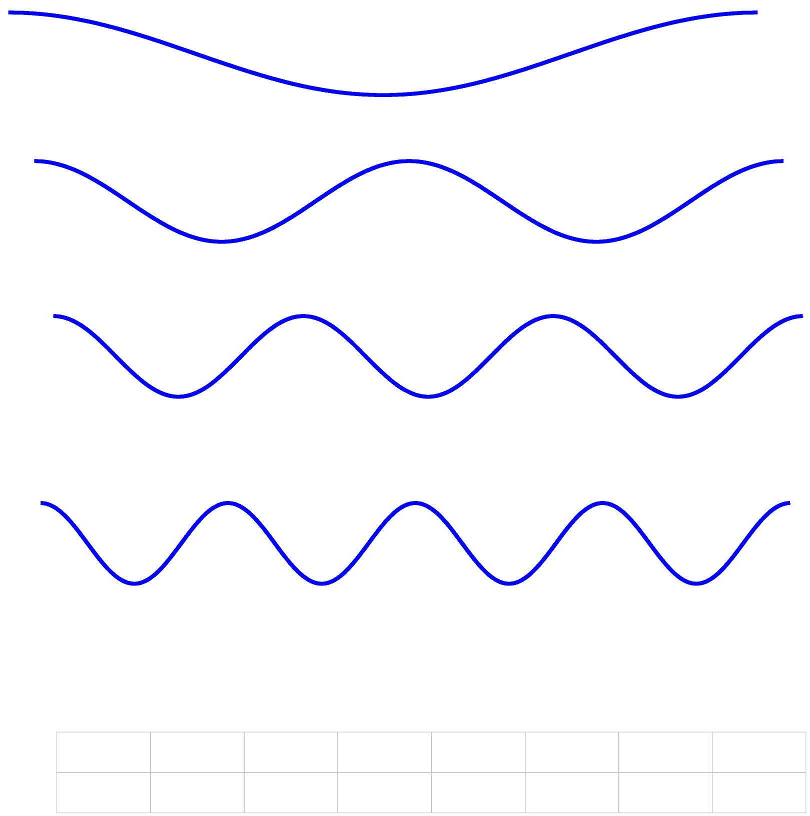
UnderSampling (2B)

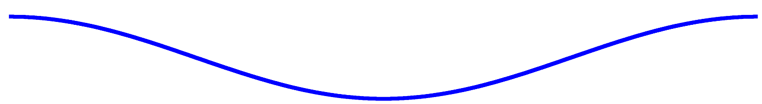
-
-

Please send corrections (or suggestions) to youngwlim@hotmail.com.

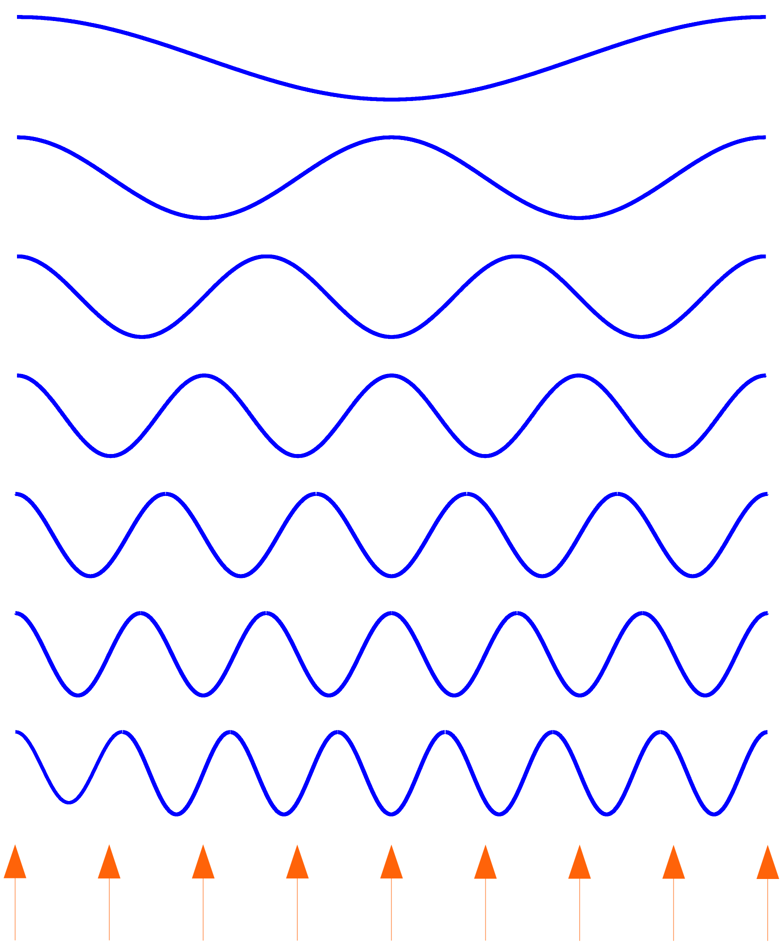
This document was produced by using OpenOffice and Octave.

Measuring Rotation Rate





Signals with Harmonic Frequencies (1)



1 Hz *1 cycle / sec*
$$\cos(1 \cdot 2\pi t) = \frac{e^{+j(1\cdot 2\pi)t} + e^{-j(1\cdot 2\pi)t}}{2}$$

2 Hz *2 cycles / sec*
$$\cos(2 \cdot 2\pi t) = \frac{e^{+j(2\cdot 2\pi)t} + e^{-j(2\cdot 2\pi)t}}{2}$$

3 Hz *3 cycles / sec*
$$\cos(3 \cdot 2\pi t) = \frac{e^{+j(3\cdot 2\pi)t} + e^{-j(3\cdot 2\pi)t}}{2}$$

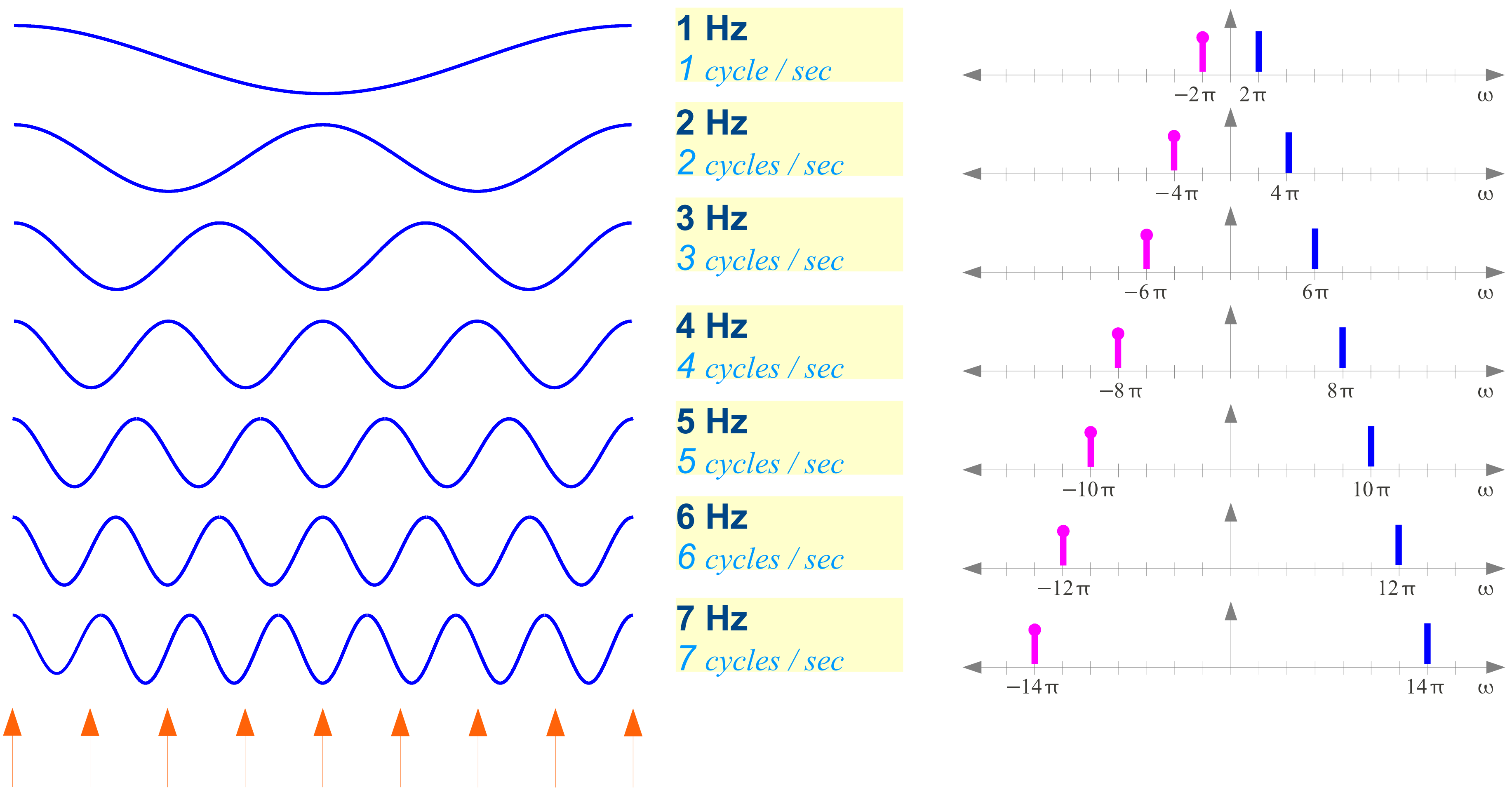
4 Hz *4 cycles / sec*
$$\cos(4 \cdot 2\pi t) = \frac{e^{+j(4\cdot 2\pi)t} + e^{-j(4\cdot 2\pi)t}}{2}$$

5 Hz *5 cycles / sec*
$$\cos(5 \cdot 2\pi t) = \frac{e^{+j(5\cdot 2\pi)t} + e^{-j(5\cdot 2\pi)t}}{2}$$

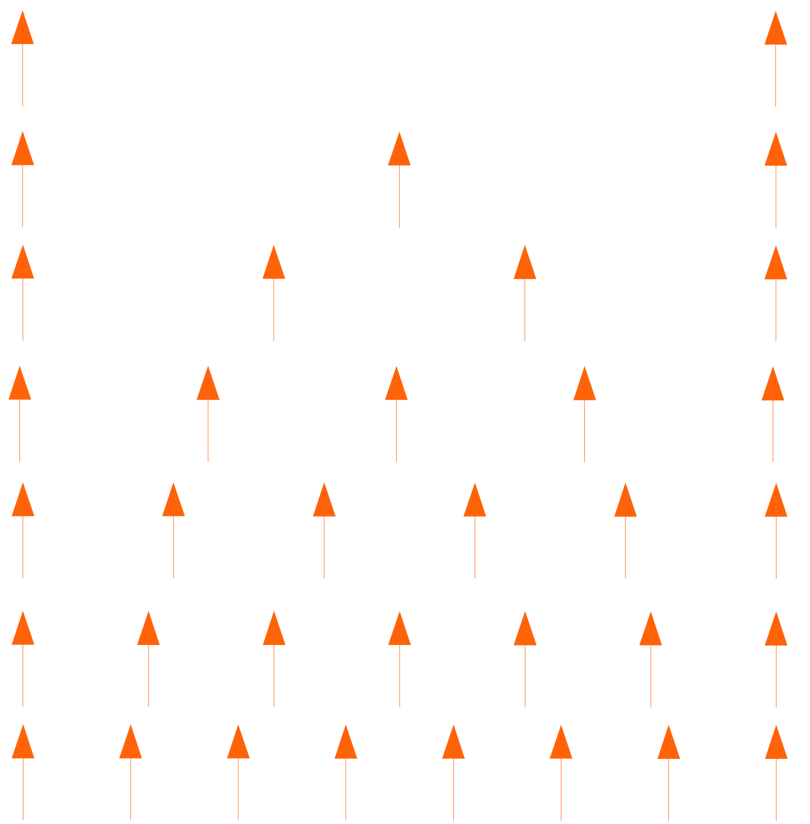
6 Hz *6 cycles / sec*
$$\cos(6 \cdot 2\pi t) = \frac{e^{+j(6\cdot 2\pi)t} + e^{-j(6\cdot 2\pi)t}}{2}$$

7 Hz *7 cycles / sec*
$$\cos(7 \cdot 2\pi t) = \frac{e^{+j(7\cdot 2\pi)t} + e^{-j(7\cdot 2\pi)t}}{2}$$

Signals with Harmonic Frequencies (2)



Sampling Frequency



1 Hz
1 sample / sec

2 Hz
2 samples / sec

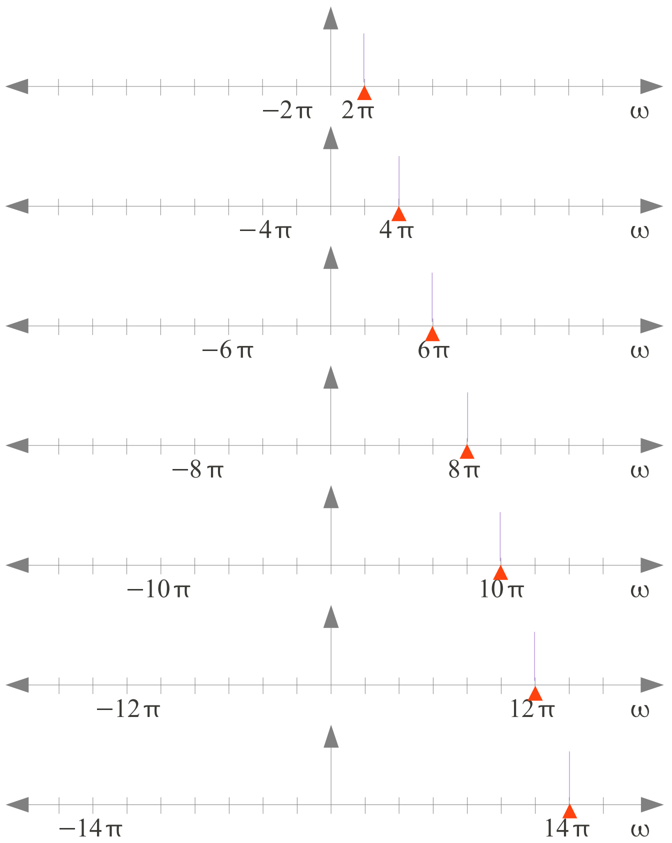
3 Hz
3 samples / sec

4 Hz
4 samples / sec

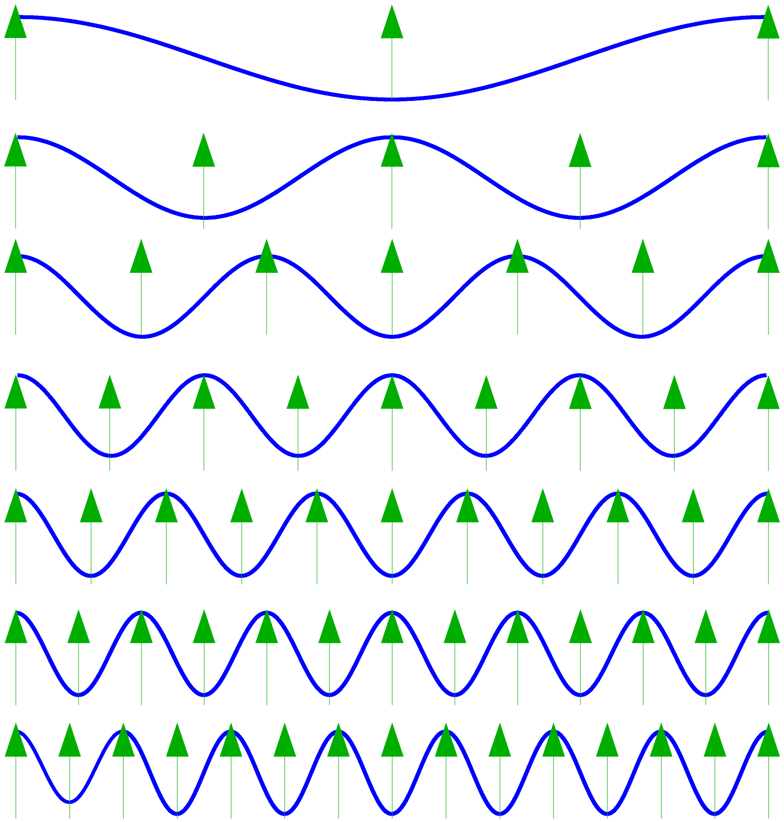
5 Hz
5 samples / sec

6 Hz
6 samples / sec

7 Hz
7 samples / sec

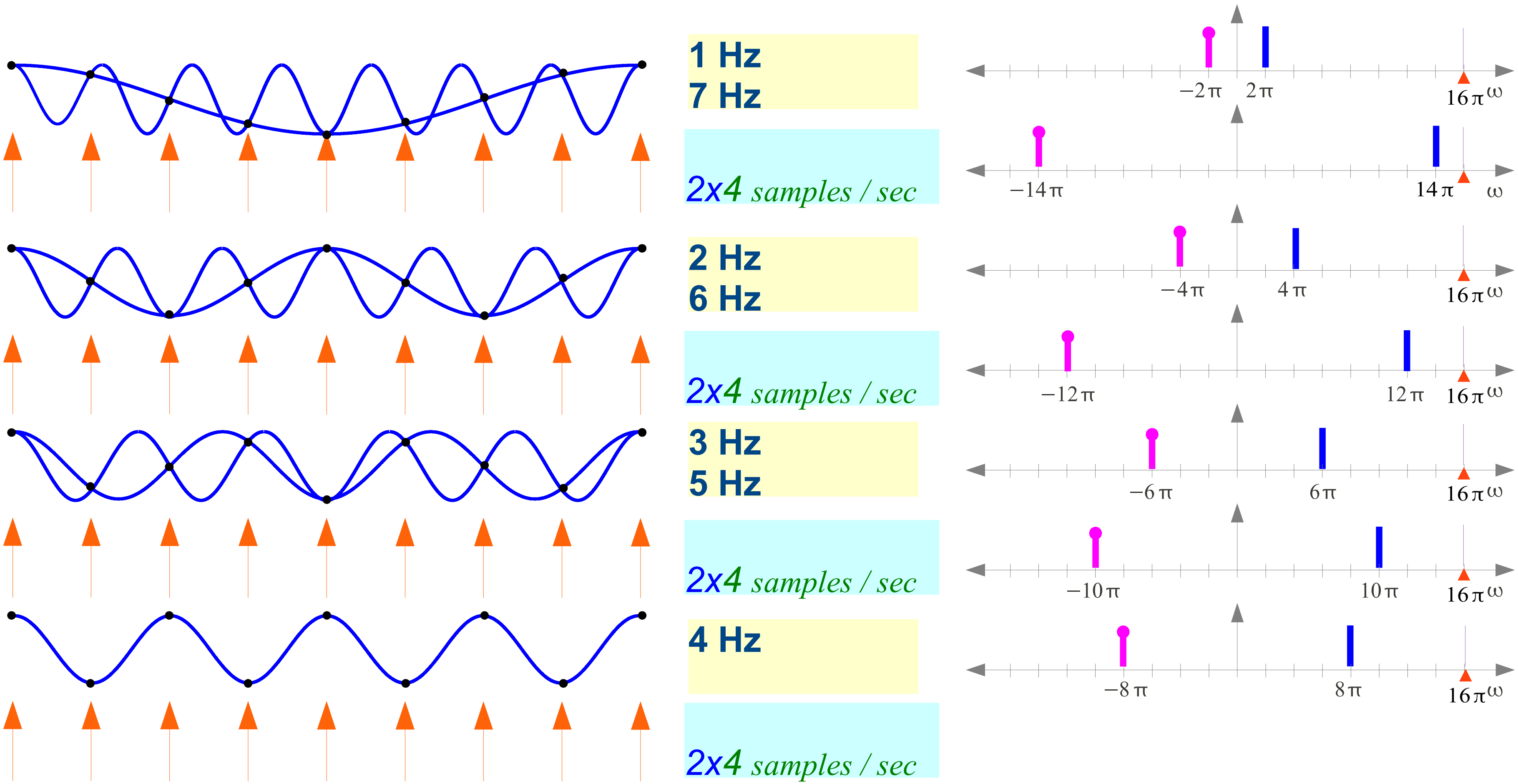


Nyquist Frequency



1 Hz 1 *cycle / sec*	2x1 *sample / sec*
2 Hz 2 *cycles / sec*	2x2 *samples / sec*
3 Hz 3 *cycles / sec*	2x3 *samples / sec*
4 Hz 4 *cycles / sec*	2x4 *samples / sec*
5 Hz 5 *cycles / sec*	2x5 *samples / sec*
6 Hz 6 *cycles / sec*	2x6 *samples / sec*
7 Hz 7 *cycles / sec*	2x7 *samples / sec*

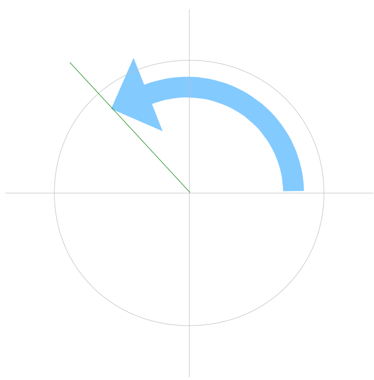
Aliasing



1 Hz
7 Hz
2x4 samples / sec
2 Hz
6 Hz
2x4 samples / sec
3 Hz
5 Hz
2x4 samples / sec
4 Hz
2x4 samples / sec
−2π
2π
16π
ω
−14π
14π
ω
−4π
4π
16π
ω
−12π
12π
16π
ω
−6π
6π
16π
ω
−10π
10π
16π
ω
−8π
8π
16π
ω

Sampling

$$\omega_s = 2\pi f_s \ (rad/sec)$$



$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \ (rad/sec)$$

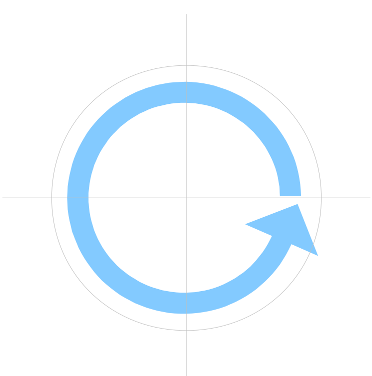
$$f_1 = \frac{f_s}{2} \ (rad/sec)$$

$$\omega_2 = 2\pi f_2$$

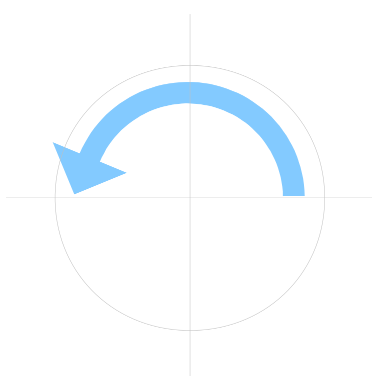
$$\omega_2 = -\frac{\omega_s}{2} \ (rad/sec)$$

$$f_2 = -\frac{f_s}{2} \ (rad/sec)$$

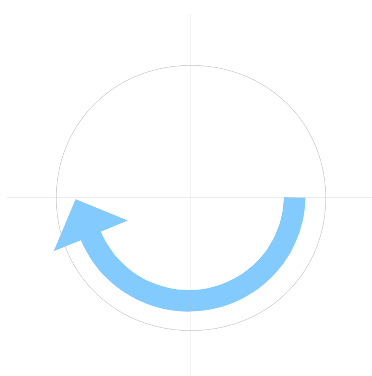
$$2\pi \ (rad) \ / \ T_s \ (sec)$$



$$\pi \ (rad) \ / \ T_s \ (sec)$$

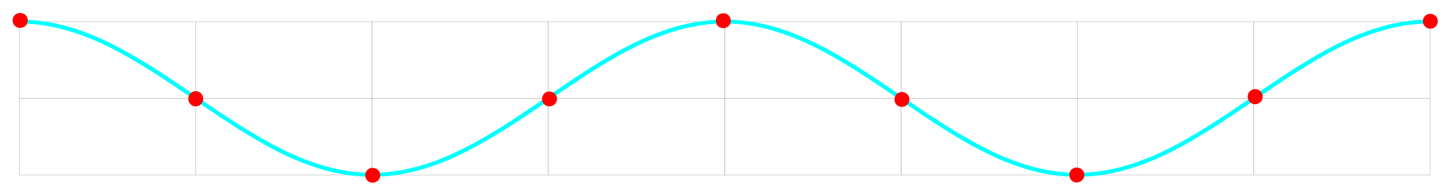


$$-\pi \ (rad) \ / \ T_s \ (sec)$$

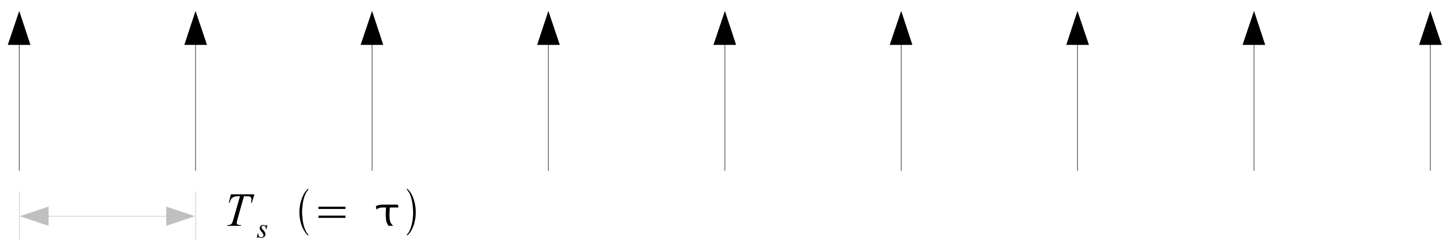


Sampling

$\omega_1 = 2\pi f_1 \; (rad/sec)$

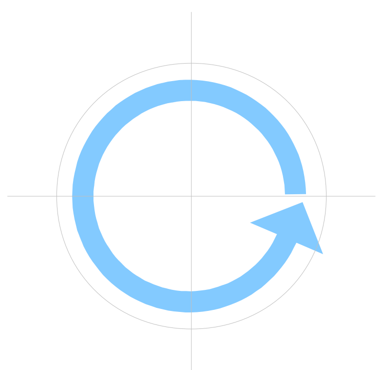


$\omega_s = 2\pi f_s \; (rad/sec)$

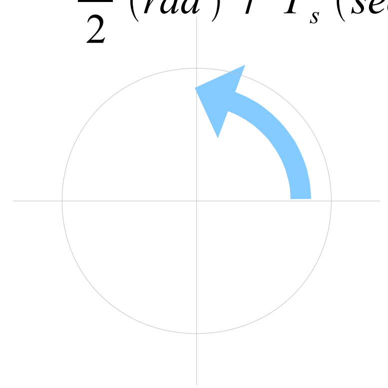


$T_s \; (= \tau)$

$2\pi \, (rad) \; / \; T_s \, (sec)$



$\frac{\pi}{2} \, (rad) \; / \; T_s \, (sec)$



For the period of T_s

Angular displacement $\frac{\pi}{2} \, (rad)$

$$
\begin{aligned}
\hat{\omega} &= \omega \cdot T_s \;\; (rad) \\
&= 2\pi f_1 \cdot T_s \;\; (rad) \\
&= 2\pi \frac{f_s}{4} \cdot T_s \;\; (rad) \\
&= \frac{\pi}{2} \;\; (rad)
\end{aligned}
$$

Angular Frequencies in Sampling

continuous-time signals

Signal Frequency

$$f_0 = \frac{1}{T_0}$$

Signal Angular Frequency

$$\omega_0 = 2\pi f_0 \quad (rad/sec)$$

sampling sequence

Sampling Frequency

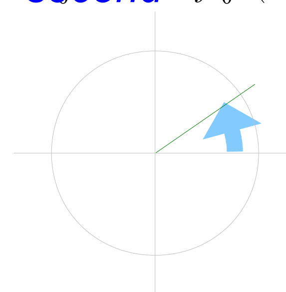
$$f_s = \frac{1}{T_s}$$

Sampling Angular Frequency

$$\omega_s = 2\pi f_s \quad (rad/sec)$$

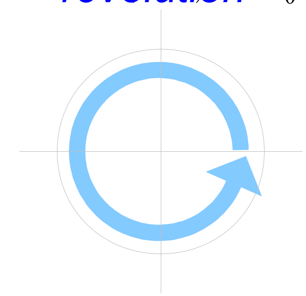
For 1 second

$\omega_0 = 2\pi f_0 \ (rad/sec)$



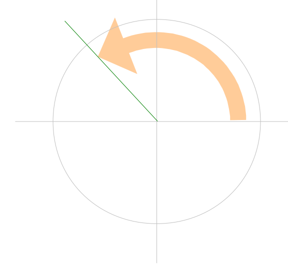
For 1 revolution

$2\pi \ (rad) \ / \ T_0 \ (sec)$



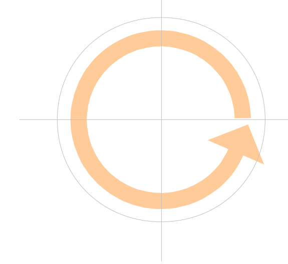
For 1 second

$\omega_s = 2\pi f_s \ (rad/sec)$



For 1 revolution

$2\pi \ (rad) \ / \ T_s \ (sec)$



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann