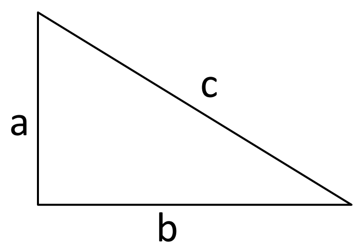
Lesson 11: The Pythagorean Theorem

Covered on 9/24

What is the Pythagorean Theorem?

- "Given a **right** triangle, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse."
  - $(\text{Leg } 1)^2 + (\text{Leg } 2)^2 = (\text{Hypotenuse})^2$
- Let's look at a picture of this:
  - a = leg 1
  - b = leg 2
  - c = hypotenuse

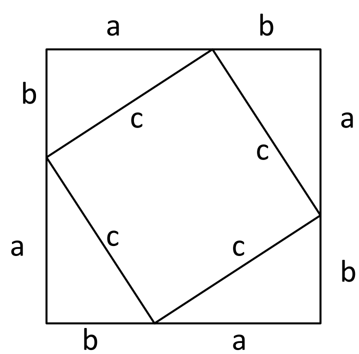


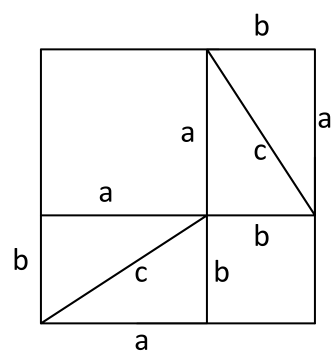
$$a^2 + b^2 = c^2$$

Now that we understand what exactly the Pythagorean Theorem is, do you think there any way to prove it?

- Using a dissection proof, which involves cutting the shapes up and then putting them back together, it is possible to prove the Pythagorean Theorem.

Look at the two squares below. From these two squares you should be able to prove that $a^2 + b^2 = c^2$.





Solution: We are trying to prove that the sum of a^2 and b^2 is equal to the sum of c^2. If you look at the two squares in comparison to one another you will see that each has the same number of right triangles, they are just arranged differently. I have colored in the right triangles below:

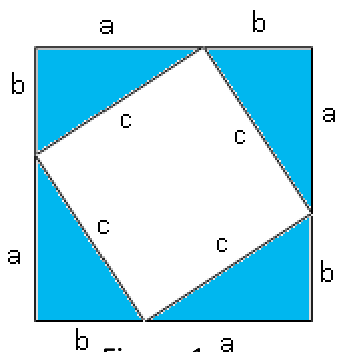


Figure 1

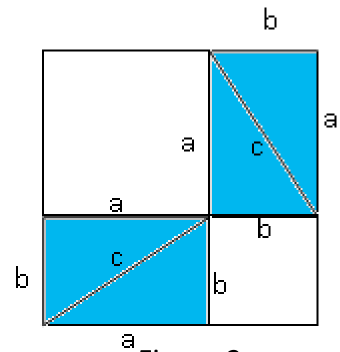
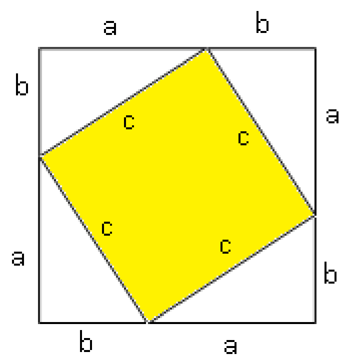
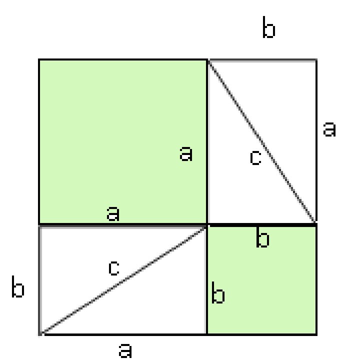


Figure 2

After you have identified the right triangles, look at the shapes you have left. Somehow you must find that $a^2 + b^2$ is equal to c^2. Look at the square remaining in figure 1:



The area of this shape is c^2. Now look at the area of the two shapes remaining in figure 2:



The area of the square on the left is a^2 and the area of the shape on the right is b^2. Is there a way that we could set these two squares equal to the square that is remaining in figure 1 to give us $a^2 + b^2 = c^2$? In order for this to work, the areas of Figure 1 and Figure 2 must be the same. Looking at the pictures below, we can calculate the area of each figure:

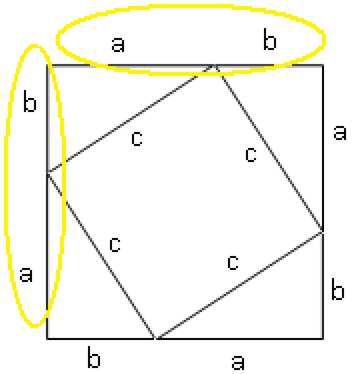


Figure 1

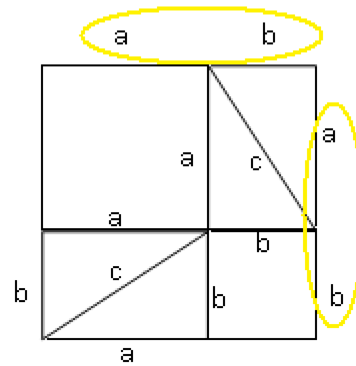


Figure 2

area = (ab)(ab) = $(ab)^2$

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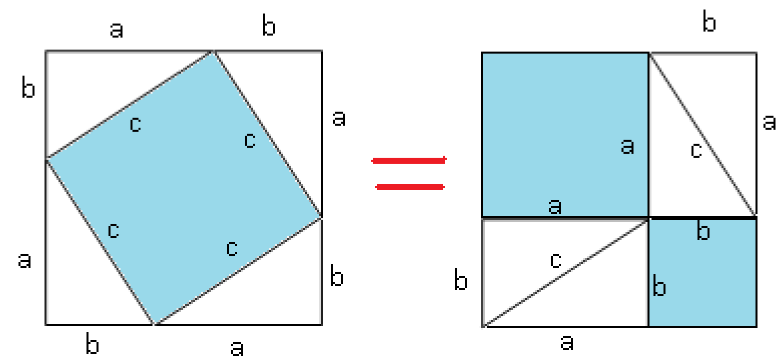
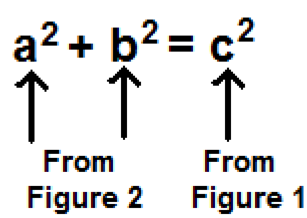
Looking at these two figures, we see that each square has the same area, making it possible to set the two smaller squares in figure 2 equal to the larger square in figure 1.



Therefore, we can see that



Congratulations! You have just proved the Pythagorean Theorem using pictures.

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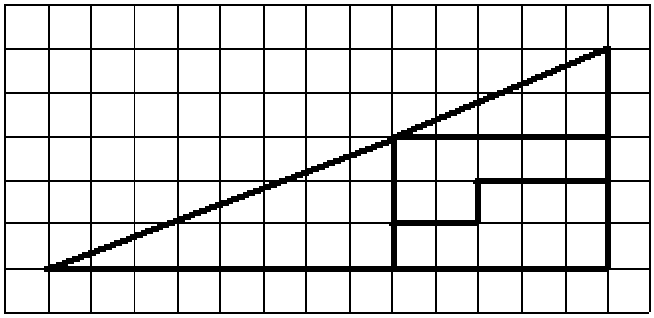
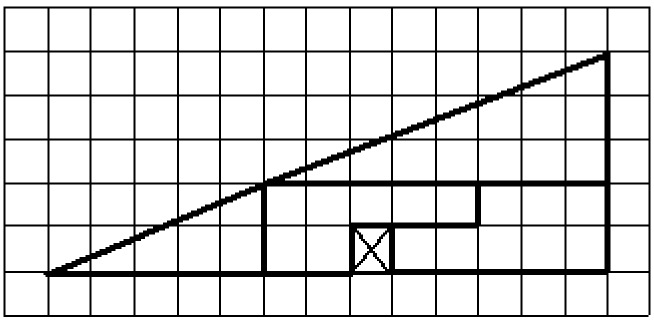
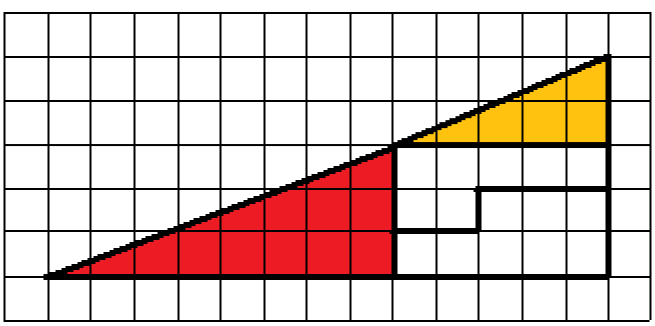
Now, let’s study a paradox involving right triangles.

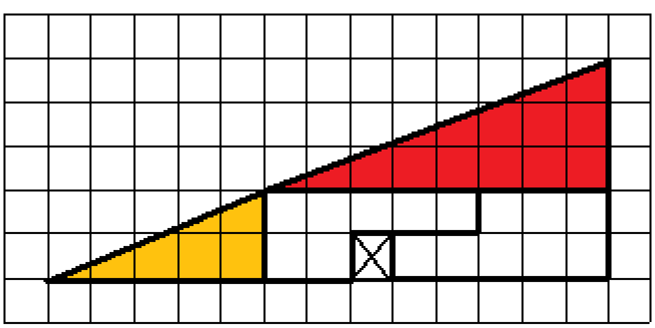
The Paradox: What is wrong with these images?





At first glance, it appears as if nothing is wrong with them and they are both right triangles. However, if you look closer, try and determine the slope of the hypotenuse of each of the two triangles in both pictures.





In order for these two figures to be considered right triangles, the slope of the two smaller triangles should be the same because they form the hypotenuse of the larger triangle. However, if you look at the slope of the yellow triangle, you will see that it is 2/5; whereas, the slope of the red triangle is 3/8. This means that these two larger figures are not triangles because they have 4 sides. By definition, triangles only have 3 sides.