CORDIC Fixed Point Simulation

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Based on the following site:

drdobbs.com

“Implementing CORDIC Algorithms”, P. Jarvis, Dr Dobb's, Oct, 1990

ANSI-C version of the above by P. Knoppers.

Circular

```
void Circular (long x, long y, long z)
{
  int i;
  X = x;
  Y = y;
  Z = z;
  for (i = 0; i <= fractionBits; ++i)
    {
      x = X >> i;
      y = Y >> i;
      z = atan[i];
      X -= Delta (y, Z);
      Y += Delta (x, Z);
      Z -= Delta (z, Z);
    }
}
```

x = X >> i; → $X \cdot 2^{-i}$

y = Y >> i; → $Y \cdot 2^{-i}$

X -= Delta (y, Z); → $X = (X - Y\,\sigma_i 2^{-i})$

Y += Delta (x, Z); → $Y = (X\,\sigma_i 2^{-i} + Y)$

x → $x'_{i+1} = (x'_i - y'_i\,\sigma_i 2^{-i})$

y → $y'_{i+1} = (x'_i\,\sigma_i 2^{-i} + y'_i)$

z → $\alpha_{i+1} = \alpha_i - \tan(\sigma_i 2^{-i})$

Delta

```
#define Delta(n, Z) (Z >= 0) ? (n) : -(n)
```

X → $x'_{i+1} = \left(x'_i - y'_i\,\sigma_i 2^{-i}\right)$

Y → $y'_{i+1} = \left(x'_i\,\sigma_i 2^{-i} + y'_i\right)$

Z → $\alpha_{i+1} = \alpha_i - \tan\left(\sigma_i 2^{-i}\right)$

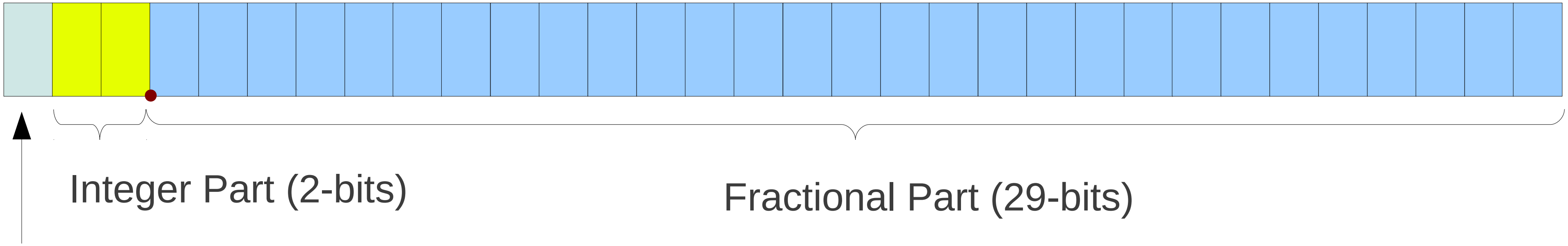
```
X -= Delta (y, Z);      (Z >= 0) X = X – y : X = X + y;
Y += Delta (x, Z);      (Z >= 0) Y = Y + x : Y = Y - x;
Z -= Delta (z, Z);      (Z >= 0) Z = Z – z : Z = Z + z;
```

y → $-\,y'_i\,\sigma_i 2^{-i}$

x → $+\,x'_i\,\sigma_i 2^{-i}$

z → $-\tan\left(\sigma_i 2^{-i}\right)$

Fixed Point Format



Integer Part (2-bits)

Fractional Part (29-bits)

Sign bit

Computing ATAN constants instead of LUT (1)

Use power series to calculate the incremental angles $\alpha_i = \tan^{-1} 2^{-i}$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} \cdots \qquad for \quad x^2 \le 1$$

$\tan^{-1} x = x$ 32-bit precision

$x^3/3 = 2^{-32}$ $\qquad x = \sqrt[3]{6 \cdot 2^{-11}}$

$$\tan^{-1} 2^{-i} = 2^{-i} \quad for \;\; i \ge 11$$

$$2^{-in} / n = 2^{-i} \quad for \;\; 1 \le i \le 10$$

$$\frac{\pi}{4} \quad for \;\; i = 0$$

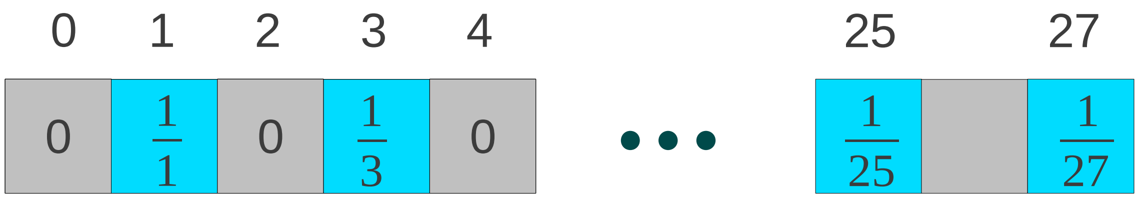
Computing ATAN constants instead of LUT (2)

Use power series to calculate the incremental angles $\alpha_i = \tan^{-1} 2^{-i}$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} \cdots \qquad for \quad x^2 \le 1$$

To compute the coefficients $\frac{1}{k}$ (k = 3, 5, 7, … , 27)

Reciprocal (**k**, **n**)
k: integer to be inversed,
n: precision for the desired fractional part



0	1	2	3	4		25		27
0	$\frac{1}{1}$	0	$\frac{1}{3}$	0	• • •	$\frac{1}{25}$		$\frac{1}{27}$

long a[28]

Restoring Division Alg

Computing ATAN constants instead of LUT (3)

Use power series to calculate the incremental angles $\alpha_i = \tan^{-1} 2^{-i}$

$$\tan^{-1} x = \frac{\pi}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} \cdots \right) \qquad for \quad x^2 \geq 1$$

atan[fractionBits + 1]

Poly2 (**k**, **n**)
Computes the power series for the specified number of terms for the specified power of two using **Horner's rule**

$$a_n\, x_n + a_{n-1}\, x_{n-1} + a_{n-2}\, x_{n-2} + \cdots + a_1\, x_1 + a_0$$

$$(\cdots(((a_n\, x + a_{n-1})\, x) + a_{n-2}\, x) + \cdots + a_1) x + a_0$$

Restoring Division Alg

Declarations

```
#define      fractionBits      29
#define      longBits          32
#define      One               (010000000000L>>1)
#define      HalfPi            (014441766521L>>1)


long X0C, X0H, X0R;    /* seed for circular, hyperbolic, and square root */
long OneOverE, E;      /* the base of natural logarithms */
long HalfLnX0R;        /* constant used in simultanous sqrt, ln computation */


static unsigned terms[11] = {0, 27, 14, 9, 7, 5, 4, 4, 3, 3, 3};
static long a[28];
static long atan[fractionBits + 1];
static long atanh[fractionBits + 1];

static long X;
static long Y;
static long Z;
```

$$2^{-in} / n = 2^{-i} \quad for \;\; 1 \le i \le 10$$

i= 1 → n= 27
i= 2 → n= 14
i= 3 → n= 9
i= 4 → n= 7
i= 5 → n= 5
i= 6 → n= 4
i= 7 → n= 4
i= 8 → n= 3
i= 9 → n= 3
i=10 → n= 3

Preparing atan[30] array

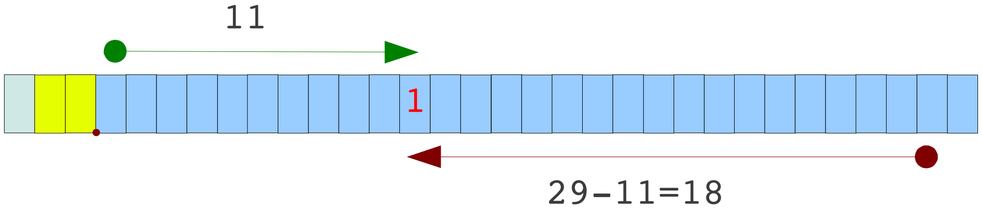
```
for (i = 0; i <= 13; ++i) {
    a[2*i] = 0;
    a[2*i+1] = Reciprocal (2*i+1, fractionBits);
 }
atan[0] = HalfPi / 2;     /* atan(2^0)= pi / 4 */
for (i = 1; i <= 7; ++i)
  a[4*i-1] = -a[4*i-1];
for (i = 1; i <= 10; ++i)
  atan[i] = Poly2 (-i, terms[i]);
for (i = 11; i <= fractionBits; ++i)
  atan[i] = atanh[i] = 1L << (fractionBits - i);

printf ("\n\natan(2^-n)\n");
for (i = 0; i <= 10; ++i) {
    printf ("%2d ", i);
    WriteVarious (atan[i]);
}

r = 0;
for (i = 0; i <= fractionBits; ++i)
  r += atan[i];
printf ("radius of convergence");
WriteFraction (r);
```

$$\left\{0, \frac{1}{1}, 0, \frac{1}{3}, 0, \frac{1}{5}, \cdots, 0, \frac{1}{27}\right\}$$

$$\frac{-1}{3}, \frac{-1}{7}, \frac{-1}{11}, \frac{-1}{15}, \frac{-1}{19}, \frac{-1}{23}, \frac{-1}{27}$$



Making Power Series – Poly2( )

```
static unsigned terms[11] =
                 {0, 27, 14, 9, 7, 5, 4, 4, 3, 3, 3};

for (i = 1; i <= 10; ++i)
  atan[i] = Poly2 (-i, terms[i]);


long Poly2 (int log, unsigned n)
{
  long r = 0;
  int i;
  for (i = n; i >= 0; --i)
    r = (log < 0 ? r >> -log : r << log) + a[i];
  return (r);
}
```

```
atan[ 1] = Poly2 ( -1, 27);
atan[ 2] = Poly2 ( -2, 14);
atan[ 3] = Poly2 ( -3,  9);
atan[ 4] = Poly2 ( -4,  7);
atan[ 5] = Poly2 ( -5,  5);
atan[ 6] = Poly2 ( -6,  4);
atan[ 7] = Poly2 ( -7,  4);
atan[ 8] = Poly2 ( -8,  3);
atan[ 9] = Poly2 ( -9,  3);
atan[10] = Poly2 (-10,  3);
```

WriteFraction (1)

```
void WriteFraction (long n)
{
  unsigned short i;
  unsigned short low;
  unsigned short digit;
  unsigned long k;
  putchar (n < 0 ? '-' : ' ');
  n = abs (n);
  putchar ((n >> fractionBits) + '0');
  putchar ('.');
  low = k = n << (longBits - fractionBits);
  /* align octal point at left */
  k >>= 4;
  /* shift to make room for a decimal digit */
  for (i = 1; i <= 8; ++i)
    {
      digit = (k *= 10L) >> (longBits - 4);
      low = (low & 0xf) * 10;
      k +=   ((unsigned long) (low >> 4)) -
             ((unsigned long) digit << (longBits - 4));
      putchar (digit + '0');
    }
}
```

32 – 29 = 3
k= frac part * 2^3

k= only frac part

32 – 4 = 28

WriteFraction (2)

```
putchar ((n >> fractionBits) + '0');
```

'0' → 0x30 = 48

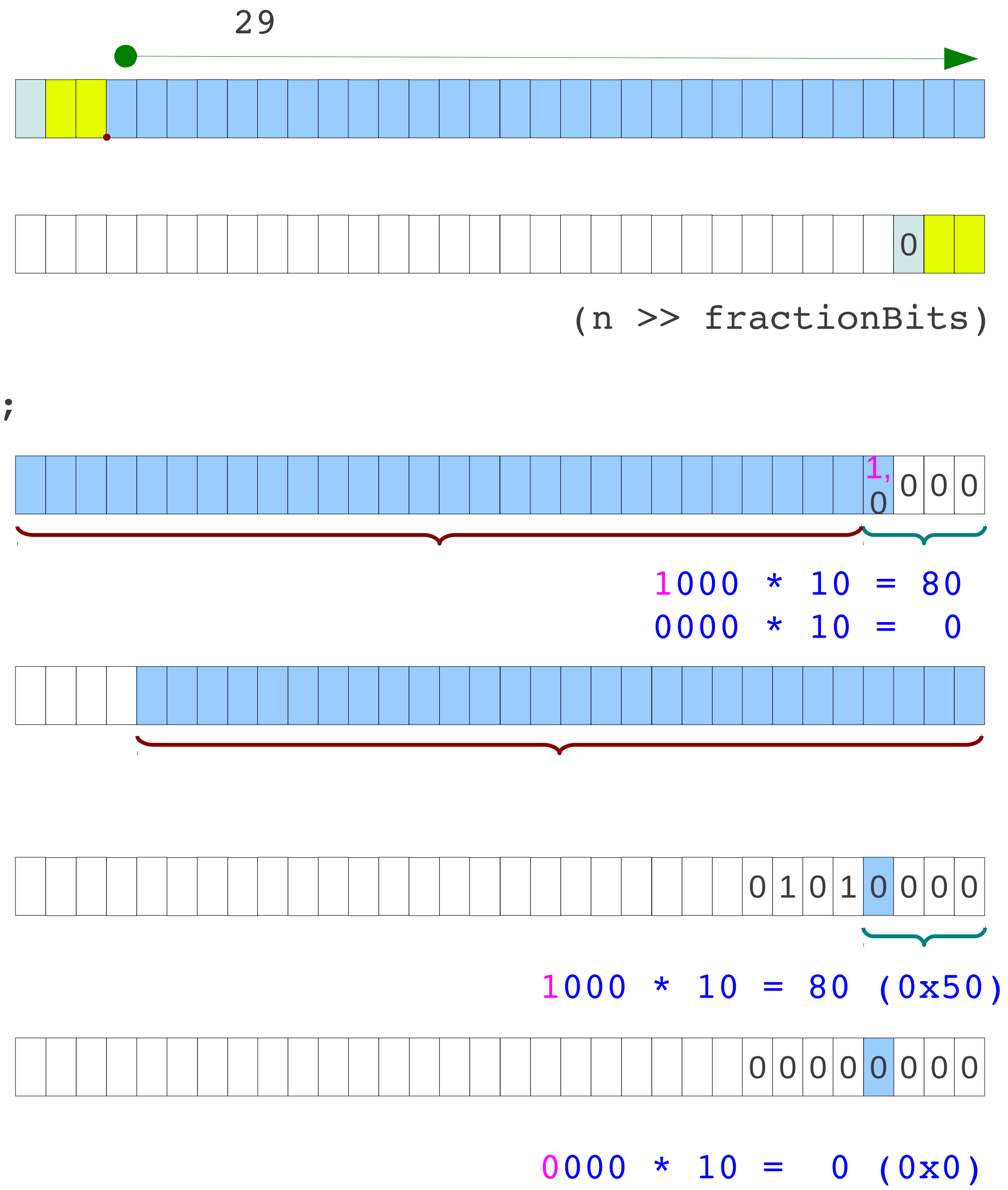
```
low = k = n << (longBits - fractionBits);
```

32 – 29 = 3

```
k >>= 4;
```

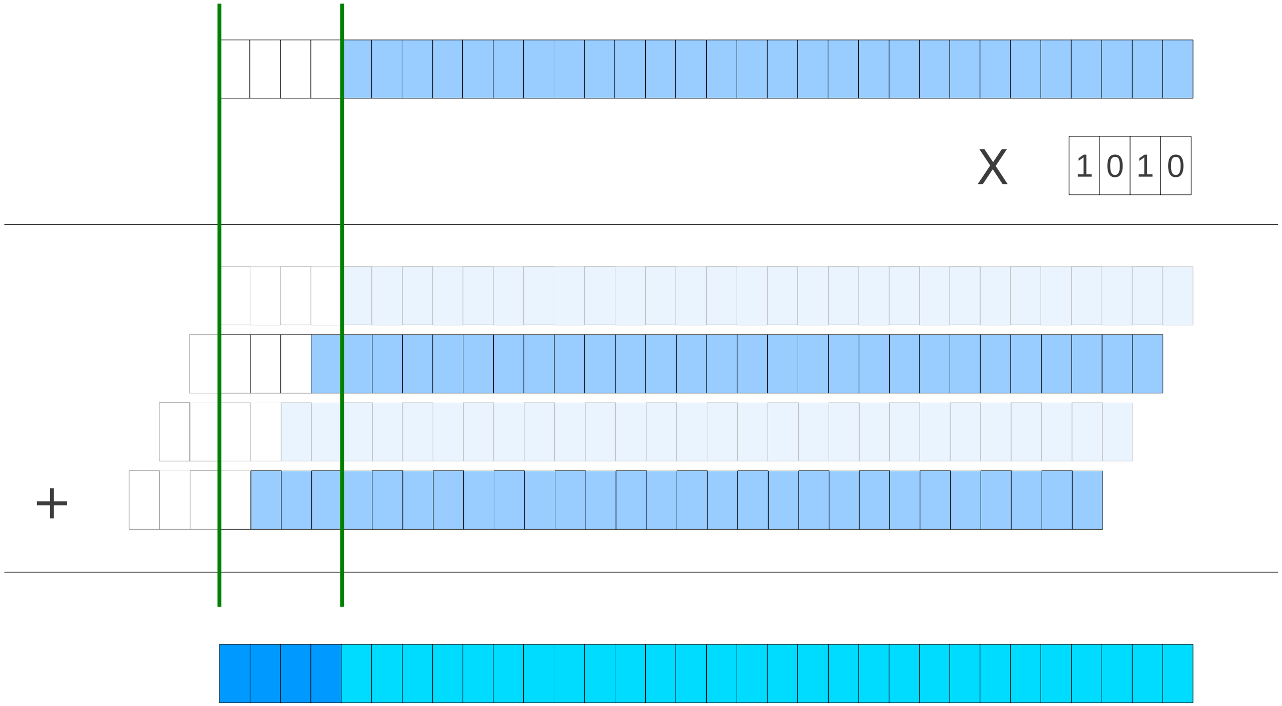
```
low = (low & 0xf) * 10;
```

```
low = (low & 0xf) * 10;
```

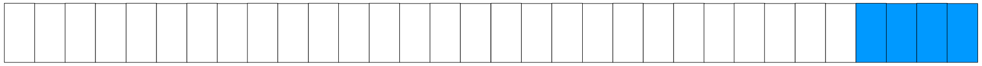


WriteFraction (3)

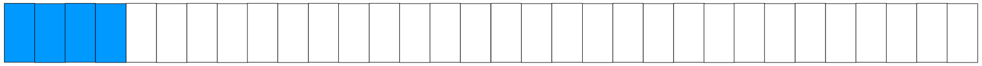
`k * 10L`



`digit = (k *= 10L) >> (longBits - 4);`



`digit << (longBits - 4)`



WriteFraction (4)

```
k = k + (low >> 4) -  (digit << (longBits - 4));
```

Computing ATAN constants instead of LUT

Computing ATAN constants instead of LUT

Computing ATAN constants instead of LUT

References

[1] http://en.wikipedia.org/
[2] "Implementing CORDIC Algorithms", P. Jarvis, Dr Dobb's
[3] ANSI-C version of [2] by P. Knoppers.