Mtg 28: Mon, 30 Oct 06 + 5min
(Lect 4)

Plan:
- Eigenvalue pb: Rigid-body modes, mechanisms
- Beams: codes, distributed load, superposition.

Eval pbs:

Appl (previously seen):

- Vibrations $\left(\underbrace{\text{vib. freq}}_{\text{evals.}}\text{. \& mode } \underbrace{\text{shapes}}_{\text{eigenvectors (evects)}}\right)$
- Control: Stability.
- FEAD: Rigid-body modes, mechanisms (Stability)
- IEA: math

$\underset{n\times n}{\underline{A}}$, eval pb: $\boxed{\underline{A}\,\underline{x} = \lambda\,\underline{x}}$

standard eval pb.

Pam:
Philip }

Eq. of motion

$$\boxed{\underset{n\times n}{\underline{M}}\,\underset{n\times 1}{\underline{\ddot{d}}} + \underset{n\times n}{\underline{C}}\,\underset{n\times 1}{\underline{\dot{d}}} + \underset{n\times n}{\underline{K}}\,\underset{n\times 1}{\underline{d}} = \underset{n\times 1}{\underline{F}}(t)} \quad (1)$$

$\underline{M}$ = mass matrix

$\underline{C}$ = damping mat.

$\underline{K}$ = stiffness mat.

$\underline{d}(t)$ = disp. dof's.

gen. disp (include rot's for beams)

Philip (w/ note): $\det(\lambda \underline{M} - \underline{K}) = 0$

(Russell: $\det(\lambda \underline{I} - \underline{A}) = 0$)

$$\boxed{\underline{K}\,\underline{x} = \lambda\,\underline{M}\,\underline{x}} \quad (2)$$

Generalized eval. pb.

Standard. eval pb for $\underline{A}$:

$$\boxed{\underline{A}\,\underline{x} = \lambda\,\underbrace{\underline{I}\,\underline{x}}_{\underline{x}}} \quad (3)$$

shawn: Solving <u>homog.</u> diff. eq.

($\underline{F} = \underline{0} \Rightarrow$ Free vib. pb.)

\+ undamped : ignore damping term $\underline{C}\,\dot{\underline{d}}$

Undamped free vib. :

$$\boxed{\underline{M}\,\ddot{\bar{\underline{d}}} + \underline{K}\,\bar{\underline{d}} = \underline{0}} \qquad (1)$$

<u>Goal</u>: Try to find a <u>basis of vectors</u> to <u>uncouple</u> the <u>syst. of coupled diff. eqs</u> (matrix eq. of motion) Eq. (1) on p. 28-2.

Method to find such basis: Eval pb Eq. (2) on p. 28-~~3~~2, which came from Eq. (1) on p. 28-3 (undamped free vib. pb.)

How? Assume $\bar{\underline{d}}(t)$ periodic in t (actually sinusoidal)

$$\underset{n\times 1}{\underline{d}}(t) = (A\cos\underset{\uparrow}{\omega} t + B\sin\underset{\uparrow}{\omega} t)\,\underset{n\times 1}{\underline{\phi}}$$

rel. to eval. (eigenvect)

Sam: Just plug into Eq. (1), p. 28.3:

$$\ddot{\underline{d}}(t) = -\omega^2 \underbrace{(A\cos\omega t + B\sin\omega t)\,\underline{\phi}}_{\underline{d}}$$

(Philip: Plug in init. value & solve. Not yet!)

$$\Rightarrow \boxed{\ddot{\underline{d}} = -\omega^2\,\underline{d}}$$

Eq. (1), p. 28.3: $-\omega^2\,\underline{M}\,\underline{d} + \underline{K}\,\underline{d} = 0$

$$\Rightarrow \underline{K}\,\underline{d} = \omega^2\,\underline{M}\,\underline{d}$$

or

$$\boxed{\underline{K}\,\underset{\underline{x}}{\underline{\phi}} = \underset{\lambda}{\omega^2}\,\underline{M}\,\underset{\underline{x}}{\underline{\phi}}}$$

Eq. (2), p. 28.2.

Mtg 29: Wed, 1 Nov 06 + 5 min (Lect 5)

Stability of trusses:

* Standard eval pb of $\underline{K}$:

$$\underline{K}\,\underline{x} = \lambda\,\underline{x} = \lambda\,\underline{I}\,\underline{x}$$

* Find zero evals :

$\underline{K}$ = global stiff. mat. before acct. for any ess. b.c.

Two types of zero evals:

1) rigid-body modes
2) mechanisms in trusses

→ zero-energy modes.

Type 1 zero evals : Rigid-body modes

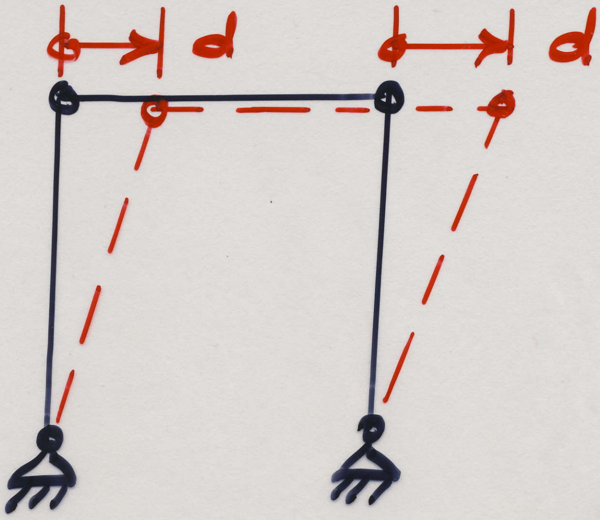
1.1. 1-D struct : 1 transl. (1 zero eval)

1.2. 2-D struct : 2 transl. + 1 rot. (3 zero eval.)

1.3. 3-D struct : $\underbrace{\text{3 transl. + 3 rot.}}_{\text{6 zero evals.}}$

Type 2 zero evals : Mechanisms

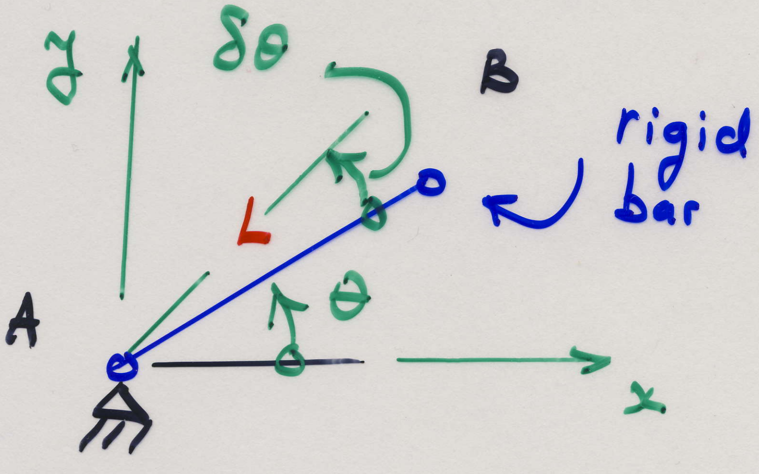
HW10 :



mechanism w/o structural defm, i.e., w/o stored strain energy

Rem: → Zero-energy modes

Virtual disp (virt. velocity)



$x_B(\theta) = L \cos\theta$

$y_B(\theta) = L \sin\theta$

$\delta\theta$ = virt. rot. (infinitesimally small)

Q: What are the disp. at B imparted by $\delta\theta$?

$$\frac{\delta x_B}{\delta\theta} = \frac{dx_B}{d\theta} = -L\sin\theta$$

$$\Rightarrow \boxed{\begin{aligned} \delta x_B &= \frac{dx_B}{d\theta}\,\delta\theta \\ &= (-L\sin\theta)\,\delta\theta \\ \text{Similarly, }\ \delta y_B &= \frac{dy_B}{d\theta}\,\delta\theta \\ &= (L\cos\theta)\,\delta\theta \end{aligned}}$$

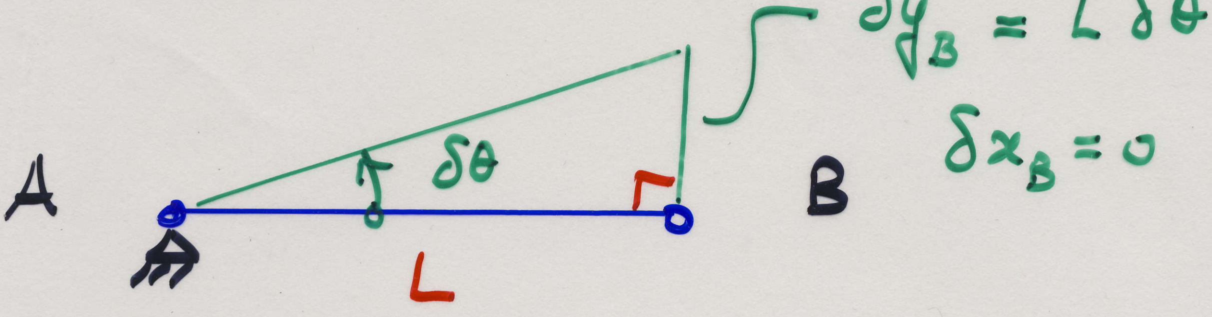
Virt disp comp. : $\delta x_B(\theta)$, $\delta y_B(\theta)$

Now consider an initially horiz bar :

$\boxed{\theta = 0}$: What are the virt. disp now?

$$\delta x_B(\theta = 0) = 0$$

$$\delta y_B(\theta = 0) = L\,\delta\theta$$



Evects corresp. to zero evals come out from eig not pure (i.e., one cannot say which evects corresp. to rigid-body modes, and which evects corresp. to mechanisms).

Consider: $\underline{K}\,\underline{v} = \lambda\,\underline{v}$ (for HW10 ex, p. 29-2)

Let

$$\underbrace{\underline{v}_1,\ \underline{v}_2,\ \underline{v}_3}_{\substack{\text{pure rigid body modes} \\ \text{(not plotted)}}},\ \underbrace{\underline{v}_4}_{\substack{\text{mechanism} \\ \text{(plotted on p. 29-2)}}}$$

be evects corresp. to the 4 zero evals.

$$\underline{K}\,\underline{v}_i = 0 \cdot \underline{v}_i \qquad i = 1, \dots, 4$$

$$= \underline{0}$$

Now consider: $\underline{w} = \sum_{i=1}^{4} c_i\,\underline{v}_i$

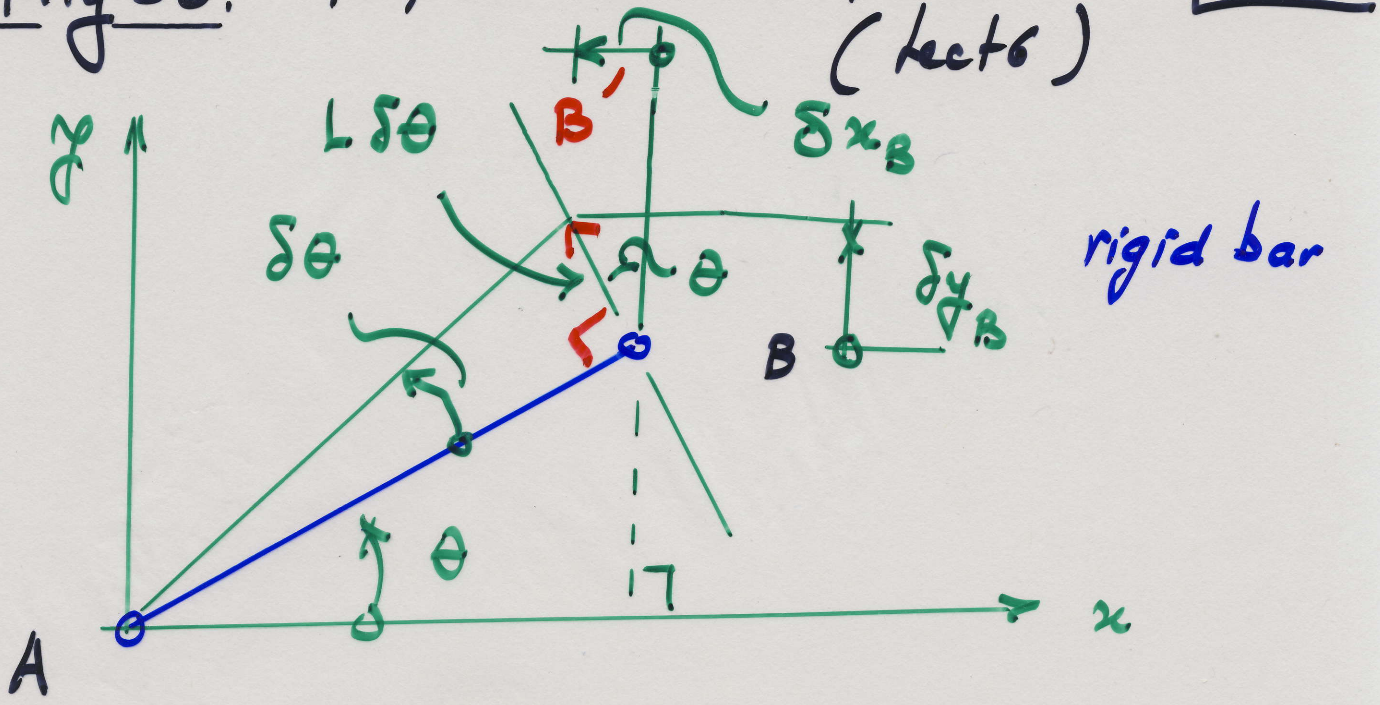
for any c_i's

Amruta : $\underline{K}\,\underline{w} = \sum_i c_i \Big(\underbrace{\underline{K}\,\underline{v}_i}_{\substack{\shortparallel \\ \underline{0}}}\Big)$

$= \underline{0}$

$\Rightarrow$ $\underline{w}$ is an evect corresp. to a zero eval.

Mtg 30: Fri, 3 Nov 06 + 5 min (lect6)



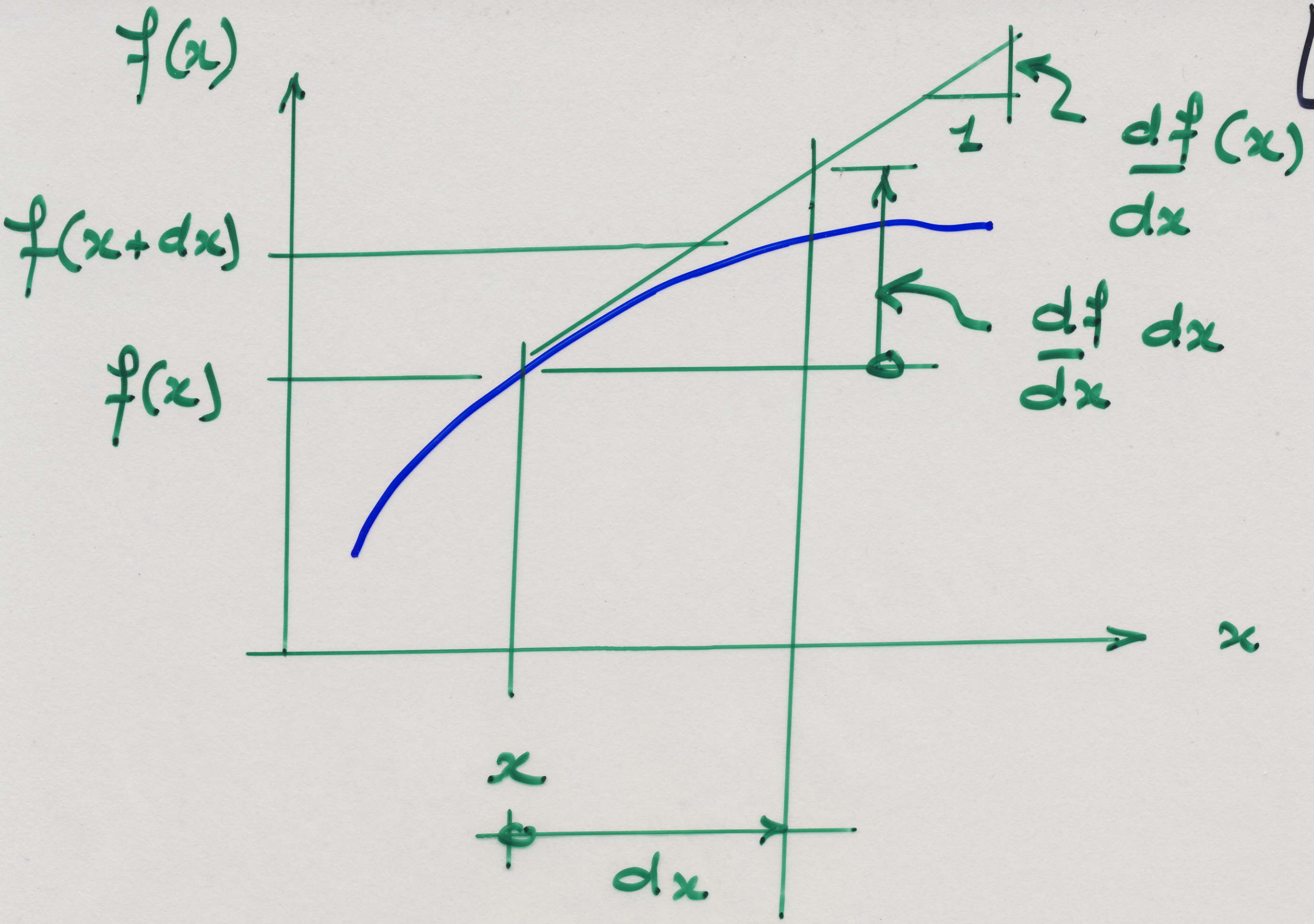
$$\begin{cases} \delta x_B = -(L\,\delta\theta)\,\cancel{\cos}\,\sin\theta \\ \delta y_B = (L\,\delta\theta)\,\cos\theta \end{cases}$$

Taylor series expans. :

$$x_B(\theta) = L\cos\theta$$

$$x_B(\theta+\delta\theta) = L\cos(\theta+\delta\theta)$$

Recall: $f(x+dx) = f(x) + \frac{df(x)}{dx}dx + \underbrace{h.o.t.}_{\text{higher order terms}}$



$$x_B(\theta+\delta\theta) = x_B(\theta) + \frac{dx_B}{d\theta}(\theta)\,\delta\theta + \text{h.o.t.}$$

$$= x_B(\theta) + \delta x_B(\theta)$$

or $$\delta x_B(\theta) = \underbrace{x_B(\theta+\delta\theta) - x_B(\theta)}_{\substack{\uparrow \\ \text{x comp. of} \\ \text{Virt. disp.} \\ \text{of B.}}}$$

$$= \frac{dx_B(\theta)}{d\theta}\,\delta\theta$$

Similarly for δy_B.

Beam theory : Justification of $\tilde{\underline{k}}^{(e)}_{B}$

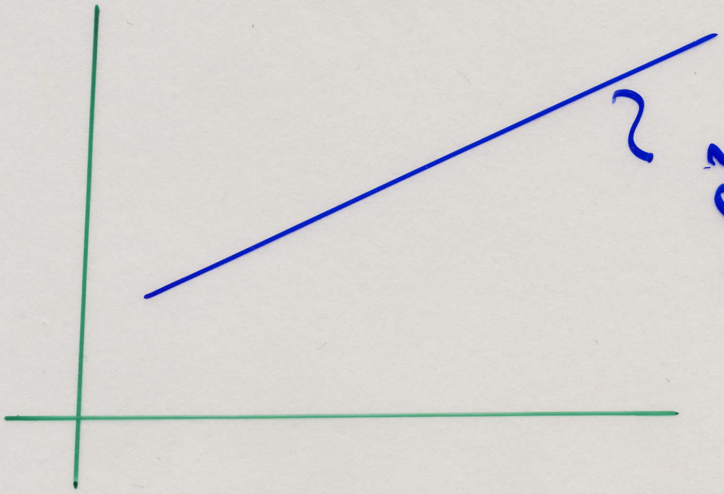
Sida : 2nd order diff eq. p.25-1

Gabriel: Defm of beam specified by poly.

Brian : soln to diff eq.

Euler-Bernoulli: beam : 4th diff. eq.
in terms of transv. disp.

Int. $\Rightarrow$ 4 const.



$y = ax + b$ (Pam)
2 const.

Peter : 4 const $\Rightarrow$ 3rd-order poly,

i.e., $c_0 + c_1 x + c_2 x^2 + c_3 x^3$

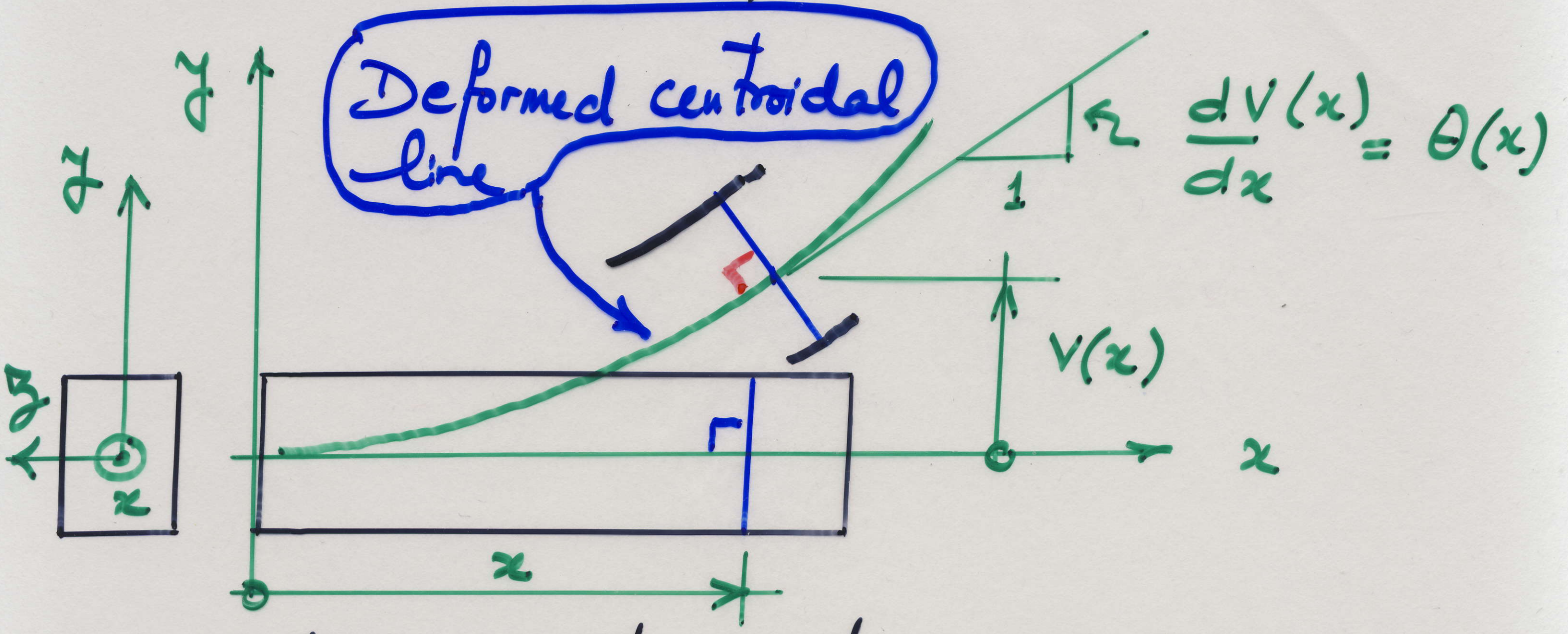
Mtg 31: Mon, 13 Nov 06 + 5 min
(lect 7)

Project (HW10, Part I) presentation.

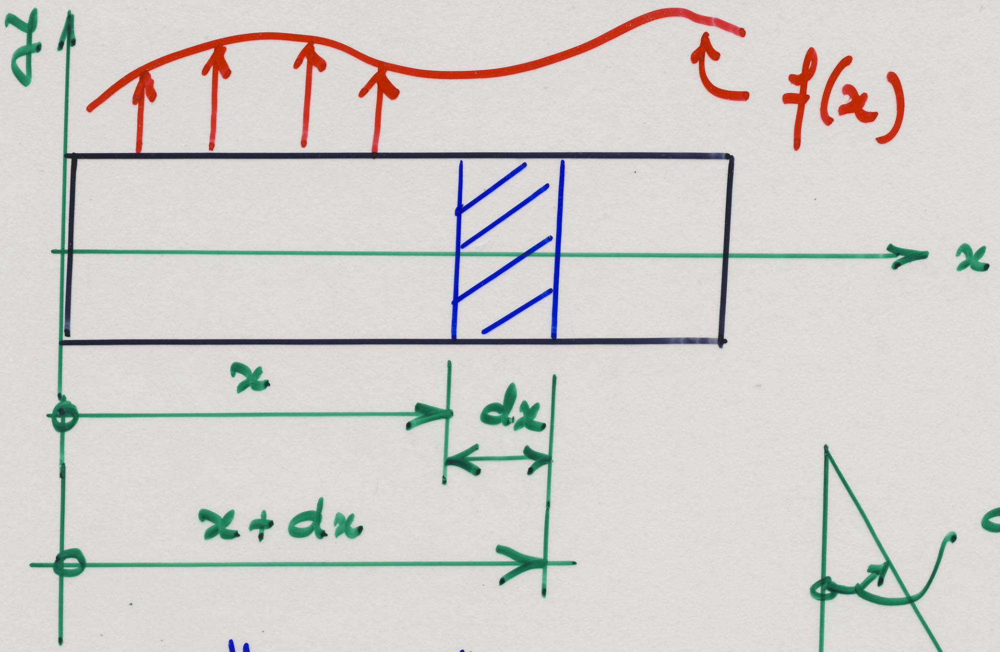
Mtg 32: Wed, 15 Nov 06 + 5 min
(Lect 8)

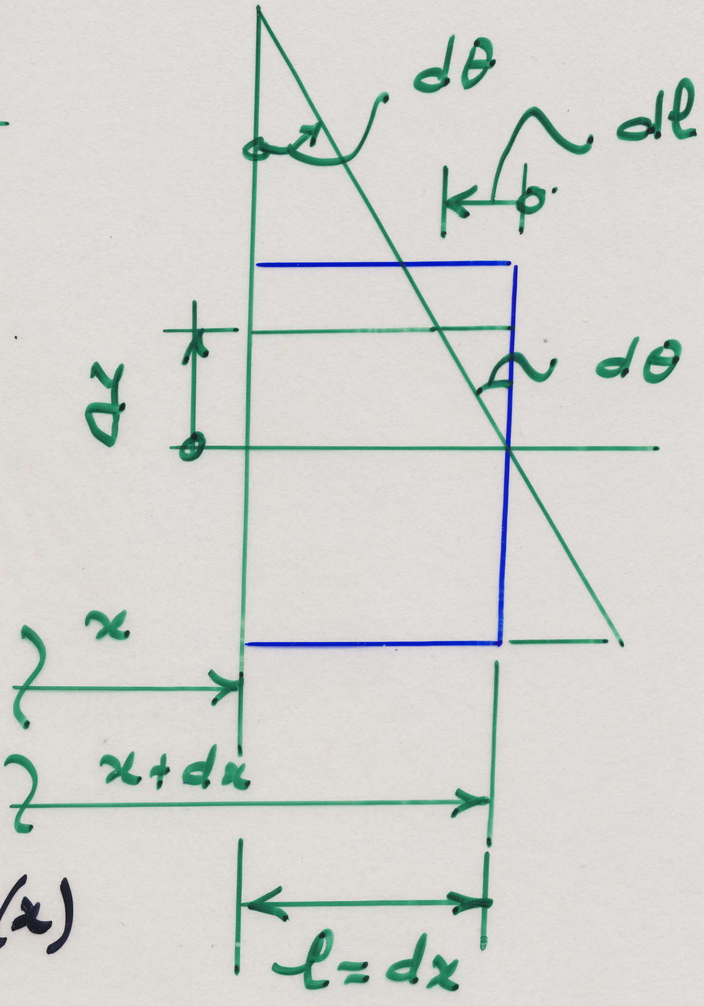
Beam theory (Cont'd)

Euler-Bernoulli beam theory, fund. assump.: Plane cross-section remains plane and perpendicular to deformed centroidal line of beam.



- * undeformed centroidal line coincides w/ x axis.
- * Hooke's law : $\sigma = E \varepsilon$
- * Q: How to relate ε to v (since the beam diff. eq. is in terms of v)?





Chris : "FBD"

$dl = -y\, d\theta$

(shortening of fiber at ordinate y)

$$\varepsilon = \frac{dl}{l} = -y \frac{d\theta(x)}{dx}$$

$$\Rightarrow \cancel{\varepsilon(x)} \quad \boxed{\varepsilon(x,y) = -y \underbrace{\frac{d^2 v(x)}{dx^2}}_{\text{David: curvature}}}$$

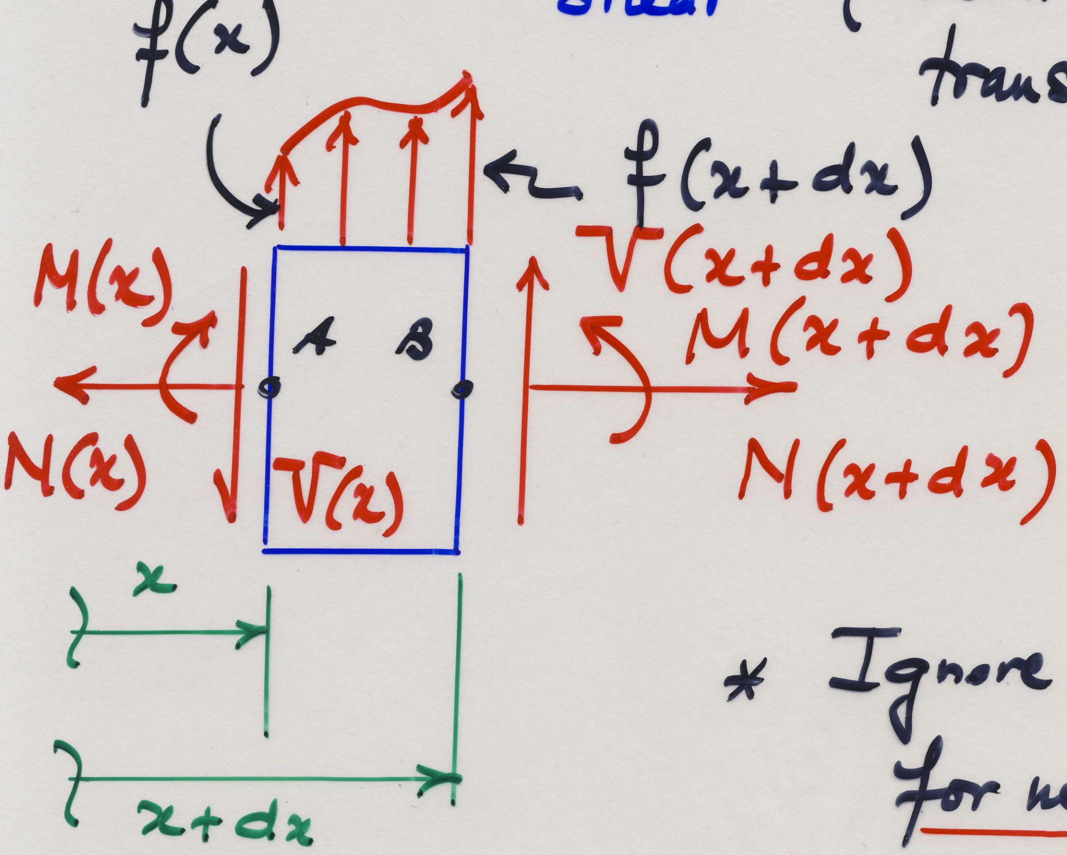
Hooke's law: $\boxed{\sigma(x,y) = -E\, y\, v''(x)}$

$$V''(x) := \frac{d^2 V}{dx^2}(x)$$ curvature

Jarita: Compute the resultants

$$\left(\underbrace{\text{force}}_{\text{shear}}, \text{mom}\right)$$

(normal force = 0
trans. force = shear)



* Ignore axial effect for now ($\Sigma F_x = 0$)

$$\Sigma F_y = 0 = -V(x) + f(x)\,dx + V(x+dx)$$

David: Taylor series

$$g(x+dx) = g(x) + \frac{dg(x)}{dx}dx + \underbrace{h.o.t.}_{\text{higher order terms}}$$

$$\Rightarrow \sum F_y = 0 = \frac{dV(x)}{dx}dx + \underbrace{h.o.t.}_{(dx^2,\ dx^3, \dots)} + f(x)\,dx$$

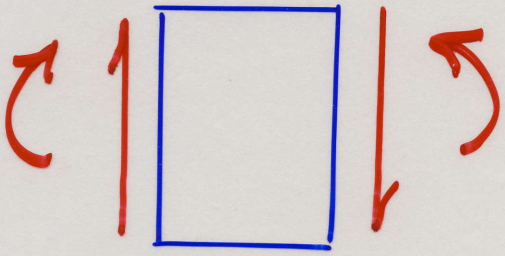
Ex: $dx = 10^{-1}$ (0.1)

$dx^2 = 10^{-2}$ $(0.01) < dx$

Neglect hot: (1) $\boxed{\frac{dV(x)}{dx} = -f(x)}$

HW: Find rel. betw. V and f for the alternative convention:

Book p. 236



* Can choose either A or B in Fig. on p. 32.3 to do mom. equil. (Convenient since eliminate as many forces as possible).

Mtg 33: Thu, 16 Nov 06

Goal: Derive beam stiffness matrix on p. 25-1.

FBD on p. 32-3 : Mom. equil. about pt A

$$\overset{+}{\circlearrowright} \sum M_A = 0 = -M(x) + f(x)\,dx \cdot \frac{dx}{2} + V(x+dx)\,dx + M(x+dx)$$

Taylor series

$$= \frac{dM}{dx}(x)\,dx + \text{hot} + f(x)\,\frac{dx^2}{2} + \left[V(x) + \frac{dV}{dx}(x)\,dx + \text{hot}\right]dx$$

$$= \frac{dM}{dx}(x)\,dx + \cancel{\text{hot}} + V(x)\,dx + \left[\frac{1}{2}f(x) + \frac{dV}{dx}(x)\right]\cancel{dx^2} + \cancel{\text{hot}}$$

neglect hot ($dx^2, dx^3, \dots$)

$$= \left[\frac{dM}{dx}(x) + V(x)\right]dx$$

$$\Rightarrow \boxed{\frac{dM}{dx}(x) = -V(x)} \quad (1)$$

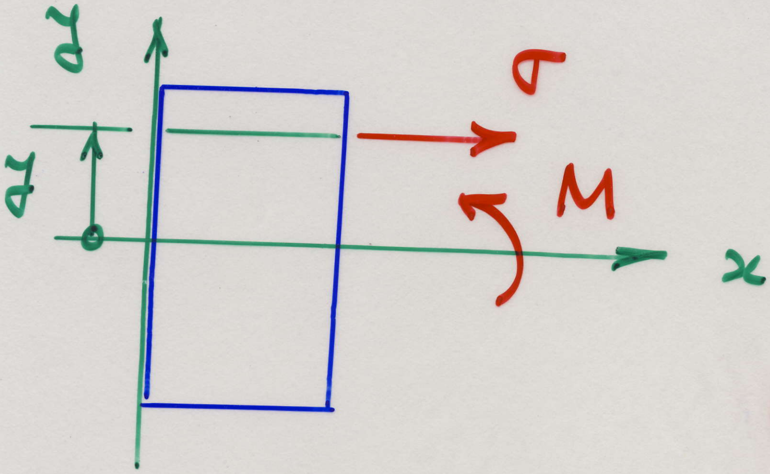
Book p. 237
diff conv.

Using Eq. (1) on p. 32-4 and
Eq. (1) on p. 33-1 :

$$\Rightarrow \frac{d^2M}{dx^2}(x) = -\frac{dV}{dx}(x) = -\left(-f(x)\right)$$

$$\Rightarrow \boxed{\frac{d^2M(x)}{dx^2} = f(x)} \quad (1)$$

* Relate M(x) to v(x) (transv. disp)



$$M = -\int_A (\sigma\, dA)\, y$$

important

$$M = -\int_A \left(-E y\, v''\, \underbrace{dA}_{dydz}\right) y$$

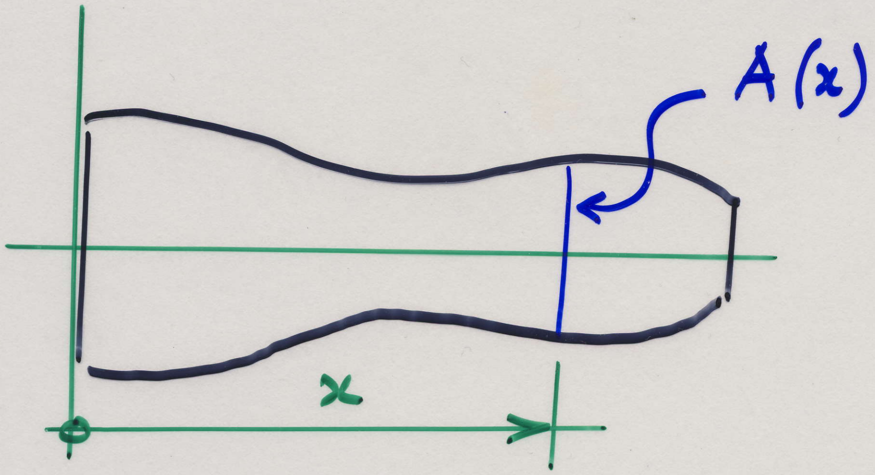
(see fig. of cross-section on p. 32-1)

$$M(x) = \int_A E\, y^2\, V''(x)\, dy\, dz$$

$$M(x) = E\left(\underbrace{\int_A y^2 \, dA}_{I}\right) v''(x)$$

$$\Rightarrow \boxed{M(x) = \underbrace{EI}_{\uparrow} \frac{d^2 V(x)}{dx^2}} \qquad (1)$$

Can be a func. of x
for beams w/ variable
cross section along x axis, or
with E
varying along
x axis.



Eq. (1), p. 33-2 :

$$\boxed{\frac{d^2}{dx^2}\left(EI(x)\frac{d^2 v(x)}{dx^2}\right) = f(x)} \qquad (2)$$

For the stiffness mat. on p. 25-1
(See Fig. on p. 24-4) :

$$f(x) = 0 \quad \text{(no dist. load)}$$

$$EI(x) = \text{const.}$$

$$\Rightarrow \quad EI \frac{d^4 v(x)}{dx^4} = 0$$

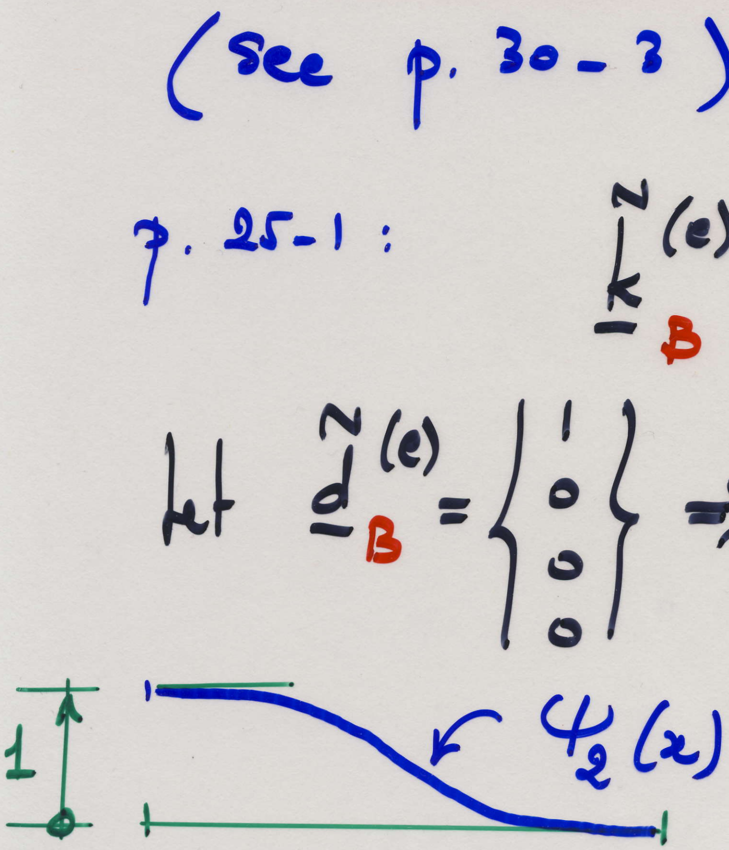
$$\Rightarrow \quad \boxed{\frac{d^4 v(x)}{dx^4} = 0} \qquad (1)$$

$$\Rightarrow \quad \boxed{v(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3}$$

(See p. 30-3) (2)

p. 25-1 : $$\underline{\tilde{k}}^{(e)}_{B} \, \underline{\tilde{d}}^{(e)}_{B} = \underline{\tilde{f}}^{(e)}_{B} \qquad (3) \ \cancel{4}$$

let $$\underline{\tilde{d}}^{(e)}_{B} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Rightarrow \underline{\tilde{f}}^{(e)}_{B} = \underbrace{\begin{Bmatrix} \tilde{k}^{(e)B}_{11} \\ \vdots \\ \tilde{k}^{(e)B}_{41} \end{Bmatrix}}_{\text{1st col.}} \qquad (4)$$



Mtg 34: Fri, 17 Nov 06 + 5 min
(Lect 9)

p. 33-4 : $\begin{cases} \tilde{d}_2 = 1 & (\text{trans. disp @ node } \boxed{1}) \\ \tilde{d}_3 = 0 & (\text{rot @ node } \boxed{1} \\ & \Rightarrow \text{zero slope}) \end{cases}$

Similarly for node $\boxed{2}$

$$\begin{cases} \tilde{d}_5 = 0 & (\text{trans. disp.}) \\ \tilde{d}_6 = 0 & (\text{rot. or slope}) \end{cases}$$

Comp. $c_0, \ldots, c_3$

$$\begin{cases} \tilde{d}_2 = \boxed{1 = V(0) = c_0} & (1) \\ \tilde{d}_5 = 0 = V(L) = 1 + c_1 L + c_2 L^2 + c_3 L^3 & (2) \end{cases}$$

$$V'(x) = c_1 + 2c_2 x + 3c_3 x^2$$

$$\begin{cases} \tilde{d}_3 = \boxed{0 = V'(0) = c_1} & (3) \\ \tilde{d}_6 = 0 = V'(L) = 2c_2 L + 3c_3 L^2 & \end{cases}$$

$$\Rightarrow \quad c_2 = -\frac{3}{2} c_3 L \qquad (4)$$

Using (3), (4) in (2) p. 34-1 :

$$0 = 1 + 0 + c_3 \left(\underbrace{-\frac{3}{2} L^3 + L^3}_{-\frac{L^3}{2}} \right)$$

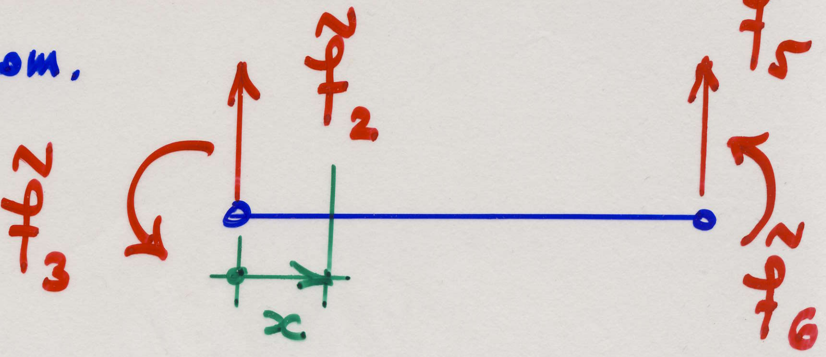
$$\Rightarrow \boxed{\begin{array}{l} c_3 = \dfrac{2}{L^3} \\ \hline c_2 = -\dfrac{3}{L^2} \end{array}}$$

$$\boxed{V(x) = 1 - \frac{3}{L^2} x^2 + \frac{2}{L^3} x^3}$$

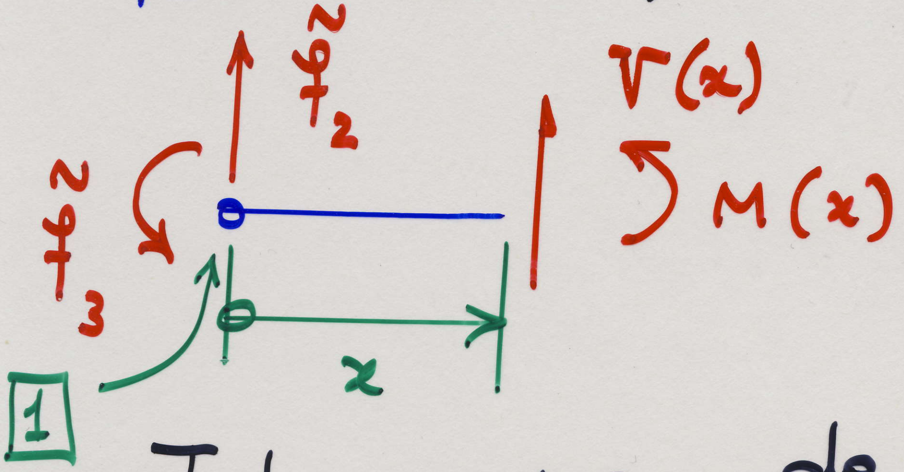
$$V''(x) = 2c_2 + 6c_3 x$$

$$\boxed{\begin{aligned} M(x) &= EI\, V''(x) \\ &= EI \left[-\frac{6}{L^2} + \frac{12}{L^3} x \right] \end{aligned}}$$

internal mom.



Pam: $x = 0$, $x = L$



Jarita:

$\Rightarrow \tilde{f}_2 = -V(0)$

Take $x \to 0$, do equil. of mom:

about $\boxed{1}$:

$$\overset{+}{\curvearrowright} \sum M_{\boxed{1}} = 0 = \tilde{f}_3 + M(0)$$

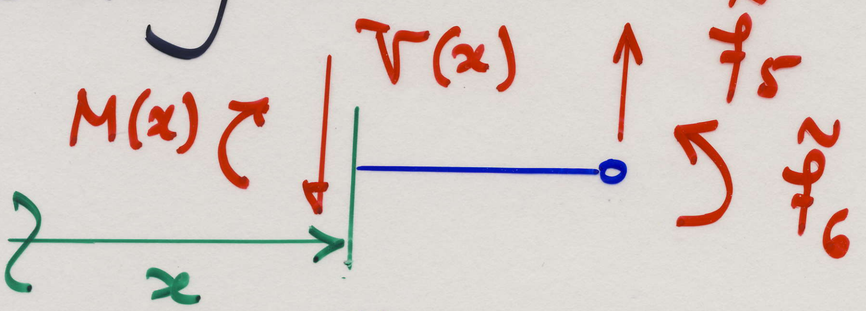
$$\Rightarrow \tilde{f}_3 = -M(0) = +\frac{6EI}{L^2}$$

$$\|$$

$$\tilde{k}^B_{21} = \tilde{k}_{32}$$

See $\tilde{\underline{k}}_B$ on p. 25-1

Similarly at $x = L$:



David:

Take $x \to L$,

$\tilde{f}_6 = M(L)$

$//$

$\tilde{k}^B_{41} = \tilde{k}_{62} = \frac{6EI}{L^2}$

$\Rightarrow \tilde{f}_5 = +V(L)$

Using (1) of p. 33-1 and
(1) of p. 33-3 :

$$\boxed{V(x) = -\frac{dM}{dx} = -EI\,\frac{d^3 v(x)}{dx^3}}$$

$$V(x) = -12\,\frac{EI}{L^3} \quad \text{for all } x\,.$$

$$\Rightarrow \quad \tilde{f}_2 = 12\,\frac{EI}{L^3} = -\tilde{f}_{\underset{5}{\cancel{6}}}$$

Mtg 35: Mon, 20 Nov 06 + 5 min
(Lect 10)

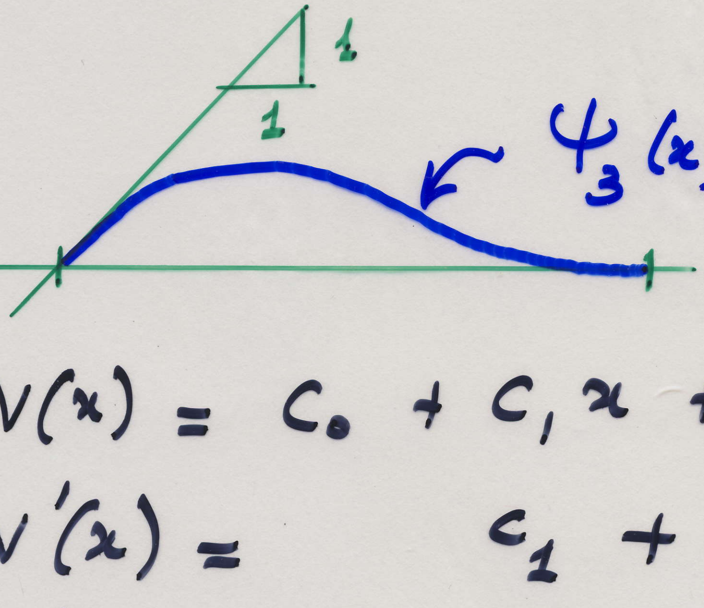
Goal: Explain 3rd col (also 3rd row) corresp. $\tilde{d}_3$ in $\underline{k}^{(e)}_{6\times 6}$, p. 25-3.

Method: (See p. 33-4)

Select

$$\underline{\tilde{d}}^{(e)T} = \{0 \quad 0 \quad \underset{\tilde{d}_3}{\overset{}{\underset{\shortparallel}{1}}} \quad 0 \quad 0 \quad 0\}$$

Eqs (1), (2), p. 33-4.



$$v(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$$

$$v'(x) = c_1 + 2c_2 x + 3c_3 x^2$$

$$v''(x) = 2c_2 + 6c_3 x$$

$$V'''(x) = 6c_3$$

$$\begin{array}{l|l} V(0) = \tilde{d}_2 = 0 & V(L) = \tilde{d}_5 = 0 \\ V'(0) = \tilde{d}_3 = 1 & V'(L) = \tilde{d}_6 = 0 \end{array}$$

Larry:

$$c_0 = 0 \qquad c_2 = -2/L$$

$$c_1 = 1 \qquad c_3 = 1/L^2$$

$$V(x) = \underbrace{1 \cdot x}_{\substack{\shortparallel \\ x}} - \frac{2}{L}x^2 + \frac{1}{L^2}x^3$$

Tony:

$$M(x) = EI\,V''(x)$$

$$= EI\left[-\frac{4}{L} + \frac{6}{L^2}x\right]$$

Sida:

$$\mathcal{V}(x) = -\frac{dM(x)}{dx} = -EI\,V'''(x)$$

$$= -\frac{6EI}{L^2}$$

Jarita: FBD's, p. 34-3

At node $\boxed{1}$, as $x \rightarrow 0$:

Shear: $\tilde{f}_2 = -V(0) = +\frac{6EI}{L^2} = \tilde{k}_{23}$

Mom: $\tilde{f}_3 = -M(0) = \frac{4EI}{L} = \tilde{k}_{33}$

At node $\boxed{2}$, as $x \rightarrow L$:

Shear: $\tilde{f}_5 = +V(L) = -\frac{6EI}{L^2} = \tilde{k}_{53}$

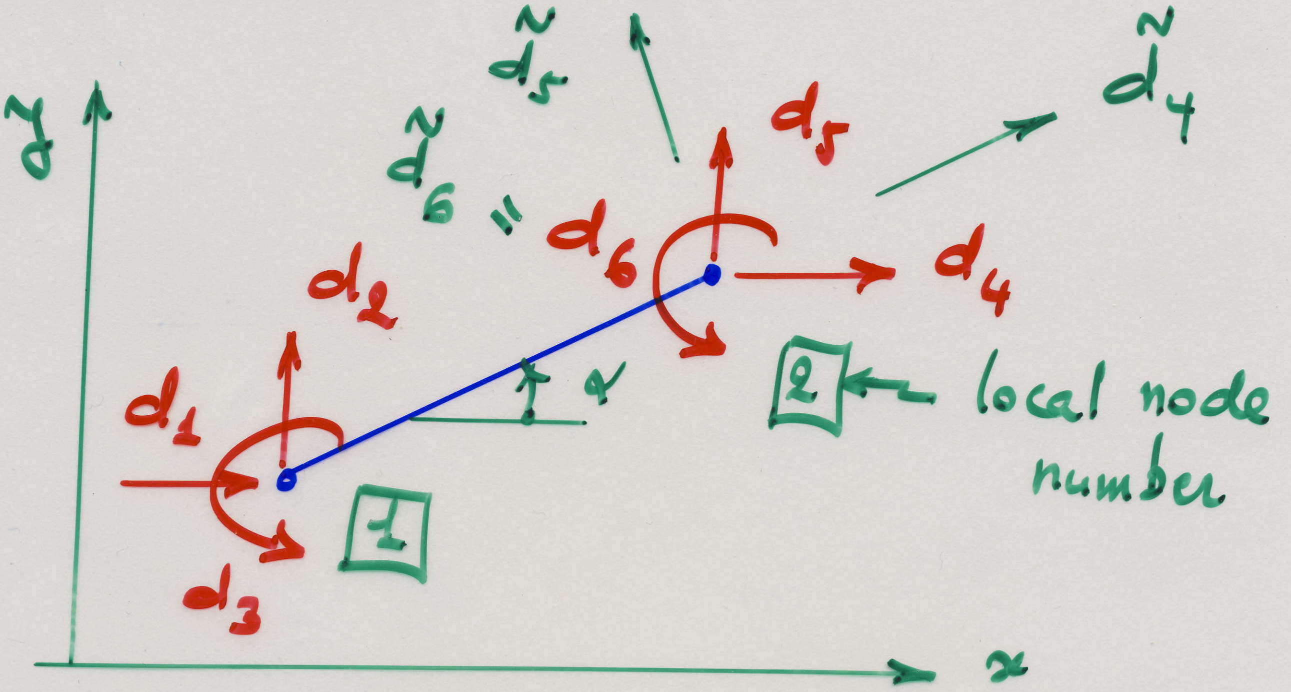
Mom: $\tilde{f}_6 = +M(L) = \frac{2EI}{L} = \tilde{k}_{63}$

HW: Derive all coeff in 5th col. and 6th col. in $\underset{6\times 6}{\underline{\tilde{k}}^{(e)}}$.

Mtg 36: Mon, 27 Nov 06,

- Frame elem arb. oriented in space (2-D)
- Distr. load (transv.) on frame elem (HW10).

General 2-D frame elem:



In local coord: $\underset{6\times6}{\underline{\tilde{k}}^{(e)}} \; \underset{6\times1}{\underline{\tilde{d}}^{(e)}} = \underset{6\times1}{\underline{\tilde{f}}^{(e)}}$

(p. 25-3a)

p. 24-4: dof's in local coord. $\underset{6\times1}{\underline{\tilde{d}}^{(e)}}$

Chris :

Shawn :

$$\underset{6\times1}{\tilde{\underline{d}}^{(e)}} = \underset{6\times6}{\underline{T}^{(e)}}\ \underset{6\times1}{\underline{d}^{(e)}}$$

$$\underset{6\times6}{\underline{T}^{(e)}} = \begin{bmatrix} \underset{3\times3}{\underline{R}^{(e)}} & \underset{3\times3}{\underline{0}} \\ \underset{3\times3}{\underline{0}} & \underset{3\times3}{\underline{R}^{(e)}} \end{bmatrix}_{6\times6}$$

Reter

$$\underset{3\times3}{\underline{R}^{(e)}} = \left[\begin{array}{cc|c} l^{(e)} & m^{(e)} & 0 \\ -m^{(e)} & l^{(e)} & 0 \\ \hline 0 & 0 & 1 \end{array}\right]_{3\times3}$$

(see p. 8-2, 8-3)

$\underline{R}^{(e)}$ is orthog. mat. ⇒

$$\underline{R}^{(e)-1} = \underline{R}^{(e)T}$$

$$\underline{T}^{(e)-1} = \begin{bmatrix} \underline{R}^{(e)-1} & \underline{0} \\ \underline{0} & \underline{R}^{(e)-1} \end{bmatrix}$$

(p. 9-2)

$$\Rightarrow \underline{T}^{(e)-1} = \begin{bmatrix} \underline{R}^{(e)T} & \underline{0} \\ \underline{0} & \underline{R}^{(e)T} \end{bmatrix} = \underline{T}^{(e)T}$$

Result:

$$\boxed{\Big[\underbrace{\underline{T}^{(e)T}\,\underline{\tilde{k}}^{(e)}\,\underline{T}^{(e)}}_{\underline{k}^{(e)}}\Big]\,\underline{d}^{(e)} = \underbrace{\underline{T}^{(e)T}\,\underline{\tilde{f}}^{(e)}}_{\underline{f}^{(e)}}}$$

obtained by 2 methods (similar to truss elem):

1) By transf. of coord.

$$\underline{\tilde{k}}^{(e)}\,\underline{\tilde{d}}^{(e)} = \underline{\tilde{f}}^{(e)}$$

$$\underline{\tilde{k}}^{(e)}\,\underline{T}^{(e)}\,\underline{d}^{(e)} = \underline{T}^{(e)}\,\underline{f}^{(e)}$$

since $\underline{\tilde{f}}^{(e)} = \underline{T}^{(e)}\,\underline{f}^{(e)}$

Same as $\tilde{\underline{d}}^{(e)} = \underline{T}^{(e)} \underline{d}^{(e)}$

$$\Rightarrow \underbrace{\underline{T}^{(e)-1}}_{\shortparallel \atop \underline{T}^{(e)T}} \tilde{\underline{k}}^{(e)} \underline{T}^{(e)} \underline{d}^{(e)} = \underline{T}^{(e)-1} \nearrow \boxed{\underline{T}^{(e)} \underline{f}^{(e)}}$$

$$\text{rhs} = \underline{f}^{(e)}$$

2) Princ. Virt. Work:

$$\tilde{\underline{k}}^{(e)} \tilde{\underline{d}}^{(e)} = \tilde{\underline{f}}^{(e)}$$

PVW: $\underline{a}^{(e)} \cdot \left[\tilde{\underline{k}}^{(e)} \tilde{\underline{d}}^{(e)} - \tilde{\underline{f}}^{(e)} \right] = 0$

for all $\underline{a}^{(e)}$

Transf. $\tilde{\underline{d}}^{(e)} = \underline{T}^{(e)} \underline{d}^{(e)}$

$$\underset{\shortparallel}{\underline{a}^{(e)}} = \underline{T}^{(e)} \underline{c}^{(e)}$$

or $\tilde{\underline{c}}^{(e)} = \underline{T}^{(e)} \underline{c}^{(e)}$

$\underline{\tilde{c}}^{(e)} = \underline{a}^{(e)}$ = virt. disp. in local coord.

$\underline{c}^{(e)}$ = virt. disp. in global coord.

$$\Rightarrow \left[\underline{T}^{(e)}\underline{c}^{(e)}\right] \cdot \left[\underline{\tilde{k}}^{(e)} \# \underline{T}^{(e)}\underline{d}^{(e)} - \underline{\tilde{f}}^{(e)}\right] = 0$$

for all $\underline{c}^{(e)}$

$$\Rightarrow \underline{c}^{(e)} \cdot \left[\underline{T}^{(e)T}\underline{\tilde{k}}^{(e)}\underline{T}^{(e)}\underline{d}^{(e)} - \underline{T}^{(e)T}\underline{\tilde{f}}^{(e)}\right] = 0$$

for all $\underline{c}^{(e)}$

$$\Rightarrow \underline{T}^{(e)T}\underline{\tilde{k}}^{(e)}\underline{T}^{(e)}\underline{d}^{(e)} = \underline{T}^{(e)T}\underline{\tilde{f}}^{(e)}$$

See mtg 21.