{t-T/2-nT}{T}\right)$$

Continuous Time Fourier Series

$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \quad \Leftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_k\, e^{+jk\omega_0 t}$$

$$C_k = \frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} x_{T_0}(t)\, e^{-jk\omega_0 t}\, dt$$

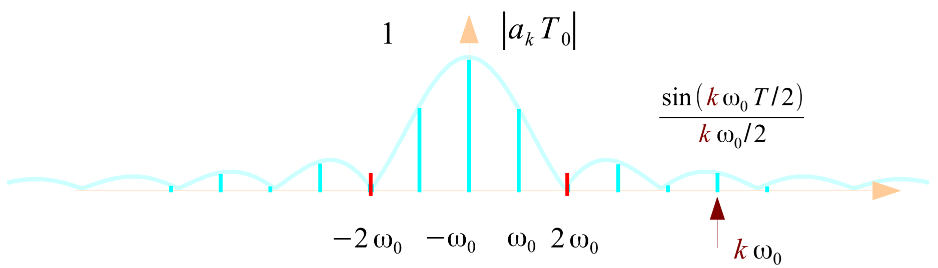
$$\omega_0 = \frac{2\pi}{T_0}$$

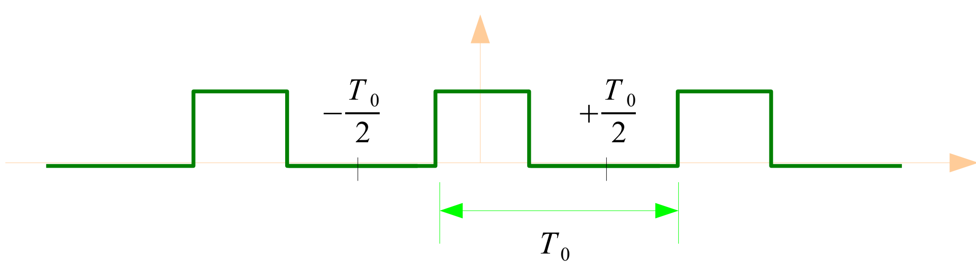
Fundamental Frequency

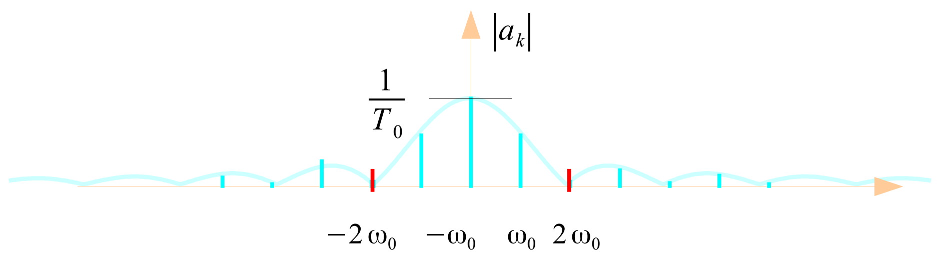
$$C_k T_0 = \int_{-T_0/2}^{+T_0/2} x_{T_0}(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \int_{-T_0/2}^{+T_0/2} e^{-jk\omega_0 t}\, dt = \left[\frac{-1}{jk\omega_0} e^{-jk\omega_0 t}\right]_{-T/2}^{+T/2}$$

$$= -\frac{e^{-jk\omega_0 T/2} - e^{+jk\omega_0 T/2}}{jk\omega_0} = \frac{\sin(k\omega_0 T/2)}{k\omega_0/2}$$





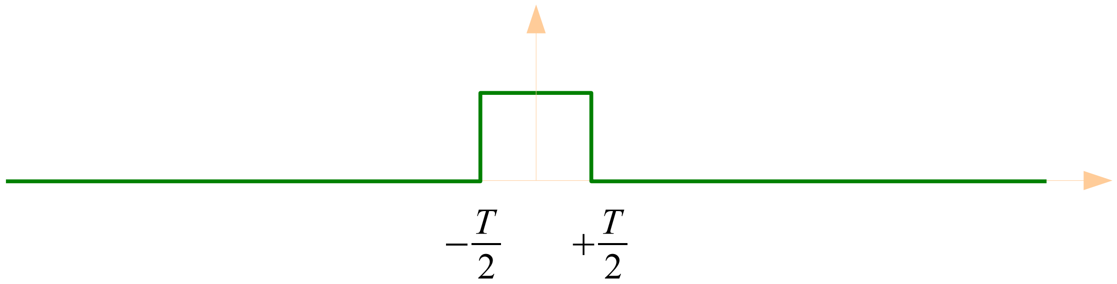


Young Won Lim
6/5/12

CTFT and CTFS

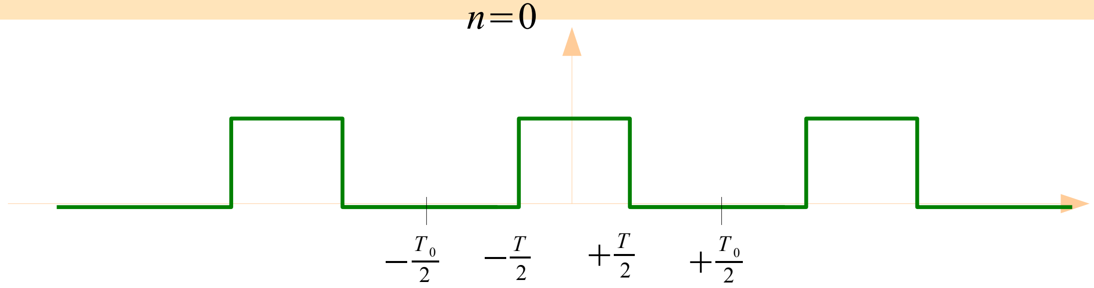
Continuous Time Fourier Transform — Aperiodic Continuous Time Signal

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Longleftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$



Continuous Time Fourier Series — Periodic Continuous Time Signal

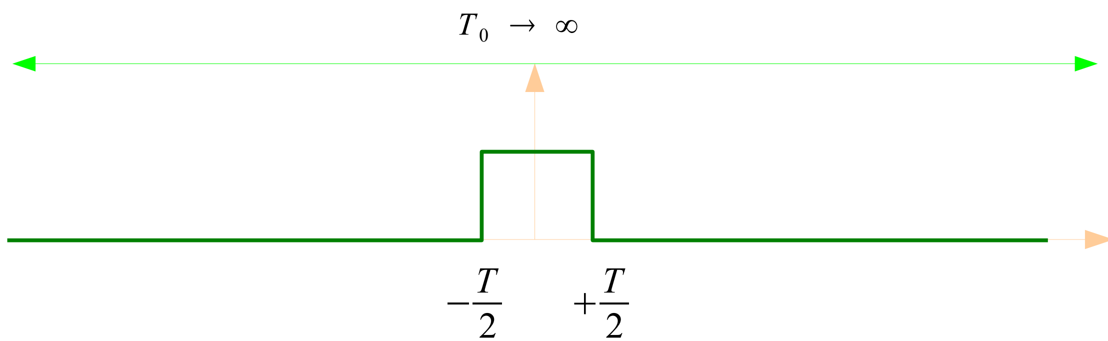
$$C_k = \frac{1}{T}\int_{0}^{T} x(t)\, e^{-jk\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_k\, e^{+jk\omega_0 t}$$



CTFT ← CTFS

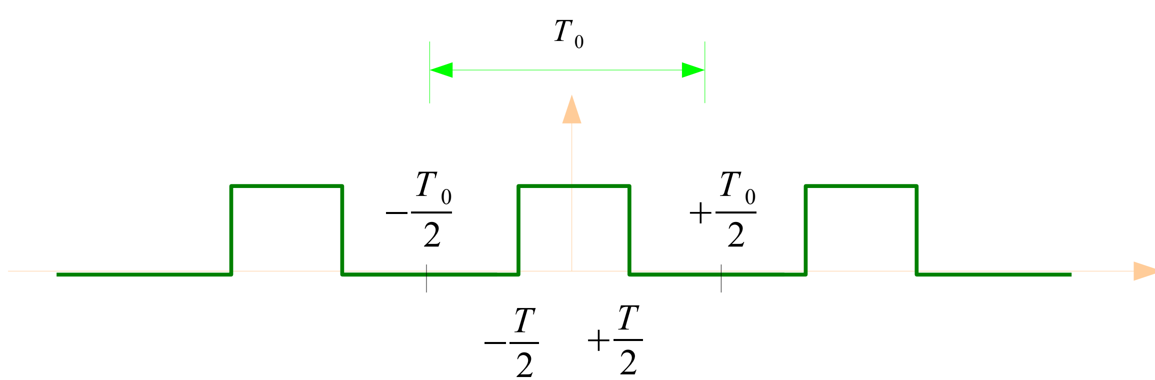
Aperiodic Continuous Time Signal

Continuous Time Fourier Transform



Periodic Continuous Time Signal

Continuous Time Fourier Series



$$x(t)$$

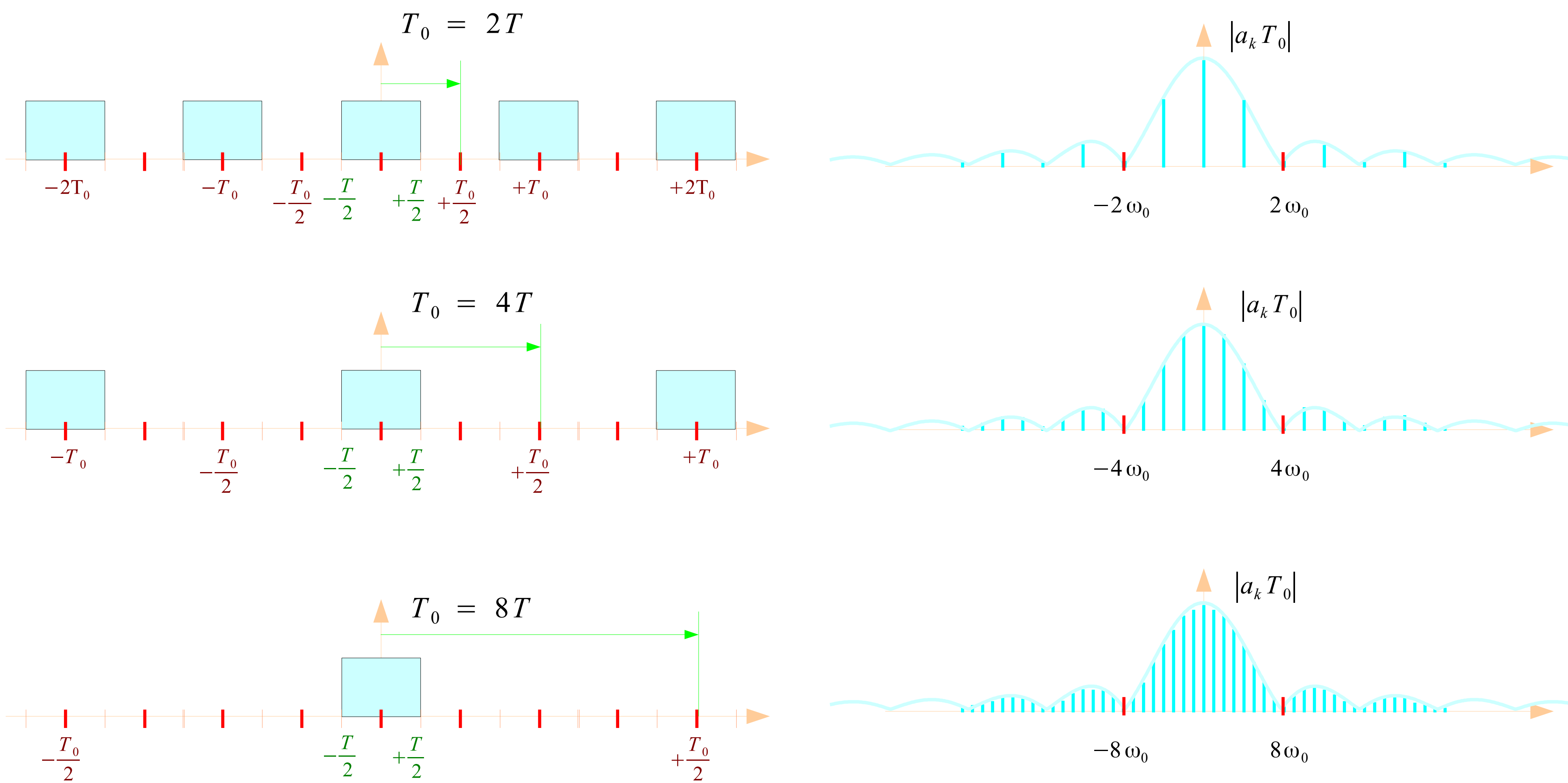
As $T_0 \to \infty$,

$$x_{T_0}(t) \to x(t)$$

$$\omega_0 = \frac{2\pi}{T_0} \to 0$$

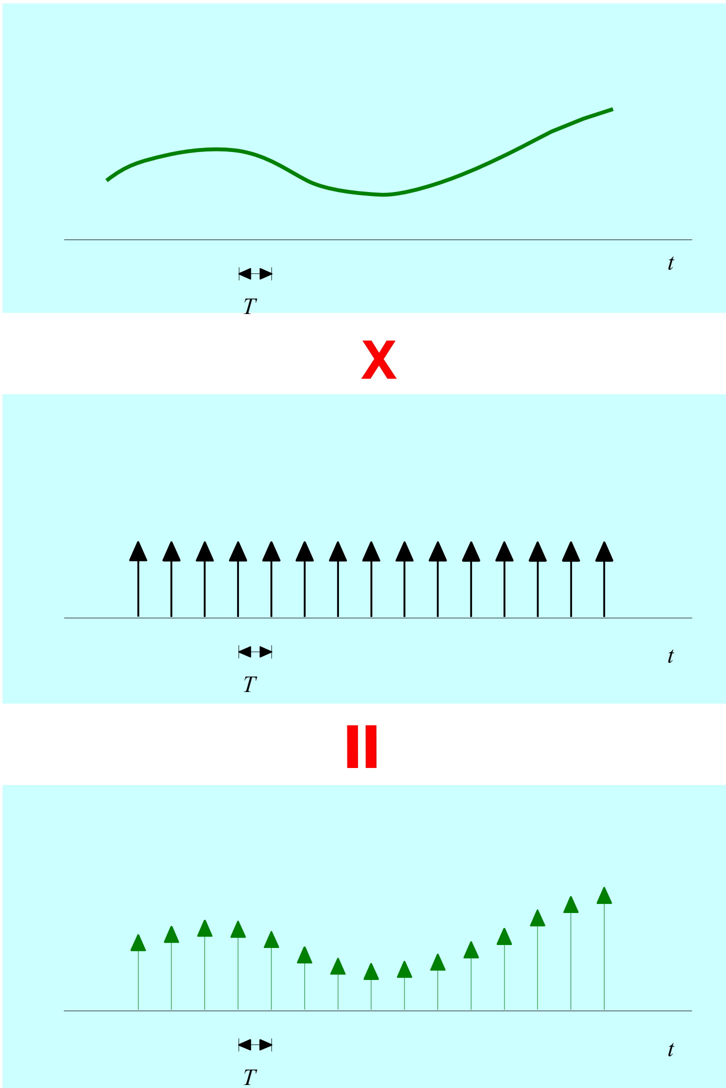
$$x_{T_0}(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0)$$

CTFT and CTFS as $T_0 \to \infty$,

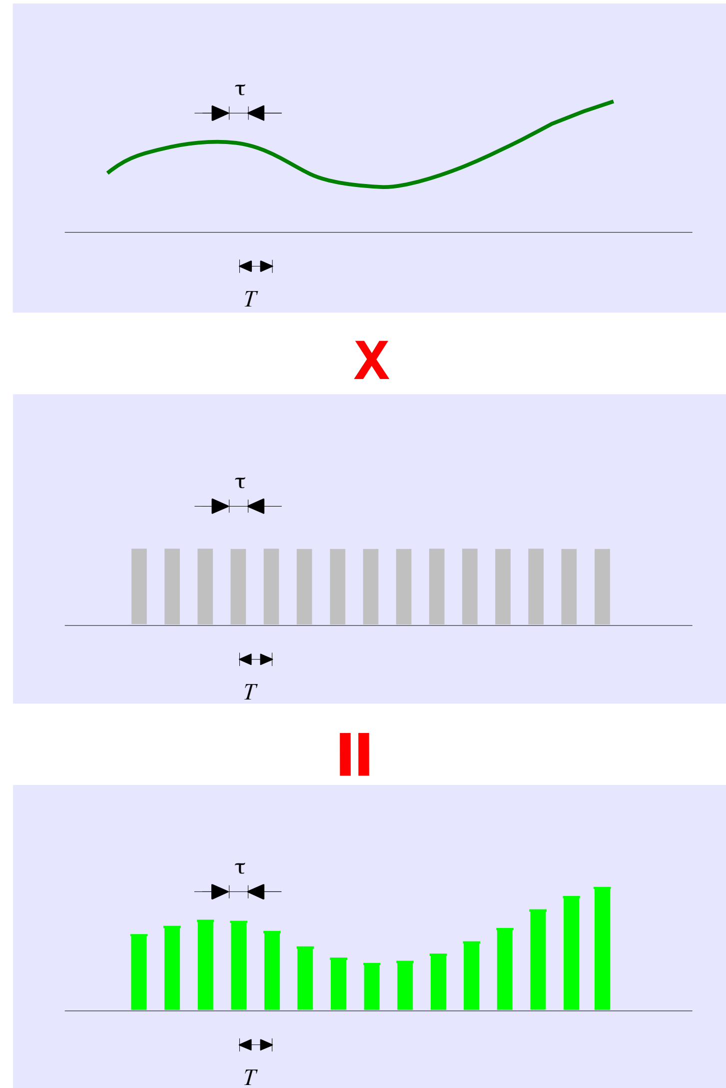


Sampling (1)

Ideal Sampling

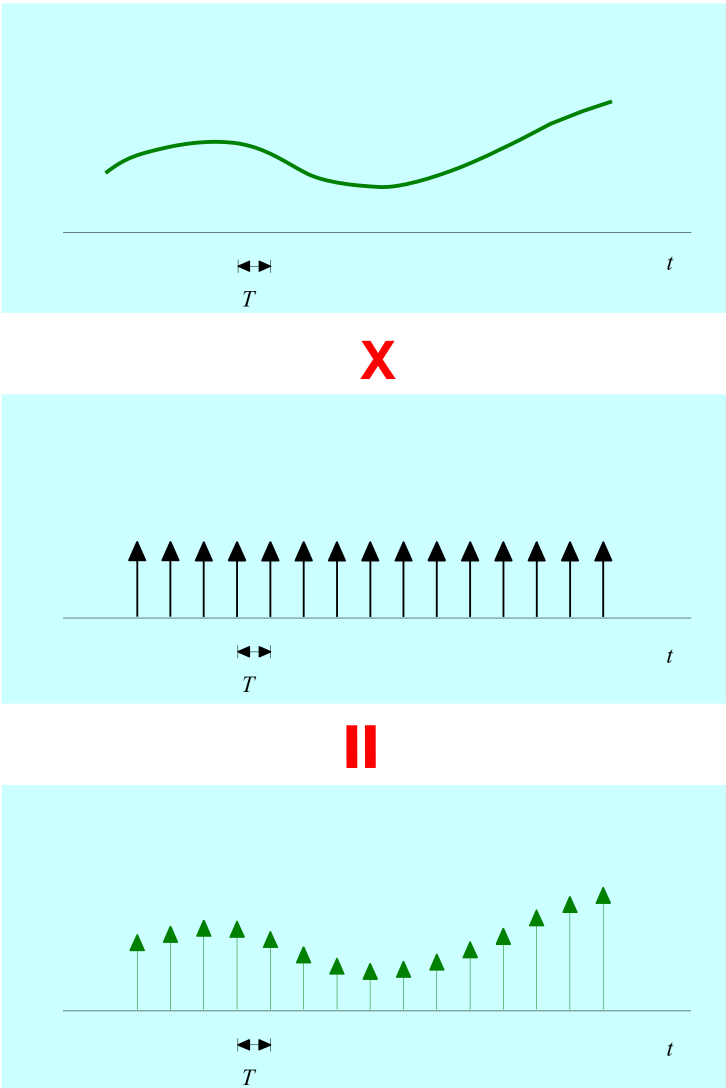


Practical Sampling

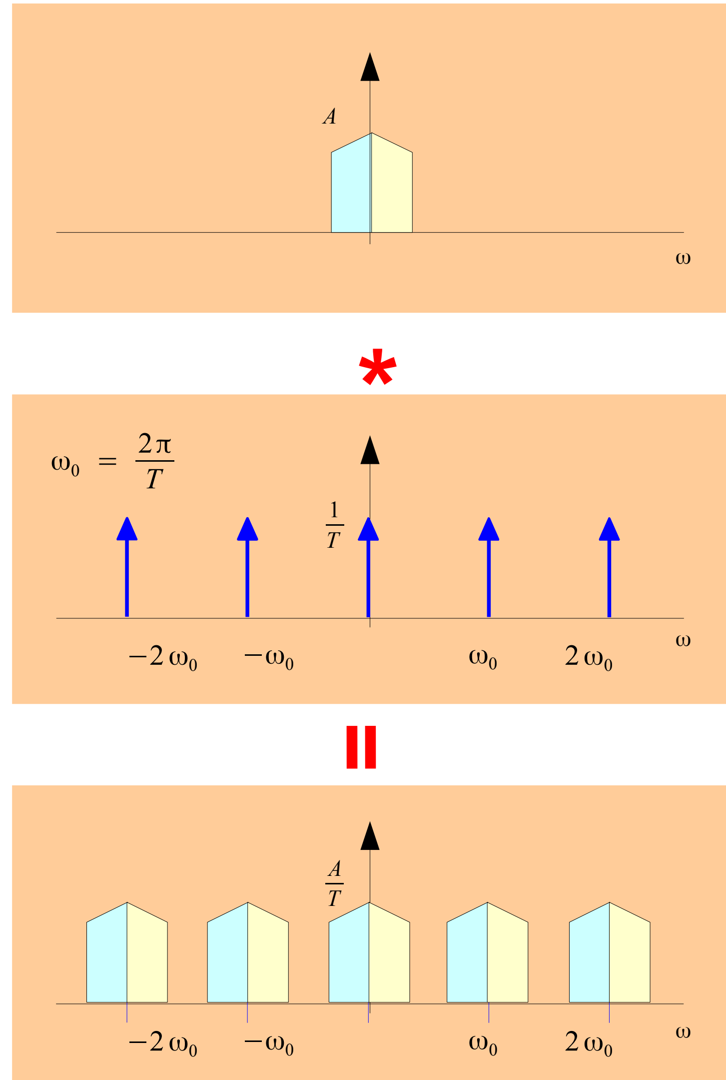


Sampling (2)

Ideal Sampling

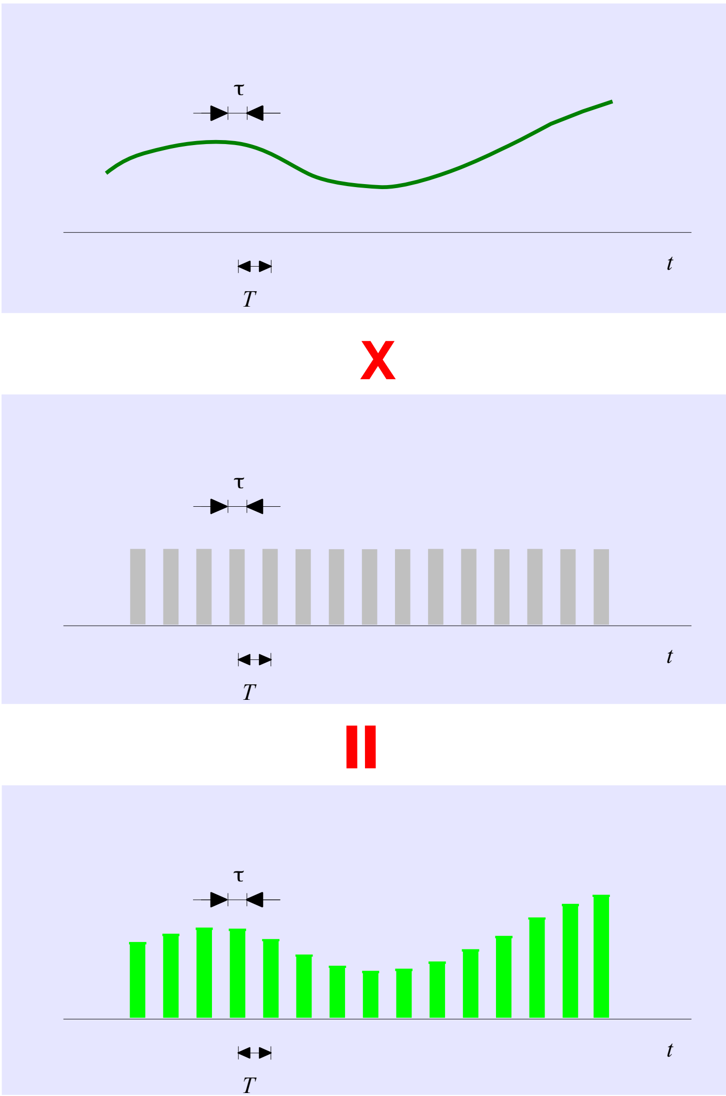


Frequency Domain

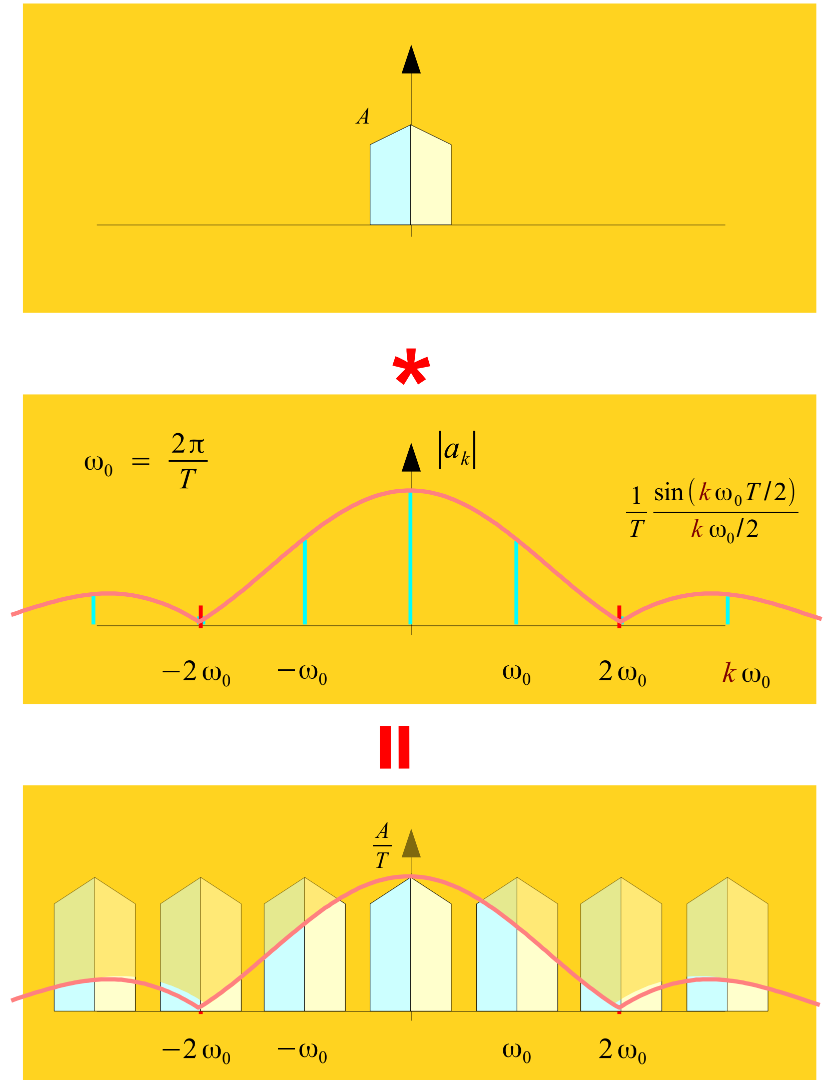


Sampling (3)

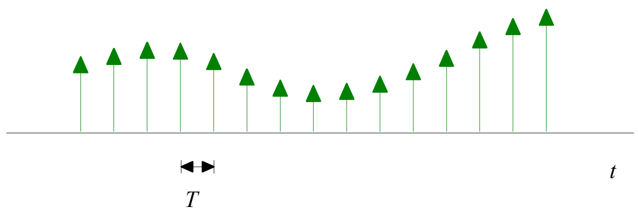
Practical Sampling

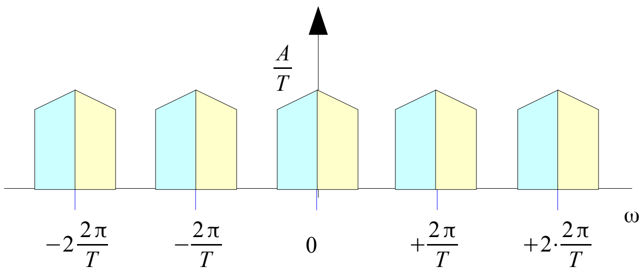


Frequency Domain



CTFT of Sampled Signal





$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

CTFT

$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)\, e^{-j2\pi f t}\, dt$$

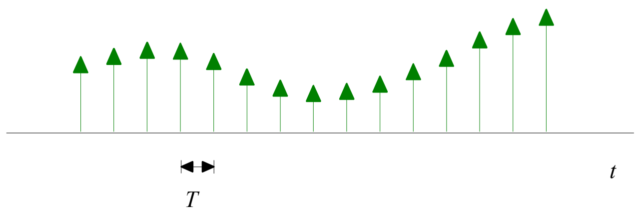
$$= \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

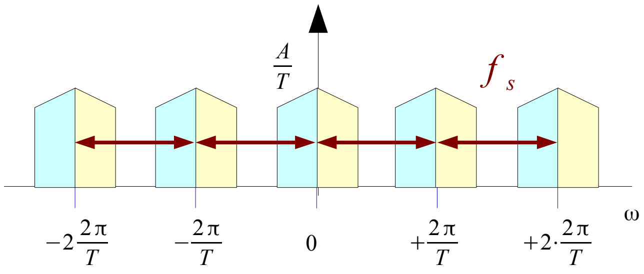
CTFT

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}$$

Periodicity in Frequency



$$f_s = \frac{1}{T} \qquad 2\pi f_s = \frac{2\pi}{T} = \omega_0$$



$$2\pi f = \omega$$

$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

CTFT

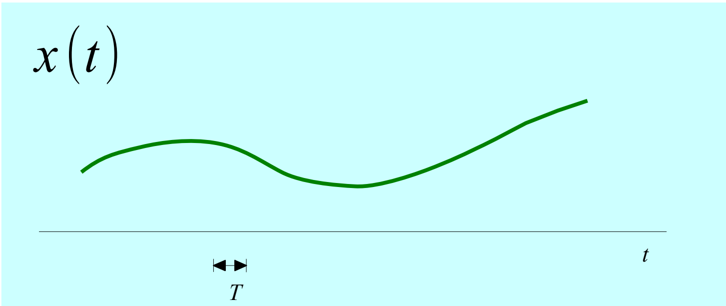
$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\,e^{-j2\pi f T n}$$

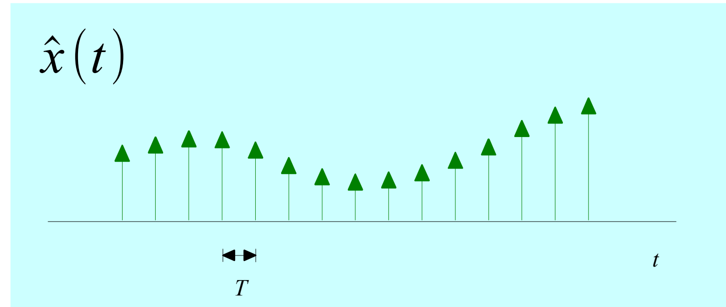
$$e^{-j2\pi(f+f_s)Tn} = e^{-j2\pi(f)Tn} \quad \Leftarrow \quad f_s T = 1$$

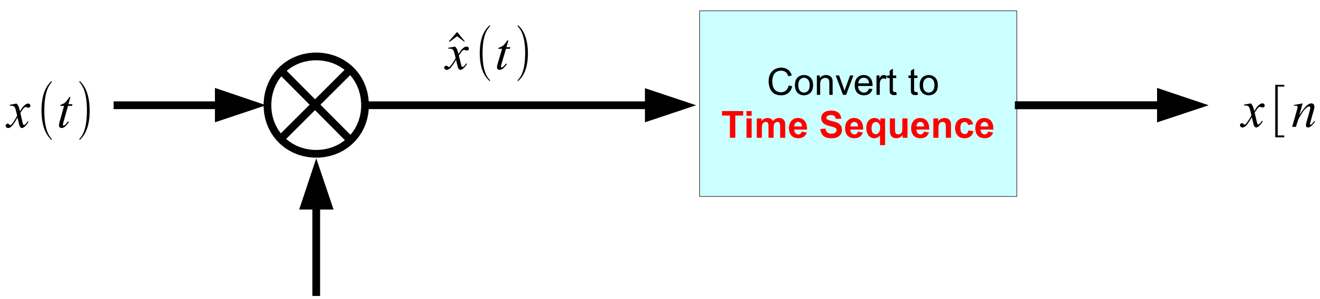
Period = Sampling Frequency f_s

$$\hat{X}(f) = \hat{X}(f+f_s)$$

Time Sequence



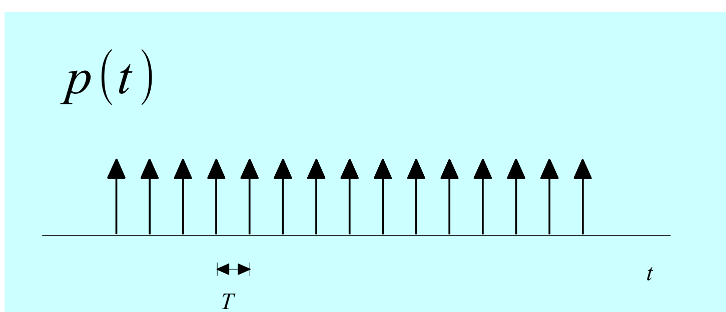


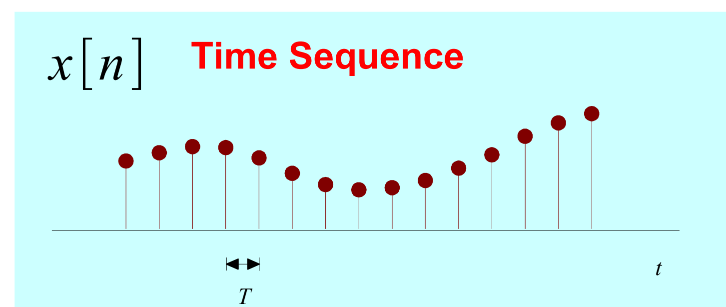


Ideal Sampling

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

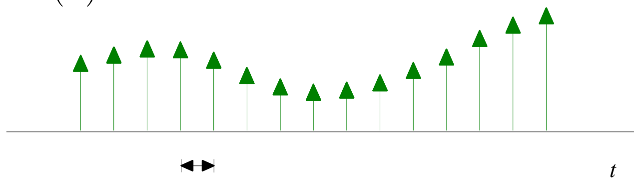
T **Sampling Period**

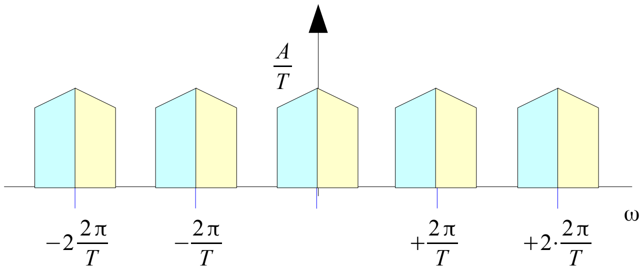




DTFT of a Time Sequence

$\hat{x}(t)$



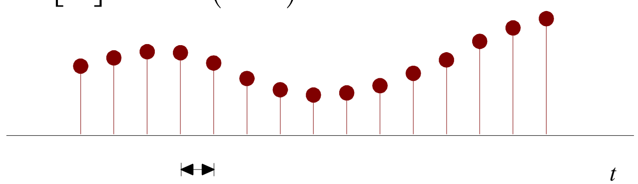


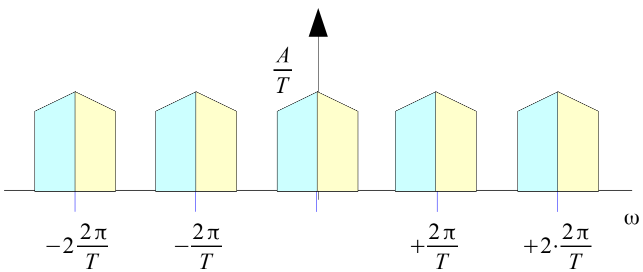
$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

CTFT

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}$$

$x[n] = x(nT)$





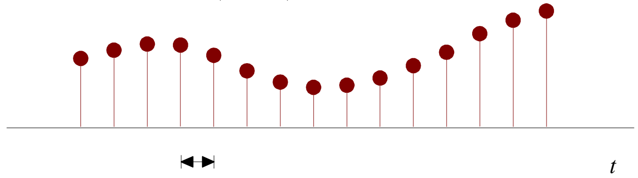
$x[n]$

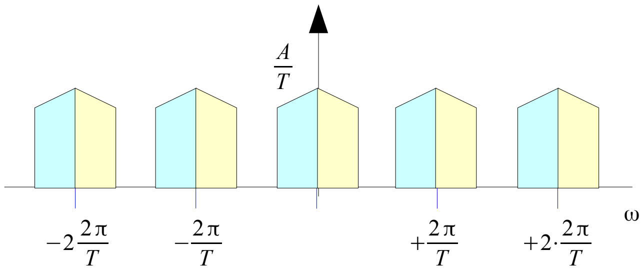
DTFT

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j2\pi f T n}$$

Discrete Time Fourier Transform (1)

$$x[n] = x(nT)$$





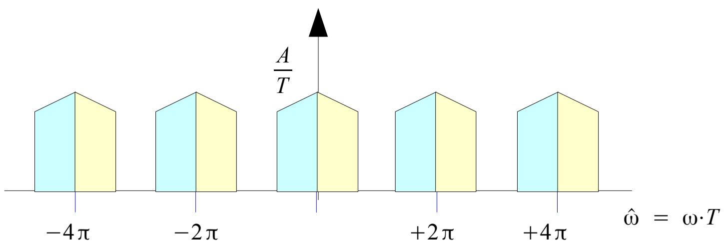
$$x[n]$$

DTFT

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j2\pi f T n}$$

Normalized Angular Frequency

$$2\pi f T = \frac{2\pi f}{1/T} = 2\pi \frac{f}{f_s} = \hat{\omega}$$

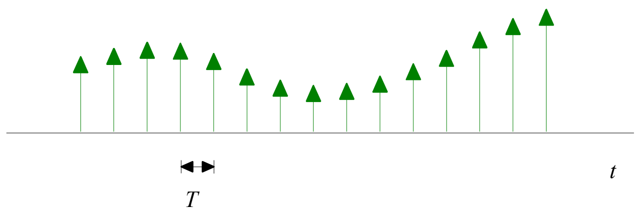


$$x[n]$$

DTFT

$$\hat{X}(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega} n}$$

Discrete Time Fourier Transform (2)



$$f_s = \frac{1}{T}$$

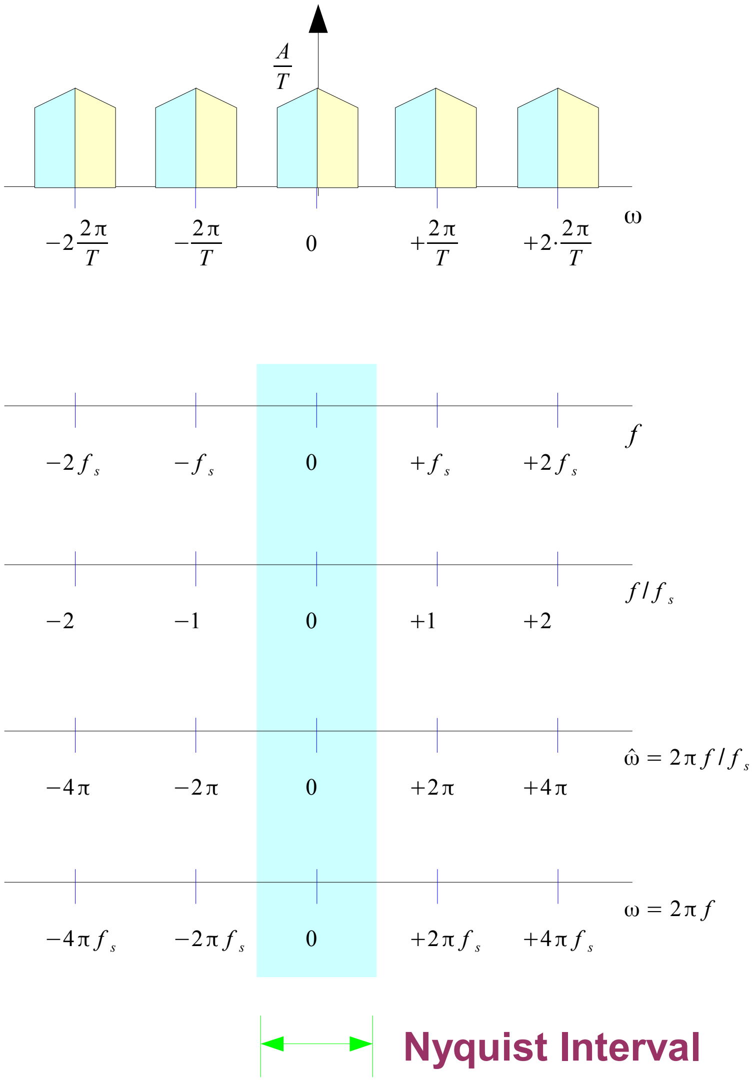
$$2\pi f_s = \frac{2\pi}{T} = \omega_0$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}$$

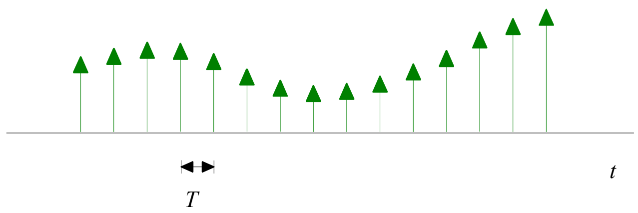
Normalized Angular Frequency

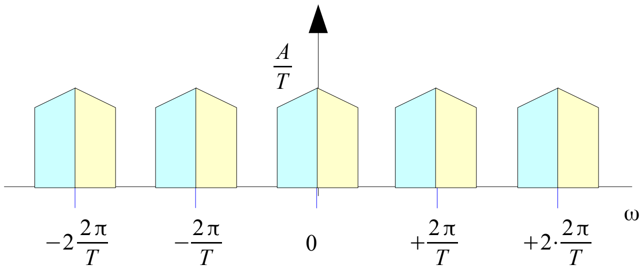
$$2\pi f T = \frac{2\pi f}{1/T} = 2\pi \frac{f}{f_s} = \hat{\omega}$$

$$\hat{X}(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi\hat{\omega} n}$$



Fourier Series





$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

CTFT

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}$$

$$x(nT) = \frac{1}{f_s} \int_{+f_s/2}^{-f_s/2} \hat{X}(f)\, e^{+j2\pi f T n}\, df$$

CTFS

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}$$

$$= \int_{-\pi}^{+\pi} \hat{X}(\omega)\, e^{+j\omega n} \frac{d\omega}{2\pi}$$

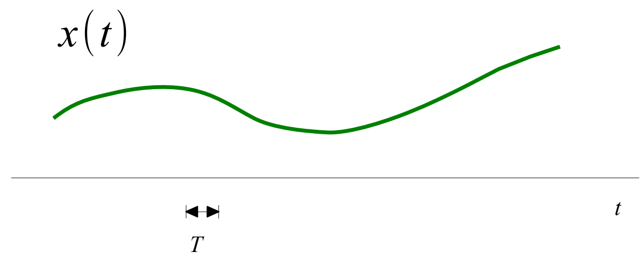
$\hat{X}(f)$ **Continuous Periodic Function**

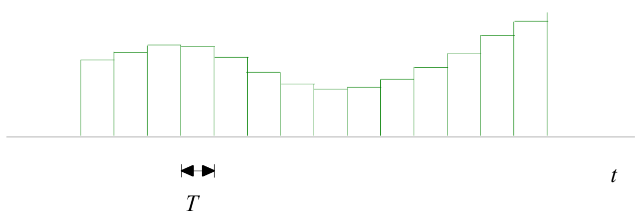
$x(nT)$ **Fourier Series Coefficients**

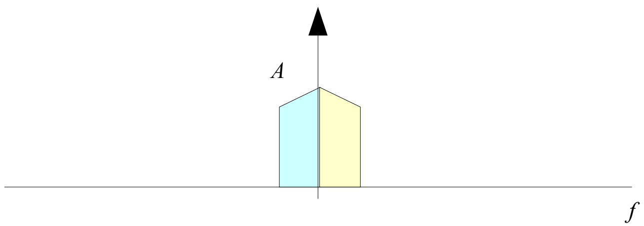
$$\omega = 2\pi f / f_s \qquad \frac{df}{f_s} = \frac{d\omega}{2\pi}$$

Numerical Approximation

$$X(f) = \lim_{T \to 0} T\,\hat{X}(f)$$





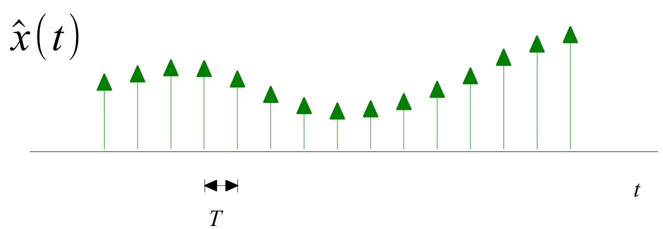


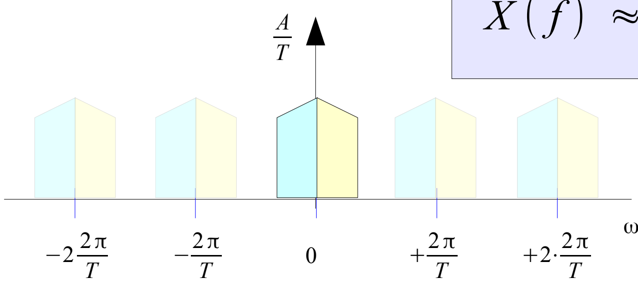
CTFT

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{+j2\pi f t}\, dt$$

$$\approx \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n} \cdot T$$

$$X(f) \approx T\,\hat{X}(f)$$





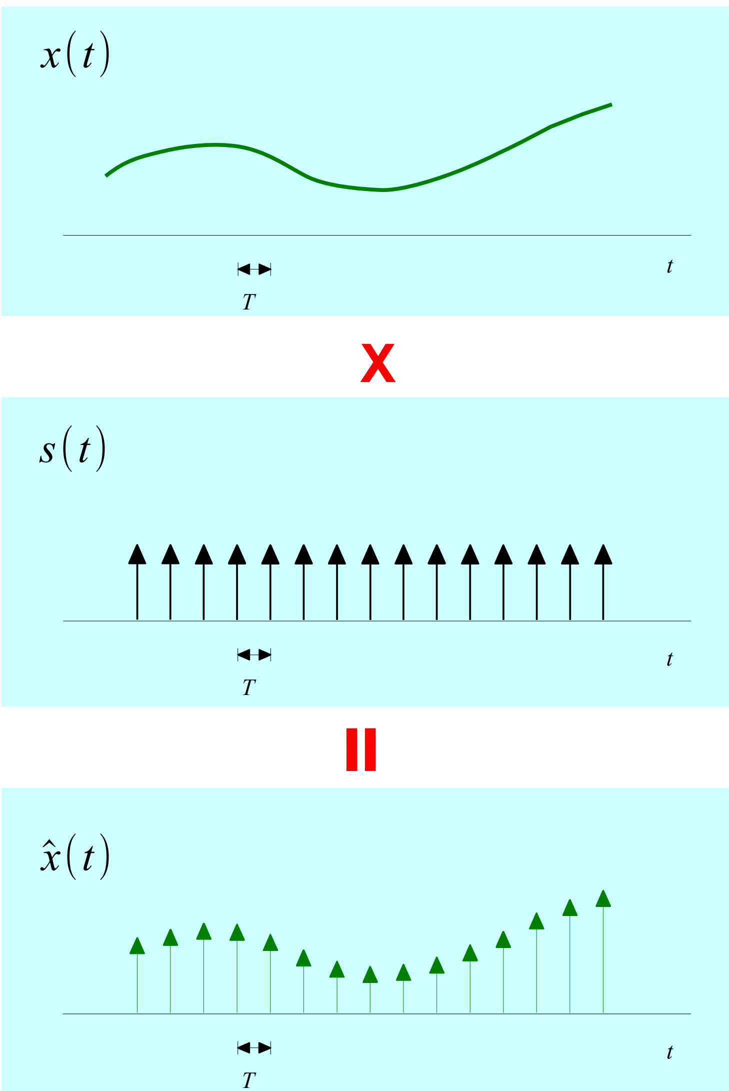
$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t - nT)$$

CTFT

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}$$

Spectrum Replication (1)

Ideal Sampling



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT)$$

$$= \frac{1}{T}\sum_{m=-\infty}^{+\infty} e^{+j2\pi m f_s t}$$

$$\hat{x}(t) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} x(t)\, e^{+j2\pi m f_s t}$$

Shift Property

$$\hat{X}(f) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X(f-m f_s)$$

Spectrum Replication (2)

$$S(f) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} \delta(f - m f_s)$$

Convolution in Frequency

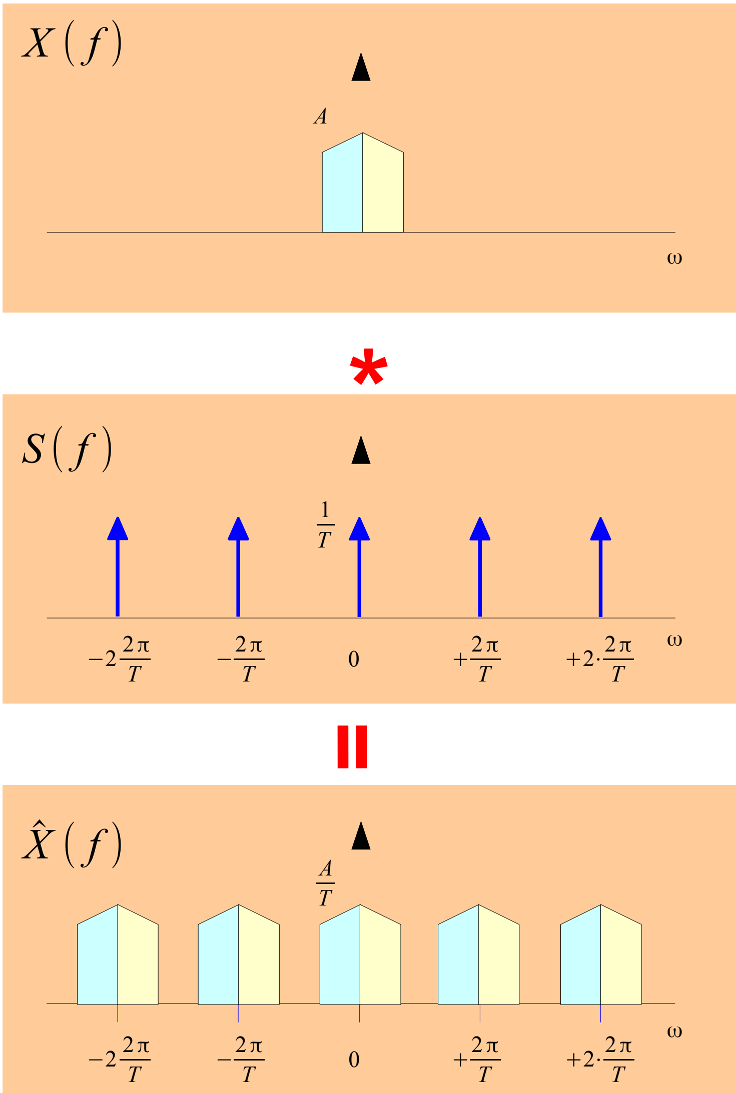
$$\hat{X}(f) = X(f) * S(f)$$

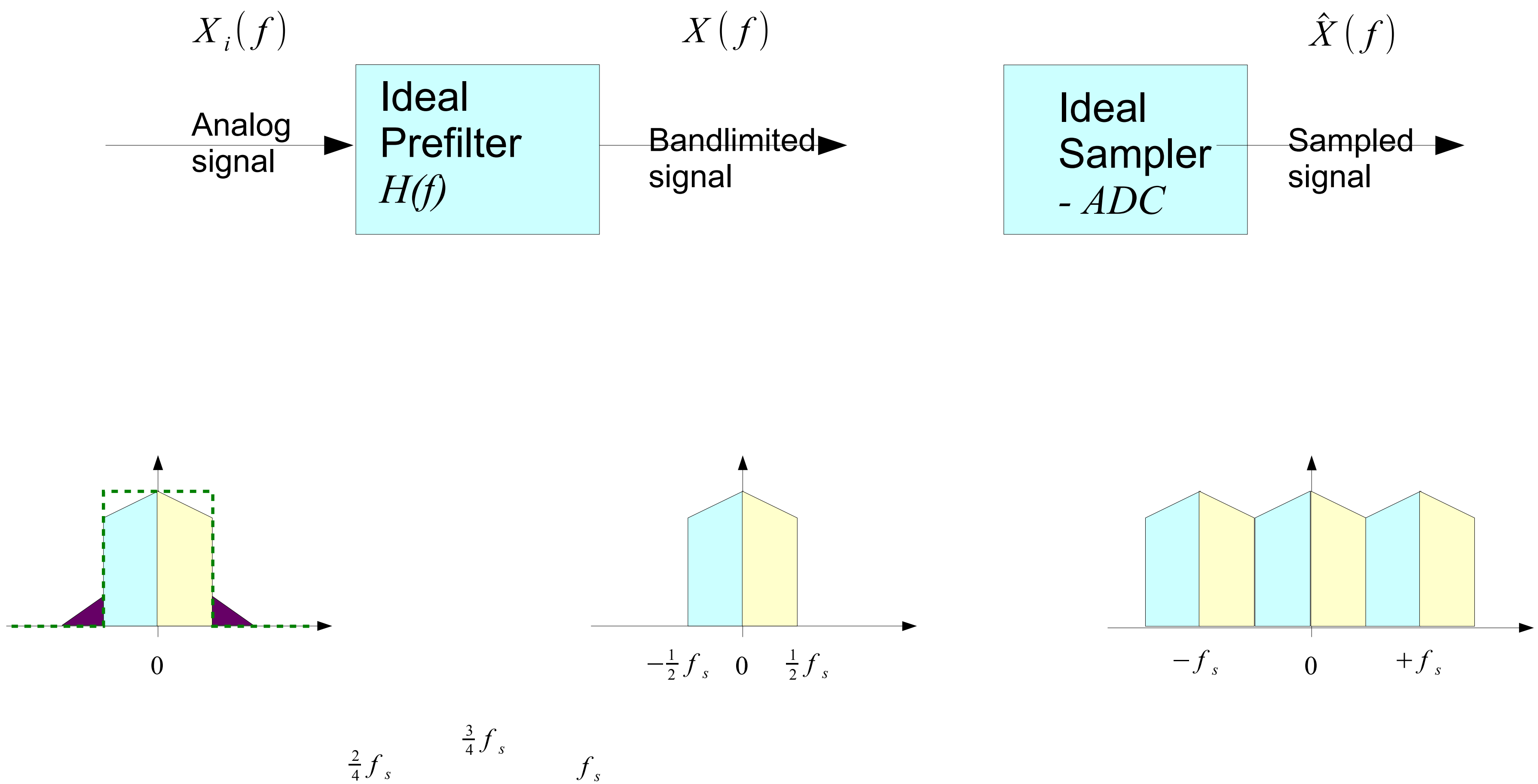
$$= \int_{-\infty}^{+\infty} X(f - f')S(f')\, df'$$

$$= \frac{1}{T}\sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} X(f - f')\delta(f' - m f_s)\, df'$$

$$\hat{X}(f) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X(f - m f_s)$$

Frequency Domain





$X_i(f)$
Analog signal
Ideal Prefilter $H(f)$
$X(f)$
Bandlimited signal
Ideal Sampler - ADC
$\hat{X}(f)$
Sampled signal
0
$-\frac{1}{2}f_s$
0
$\frac{1}{2}f_s$
$-f_s$
0
$+f_s$
$\frac{2}{4}f_s$
$\frac{3}{4}f_s$
f_s

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
[4] R. G. Lyons, Understanding Digital Signal Processing, 1997
[5] AVR121: Enhancing ADC resolution by oversampling
[6] S.J. Orfanidis, Introduction to Signal Processing
www.ece.rutgers.edu/~orfanidi/intro2sp