Problem R2.10

Problem Statement: Consider the same system as in the example p. 7-3, i.e. with the same L2-ODE-CC(4) p. 5-5 and initial conditions (2) p. 3-4, but with the following excitation:

$$r(x) = 7e^{5x} - 2x^2$$

Given Equation: $y'' - 10y' + 25y = r(x)$

Homogenous Solution: replacing y's with lambdas and setting $r(x)=0$.

$$\lambda^2 - 10\lambda + 25 = 0$$

factoring...

$$(\lambda - 5)^2$$

$\lambda = 5$ (real double root)

using the characteristic form of the solution...

$$C_1 e^{5x} + C_2 x e^{5x} = y(h)$$

Now working towards the particular solutions...

$$y(p_1) = Cx^2 e^{5x}$$

$$y(p_2) = K_2 x^2 + K_1 x + K_0$$

$$y'(p_1) = C2xe^{5x} + C5x^2 e^{5x} = 2Cxe^{5x} + 5Cx^2 e^{5x}$$

$$y''(p_1) = 2Ce^{5x} + 10Cxe^{5x} + 10Cxe^{5x} + 25Cx^2 e^{5x}$$

solving for C

$$2Ce^{5x} + 20Cxe^{5x} + 25Cx^2 e^{5x} - 20Cxe^{5x} - 50Cx^2 e^{5x} + 25Cx^2 e^{5x} = 7e^{5x}$$

$$2C = 7$$

$$C = 7/2$$

$y(p_2) = K_2x^2 + K_1x + K_0$

$y'(p_2) = 2K_2x + K_1$

$y''(p_2) = 2K_2$

$$2K_2 - 20K_2x - 10K_1 + 25K_2x^2 + 25K_1x + 25K_0 = -2x^2$$

setting x^2 terms $= -2x^2$

$25K_2x^2 = -2x^2$

$K_2 = \frac{-2}{25}$

setting x^1 terms $= 0$

$-20K_2x + 25K_1x = 0$

setting $x^0 = 0$

$2K_2 - 10K_1 + 25K_0 = 0$

solving for K_1...

$\frac{40}{25} + 25K_1 = 0$

$K_1 = \frac{-40}{625}$

solving for K_0...

$2K_2 - 10K_1 + 25K_0 = 0$

$-\frac{4}{25} + \frac{400}{625} + 25K_0 = 0$

$25K_0 = -.48$

$K_0 = -.0192$ or $\frac{12}{625}$

$y(p_2) = \frac{-2}{25}x^2 - \frac{40}{625}x - .0192$

$y(p_1) = \frac{7}{2}x^2e^{5x}$

$y(h) = C_1e^{5x} + C_2xe^{5x}$

$y(gen) = C_1e^{5x} + C_2xe^{5x} - \frac{2}{25}x^2 - \frac{40}{625}x - .0192 + \frac{7}{2}x^2e^{5x}$

evaluating at initial conditions...

$y(0) = 4$

$C_1 + 0 - 0 - 0 - .0192 + 0 = 4$

$C_1 = 4.0192$

$y'_{(gen)} = 5C_1e^{5x} + C_2e^{5x} + 5C_2xe^{5x} - .16x - \frac{40}{625} + \frac{7}{2}xe^{5x} + \frac{35}{2}x^2e^{5x}$

$y'(0) = -5$

$y'(0) = 5(4.0192)e^{5(0)} + C_2e^{5(0)} + 5C_2(0)e^{5(0)} - .16(0) - \frac{40}{625} + \frac{7}{2}(0)e^{5(0)} + \frac{35}{2}(0)e^{2\,5(0)}$

$-5 = 20.096 + C_2 + 0 - 0 + 0 + 0$

$-5 = 20.096 + C_2$

$C_2 = -25.096$

$y(h) = 4.0192e^{5x} - 25.096xe^{5x}$