Sampling Basics(1B)

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3/10/11

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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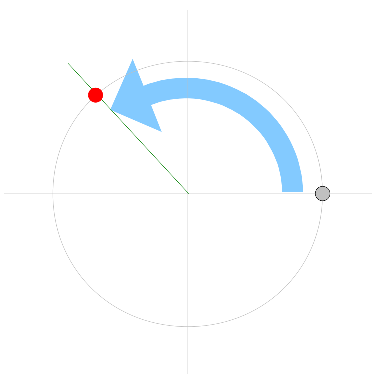
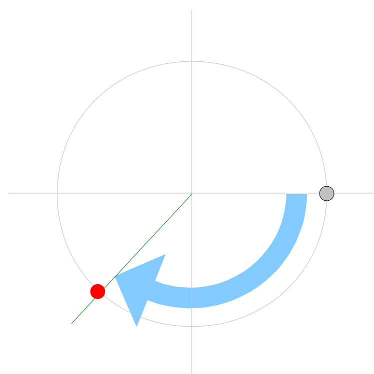
Measuring Rotation Rate

Angular Speed (Frequency)

$$\omega = \frac{2\pi}{T} = 2\pi f$$



$+\omega_0 \; rad \; / \; 1 \; sec$



$-\omega_0 \; rad \; / \; 1 \; sec$

$+\omega_0 \; (rad/sec)$

$-\omega_0 \; (rad/sec)$

- **Negative Angles**

RPM

$$rpm = revolutions \; / \; minute$$

$$\begin{aligned} 1 \; rpm &= 2\pi \; rad \; / \; 1 \; min \\ &= 2\pi \; rad \; / \; 60 \; sec \\ &= \frac{\pi}{30} \; rad/sec \end{aligned}$$

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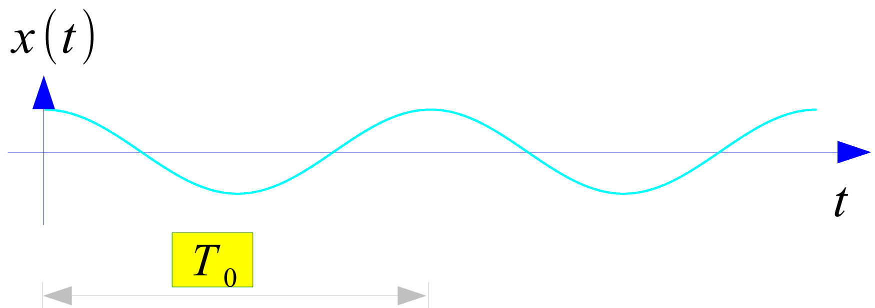
Angular Frequency and Sinusoid

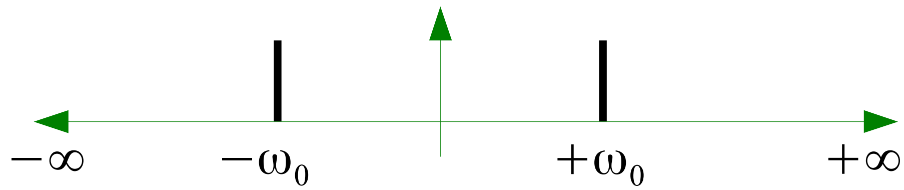
Time Domain



$$\omega_0 = \frac{2\pi}{T_0}$$

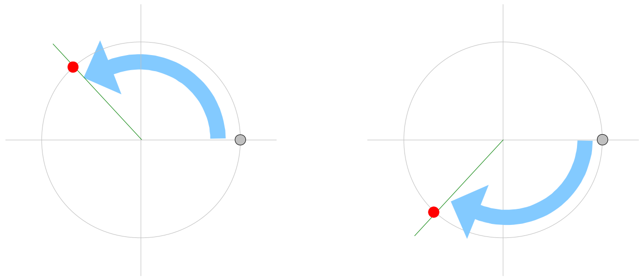
$$x(t) = A\cos(\omega_0 t)$$
$$= \frac{A}{2} e^{j\omega_0 t} + \frac{A}{2} e^{-j\omega_0 t}$$

Frequency Domain



For 1 second

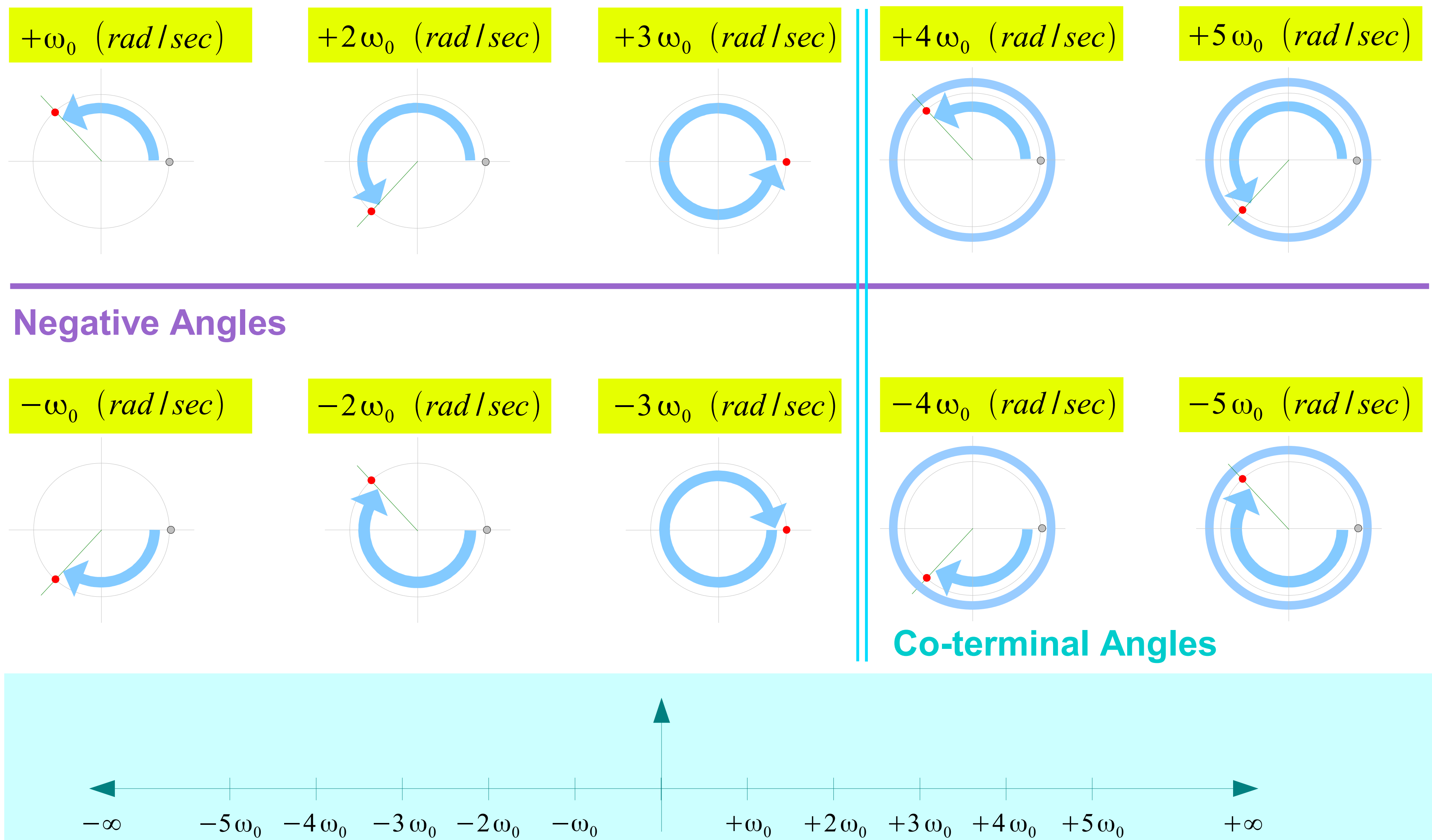
For 1 second



$+\omega_0 \ (rad/sec)$

$-\omega_0 \ (rad/sec)$

Angular Speed Examples



Angular Speed and Frequency

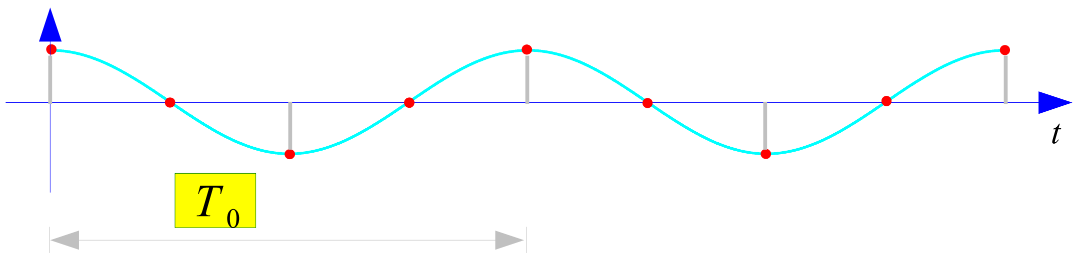
$$\omega = \frac{2\pi}{T} = 2\pi f$$

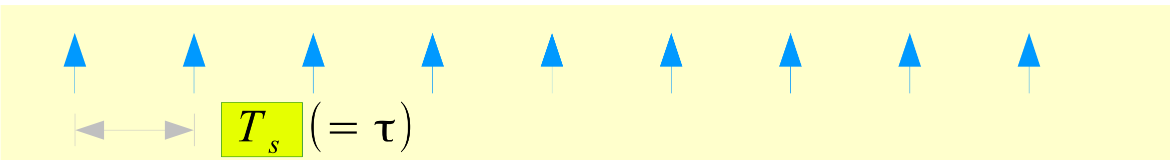
T (sec)	0.01 sec	0.1 sec	1 sec	10 sec	100 sec
f (Hz)	100 Hz	10 Hz	1 Hz	0.1 Hz	0.01 Hz
ω (rad/sec)	200π (rad/sec)	20π (rad/sec)	2π (rad/sec)	0.2π (rad/sec)	0.02π (rad/sec)
	= 628	= 62.8	= 6.28	= 0.628	=0.0628

Sampling

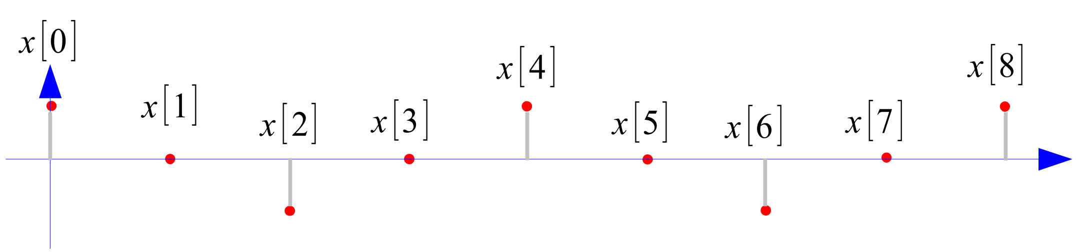
continuous-time signals

$$x(t) = A\cos(\omega_0 t)$$





discrete-time sequence



Sampling Time

$$T_s \ (= \tau)$$

Sequence Time Length

$$T = N \cdot T_s$$

Sampling Frequency

$$f_s = \frac{1}{T_s} \quad \text{(samples / sec)}$$

Signal's Frequency

$$f_0 = \frac{1}{T_0} \quad \text{(cycles / sec)}$$

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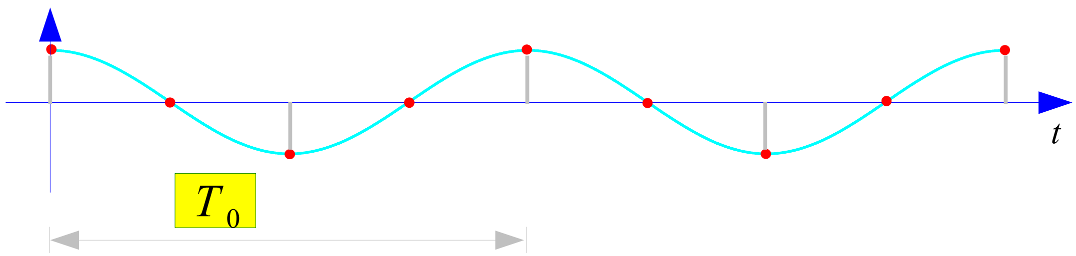
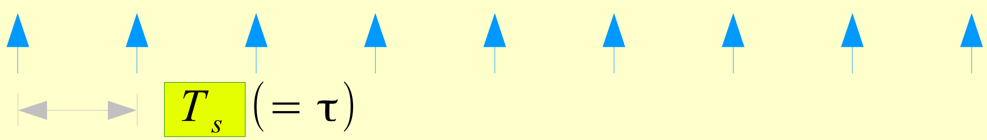
Sampling Frequency

continuous-time signals

$$x(t) = A\cos(\omega_0 t)$$





For 1 second

$\frac{1}{T_s}$ **(samples / sec)**

For 1 sample

1 **(samples)** / T_s **(sec)**

For 1 second

$\frac{1}{T_0}$ **(cycles / sec)**

For 1 cycle

1 **(cycles)** / T_0 **(sec)**

Sampling Time

$$T_s \; (= \tau)$$

Sequence Time Length

$$T = N \cdot T_s$$

Sampling Frequency

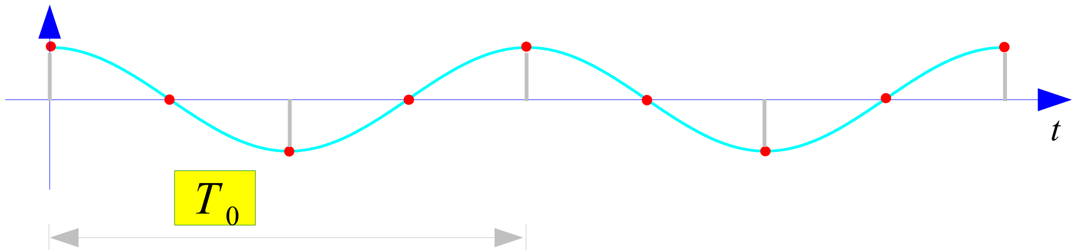
$$f_s = \frac{1}{T_s} \quad \textbf{(samples / sec)}$$

Signal's Frequency

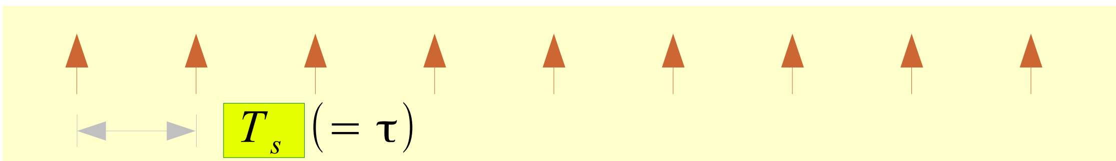
$$f_0 = \frac{1}{T_0} \quad \textbf{(cycles / sec)}$$

Angular Frequencies in Sampling

$$x(t) = A\cos(\omega_0 t)$$

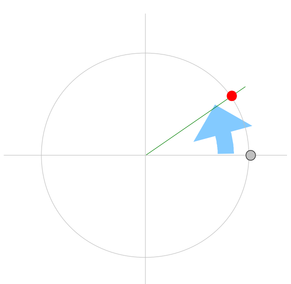


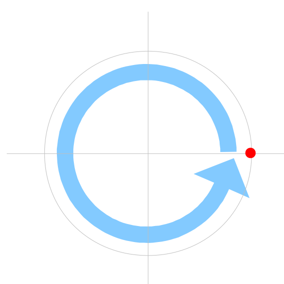
$$\omega_0 = 2\pi f_0 \qquad f_0 = \frac{1}{T_0}$$

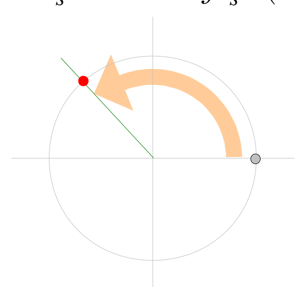


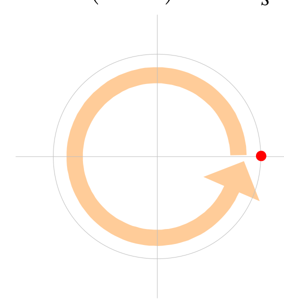
$$\omega_s = 2\pi f_s \qquad f_s = \frac{1}{T_s}$$

continuous-time signals		***sampling sequence***	
For 1 second	*For 1 revolution*	*For 1 second*	*For 1 revolution*
$\omega_0 = 2\pi f_0 \; (rad/sec)$	$2\pi \, (rad) \; / \; T_0 \, (sec)$	$\omega_s = 2\pi f_s \; (rad/sec)$	$2\pi \, (rad) \; / \; T_s \, (sec)$



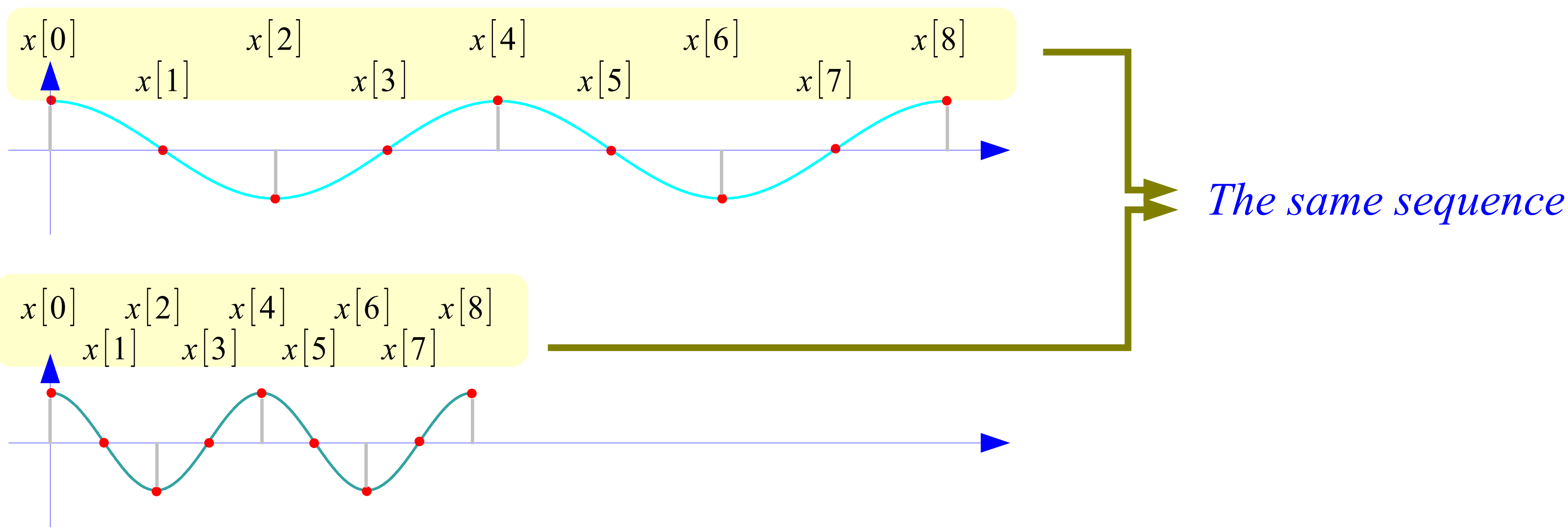






Dimensionless Sequence

$$x[n] \Rightarrow \cdots, \; x[0], \; x[1], \; x[2], \; x[3], \; x[4], \; x[5], \; x[6], \; x[7], \; x[8], \; \cdots$$



Infinite number of continuous time signals

Sampling

The same discrete-time sequence

Sampling of Sinusoid Functions

$$x(t) = A\cos(\omega t + \phi)$$

$$t \rightarrow n\,T_s$$

$$x[n] = x(n\,T_s)$$

$$= A\cos(\omega \cdot n\,T_s + \phi)$$

$$= A\cos(\omega \cdot T_s\, n + \phi)$$

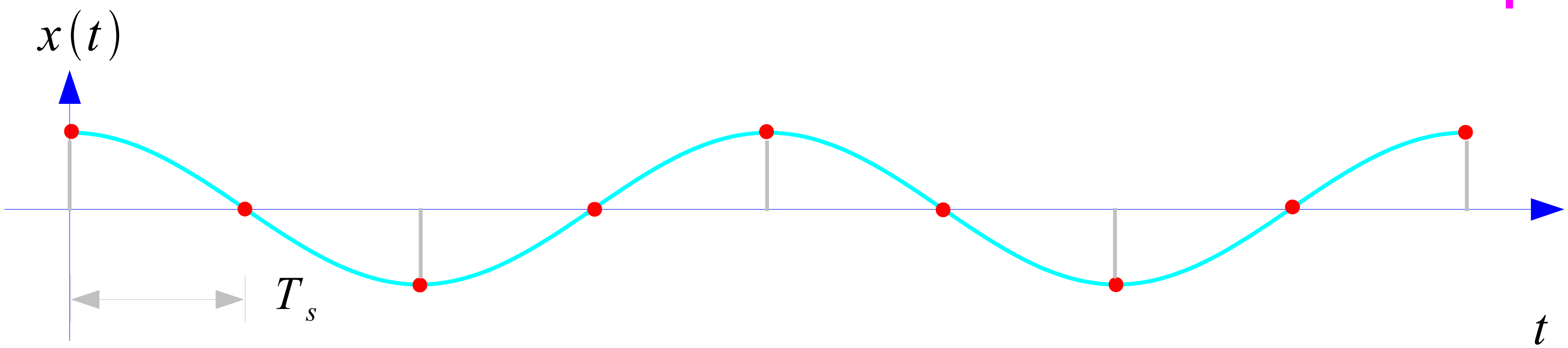
$$= A\cos(\hat{\omega} \cdot n + \phi)$$

$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s}$$

$$\hat{\omega} = \frac{\omega}{f_s} = 2\pi \frac{f}{f_s}$$

Normalized to f_s

Normalized Radian Frequency



Normalized Radian Frequency (1)

continuous-time signals	Sampling	discrete-time sequence
$x(t)$	$t \rightarrow n\,T_s$	$x[n] = x(nT_s)$

Angular Frequency	Sampling	Normalized Radian Frequency
$\omega \quad (rad/sec)$	$\times\, T_s$	$\hat{\omega} = \omega \cdot T_s \quad (rad/sample)$

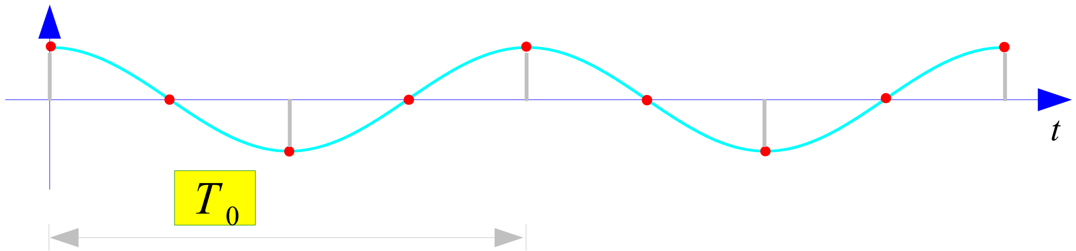
Angular Speed X Sampling Time

Normalized Radian Frequency
can be viewed as
"the <u>angular displacement</u> of a signal
during the period of its <u>sample time</u> T_s"

- Negative Angles
  → folding
- Co-terminal Angles
  → periodic

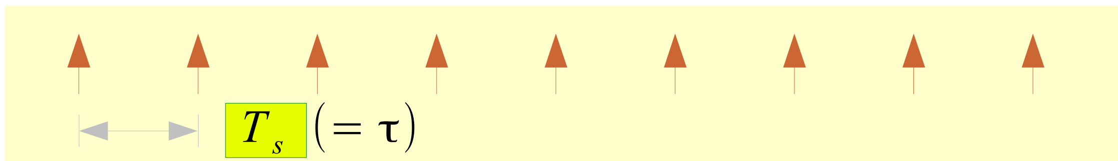
Normalized Radian Frequency (2)

$$x(t) = A\cos(\omega_0 t)$$



$$\omega_0 = 2\pi f_0$$

$$f_0 = \frac{1}{T_0}$$



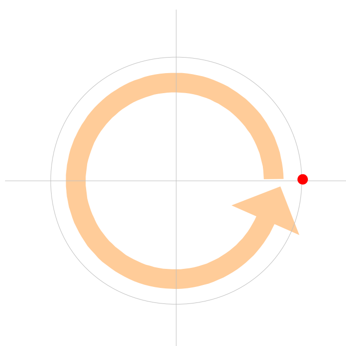
$$\omega_s = 2\pi f_s$$

$$f_s = \frac{1}{T_s}$$

sampling sequence

For 1 sample

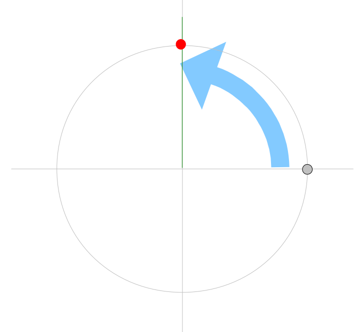
$$2\pi\ (rad)\ /\ T_s\ (sec)$$



continuous-time signals

For T_s second

$$\hat{\omega} = \omega_0 \cdot T_s \quad (rad/sample)$$



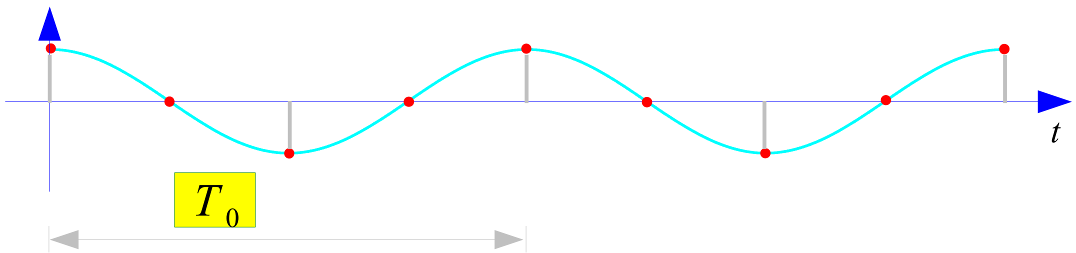
$$\hat{\omega} = \omega T_s$$

$$\hat{\omega} = \frac{\omega}{f_s}$$

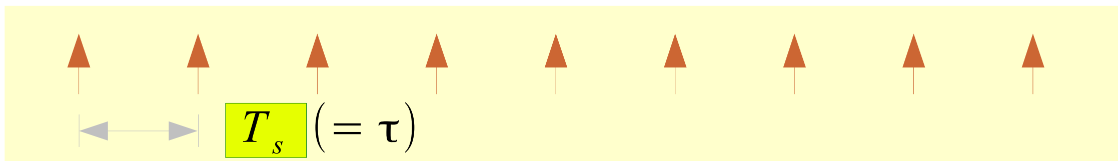
Signal's relative angle position after each of T_s second

Normalized Radian Frequency (3)

$$x(t) = A\cos(\omega_0 t)$$



$$\omega_0 = 2\pi f_0 \qquad f_0 = \frac{1}{T_0}$$



$$\omega_s = 2\pi f_s \qquad f_s = \frac{1}{T_s}$$

Normalized Frequency

$$\frac{f_0}{f_s} \ \frac{(cycle\ /\ sec)}{(sample\ /\ sec)} \Rightarrow \frac{f_0}{f_s} \ (cycle\ /\ sample)$$

Normalized Radian Frequency

$$2\pi \frac{(rad)}{(cycle)} \cdot \frac{f_0}{f_s} \frac{(cycle)}{(sample)} \Rightarrow \frac{\omega_0}{f_s} \ (rad\ /\ sample)$$

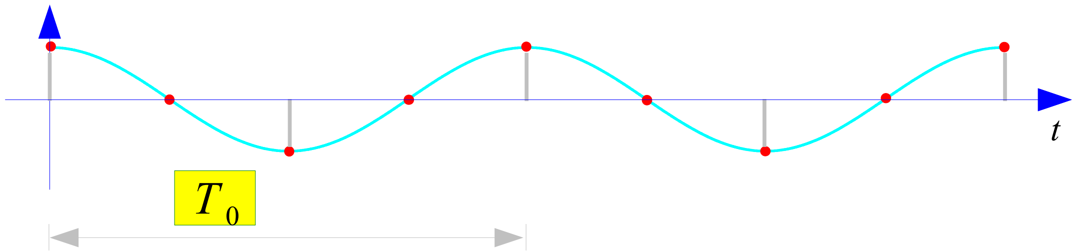
$$f \in \left(-\frac{f_s}{2}, \quad +\frac{f_s}{2} \right)$$

$$\frac{f}{f_s} \in \left(-\frac{1}{2}, \quad +\frac{1}{2} \right)$$

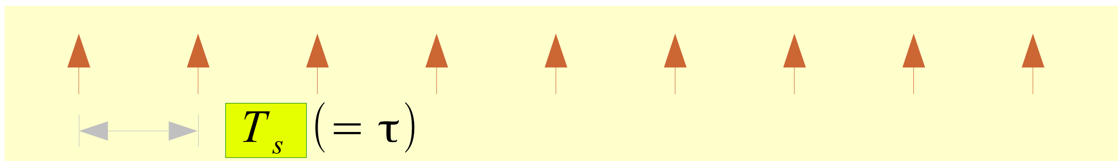
$$\hat{\omega} \in \left(-\pi, \quad +\pi \right)$$

Normalized Radian Frequency (4)

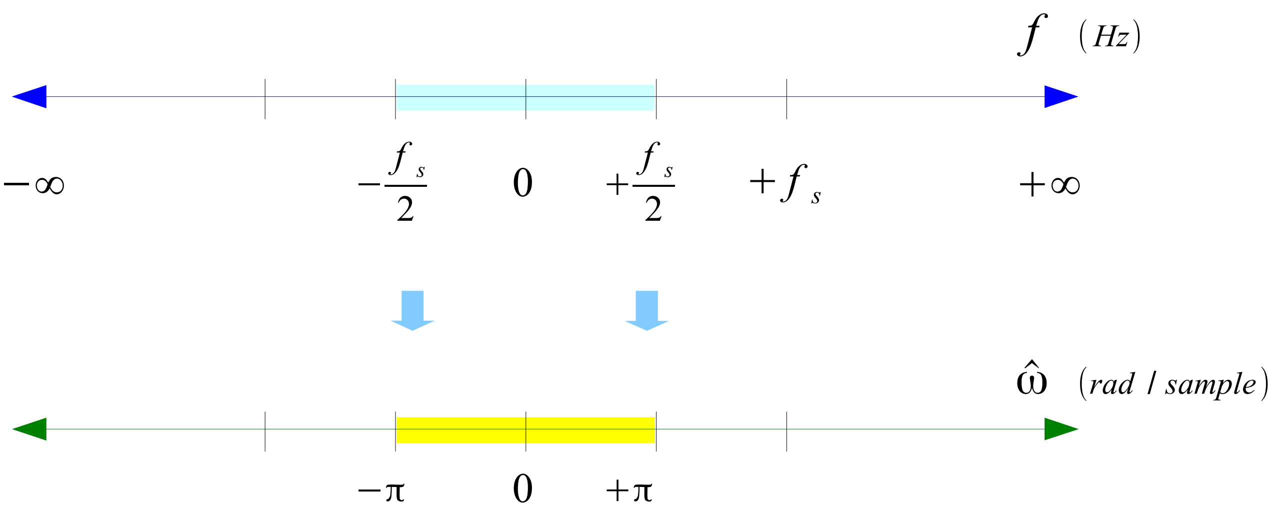
$$x(t) = A \cos(\omega_0 t)$$

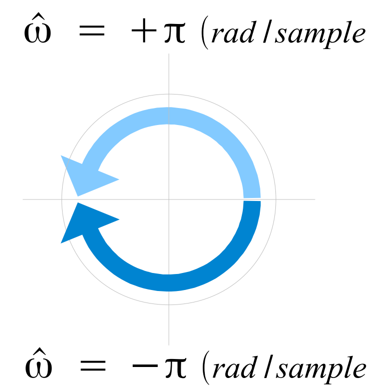


$$\omega_0 = 2\pi f_0 \qquad f_0 = \frac{1}{T_0}$$



$$\omega_s = 2\pi f_s \qquad f_s = \frac{1}{T_s}$$





Example (1)

$$\omega_s = 2\pi f_s \ (rad/sec)$$

$$2\pi\,(rad) \ / \ T_s\,(sec)$$

$$\hat{\omega}_1 = \omega_1 \cdot T_s \ (rad/sample)$$

$$\hat{\omega}_2 = \omega_2 \cdot T_s \ (rad/sample)$$

$$A\cos(\omega_1 t + \phi)$$

$$\omega_1 = \frac{\omega_s}{2}$$

$$\hat{\omega}_1 = \pi\,(rad)$$

$$\omega_1 = -\frac{\omega_s}{2}$$

$$\hat{\omega}_1 = -\pi\,(rad)$$

$$A\cos(\omega_2 t + \phi)$$

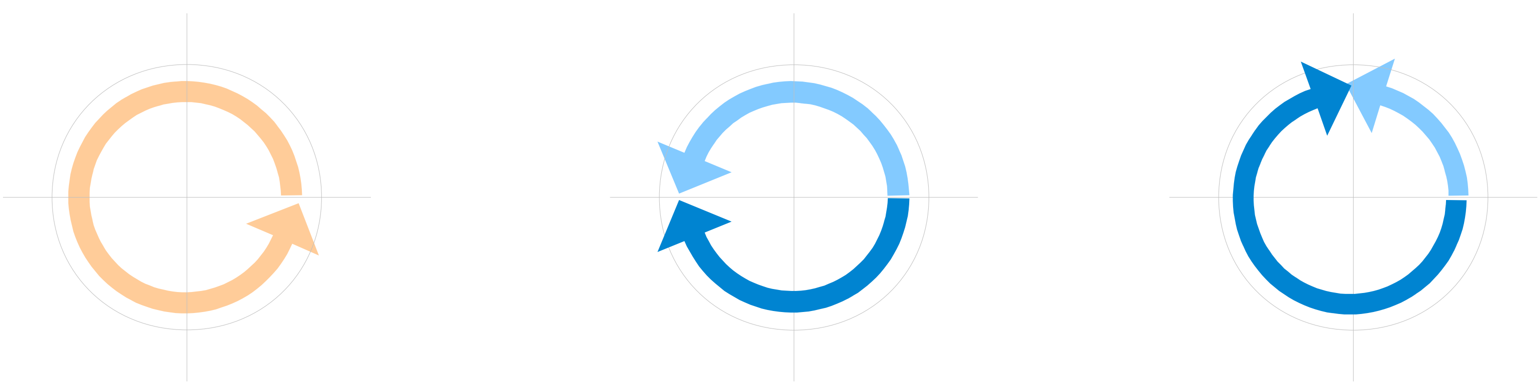
$$\omega_2 = \frac{\omega_s}{4}$$

$$\hat{\omega}_2 = \frac{\pi}{2}\,(rad)$$

$$\omega_2 = -\frac{3\omega_s}{2}$$

$$\hat{\omega}_2 = -\frac{3\pi}{2}\,(rad)$$

Negative Angles



Example (2)

$$\omega_s = 2\pi f_s \quad (rad/sec)$$

$$2\pi\,(rad) \;/\; T_s\,(sec)$$

$$\hat{\omega}_1 = \omega_1 \cdot T_s \quad (rad/sample)$$

$$\hat{\omega}_2 = \omega_2 \cdot T_s \quad (rad/sample)$$

Co-terminal Angles

$$A\cos(\omega_1 t + \phi)$$

$$\omega_1 = \frac{\omega_s}{2}$$

$$\hat{\omega}_1 = \pi\,(rad)$$

$$\omega_1 = \frac{\omega_s}{2} + \omega_s$$

$$\hat{\omega}_1 = \pi + 2\pi\,(rad)$$

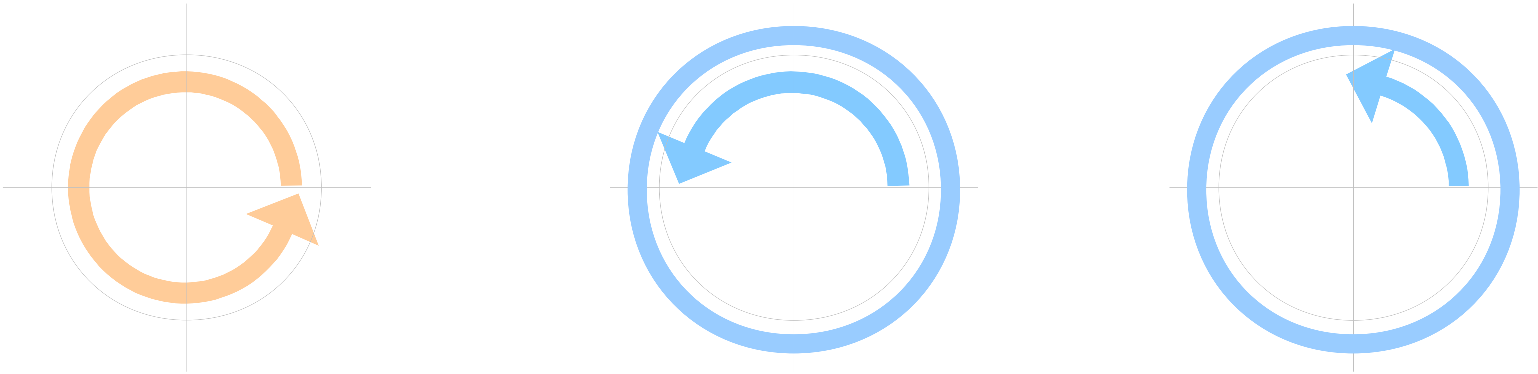
$$A\cos(\omega_2 t + \phi)$$

$$\omega_2 = \frac{\omega_s}{4}$$

$$\hat{\omega}_2 = \frac{\pi}{2}\,(rad)$$

$$\omega_2 = \frac{\omega_s}{4} + \omega_s$$

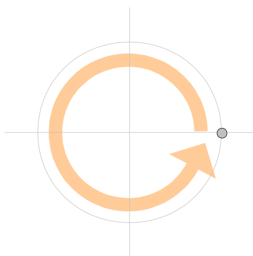
$$\hat{\omega}_2 = \frac{\pi}{2} + 2\pi\,(rad)$$



Co-terminal Angles (1)

For 1 sample

$2\pi\ (rad)\ /\ T_s\ (sec)$

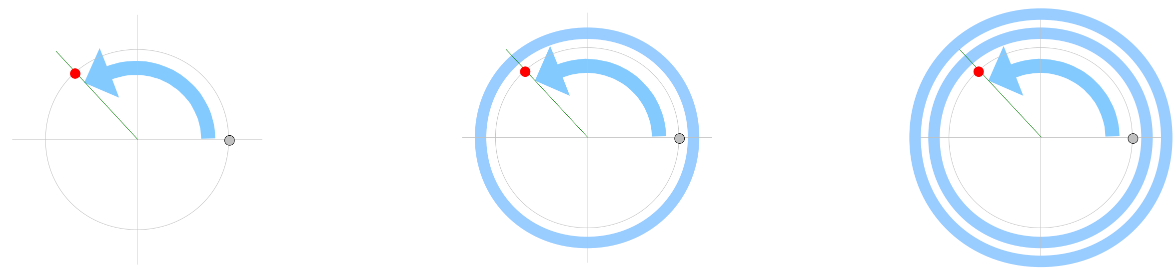


$\hat{\omega} = \omega \cdot T_s \quad (rad/sample)$

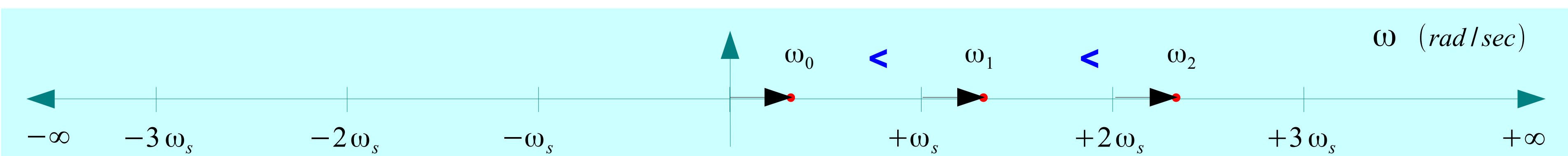
$= \omega / f_s \quad (rad/sample)$

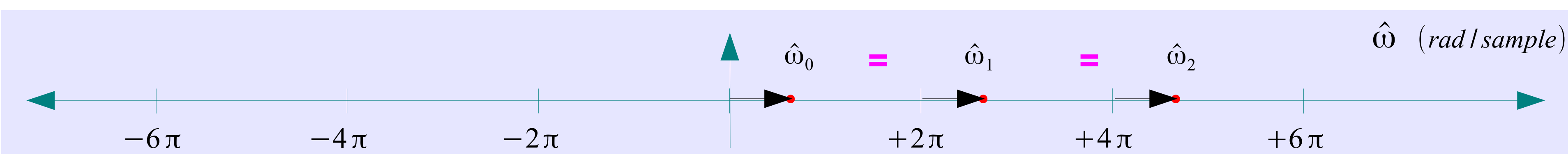
For T_s second

$\hat{\omega} = \omega \cdot T_s \quad (rad/sample)$



f_0	$f_0 + f_s$	$f_0 + 2f_s$
$\omega_0 = 2\pi f_0$	$\omega_1 = 2\pi(f_0 + f_s)$	$\omega_2 = 2\pi(f_0 + 2f_s)$
$\hat{\omega}_0 \quad (rad/sample)$	$\hat{\omega}_0 + 2\pi \quad (rad/sample)$	$\hat{\omega}_0 + 4\pi \quad (rad/sample)$

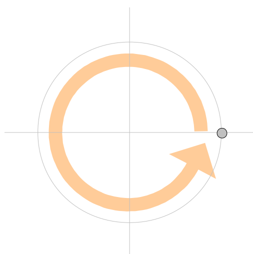




Co-terminal Angles (2)

For 1 sample

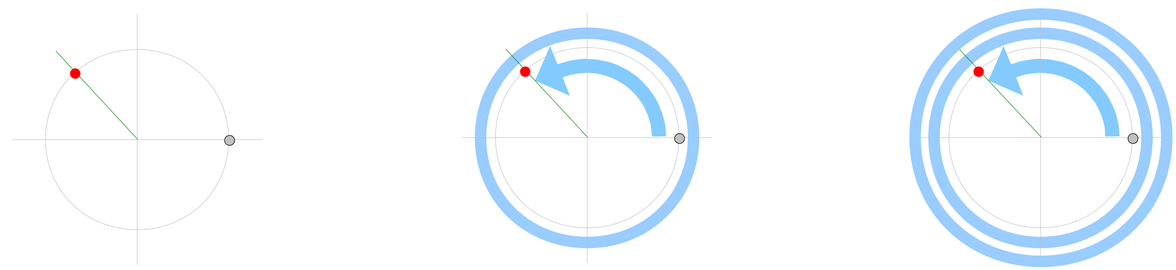
$2\pi\ (rad)\ /\ T_s\ (sec)$



$$\hat{\omega} = \omega \cdot T_s \quad (rad/sample)$$
$$= \omega / f_s \quad (rad/sample)$$

For T_s second

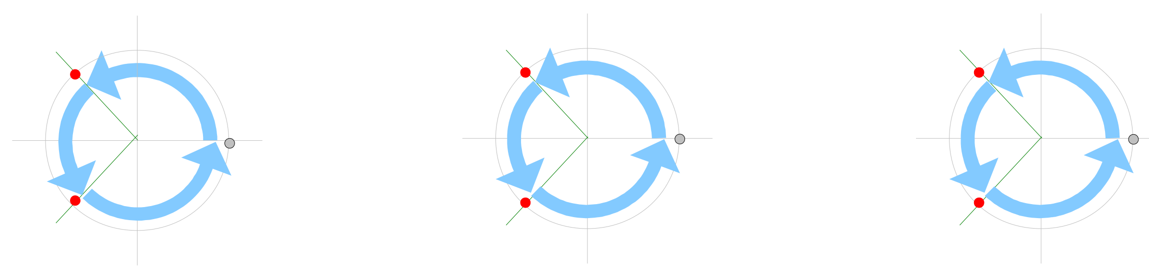
$$\hat{\omega} = \omega \cdot T_s \quad (rad/sample)$$



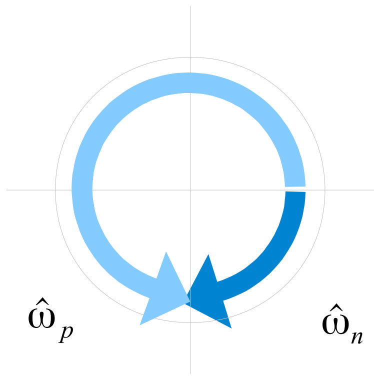
f_0	$f_0 + f_s$	$f_0 + 2f_s$
$\omega_0 = 2\pi f_0$	$\omega_1 = 2\pi(f_0 + f_s)$	$\omega_2 = 2\pi(f_0 + 2f_s)$
$\hat{\omega}_0 \quad (rad/sample)$	$\hat{\omega}_0 + 2\pi \quad (rad/sample)$	$\hat{\omega}_0 + 4\pi \quad (rad/sample)$

Co-terminal Angles

The same angular positions after each sample time.



Positive & Negative Angles (1)



$$\overset{+}{\hat{\omega}_p} - \overset{-}{\hat{\omega}_n} = 2\pi$$

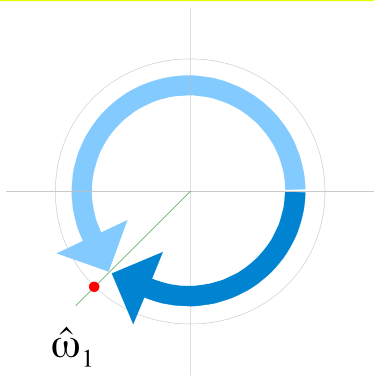
Positive Normalized Rad Freq

$$\underset{+}{\hat{\omega}_p} = 2\pi + \underset{-}{\hat{\omega}_n}$$

Negative Normalized Rad Freq

$$\underset{-}{\hat{\omega}_n} = \underset{+}{\hat{\omega}_p} - 2\pi$$

$$\frac{f_s}{2} < f_1 < f_s$$



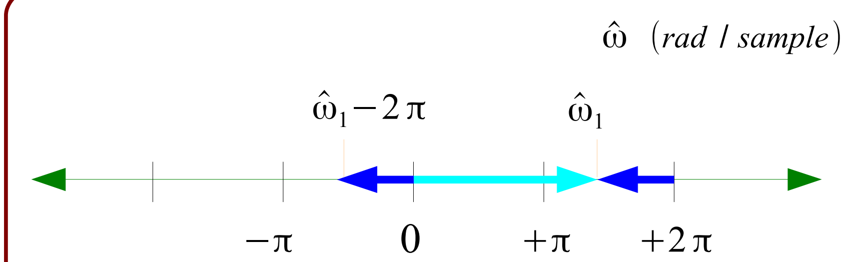
Positive Angle

$$+\pi < \hat{\omega}_1 < 2\pi$$

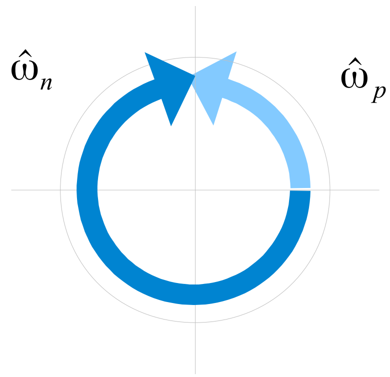
Negative Angle

$$-\pi < \hat{\omega}_1 - 2\pi < 0$$

Normalized Radian Frequency



Positive & Negative Angles (2)



$$\overset{+}{\hat{\omega}_p} - \overset{-}{\hat{\omega}_n} = 2\pi$$

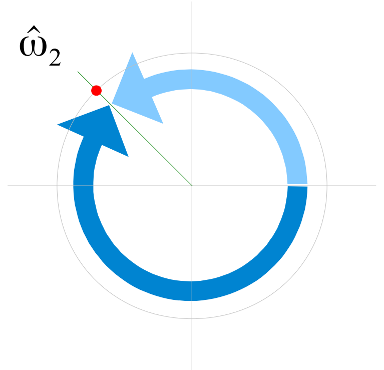
Positive Normalized Rad Freq

$$\underset{+}{\hat{\omega}_p} = 2\pi + \underset{-}{\hat{\omega}_n}$$

Negative Normalized Rad Freq

$$\underset{-}{\hat{\omega}_n} = \underset{+}{\hat{\omega}_p} - 2\pi$$

$$-f_s < f_2 < -\frac{f_s}{2}$$



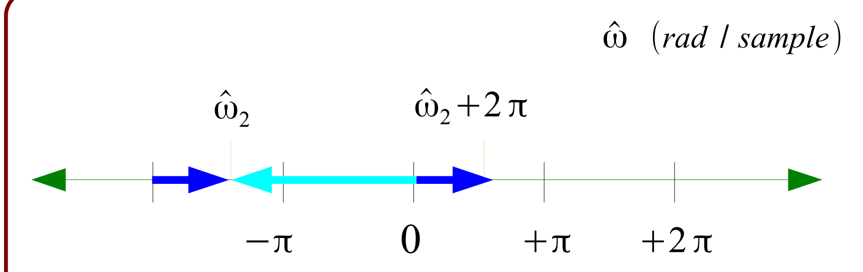
Negative Angle

$$-2\pi < \hat{\omega}_2 < -\pi$$

Positive Angle

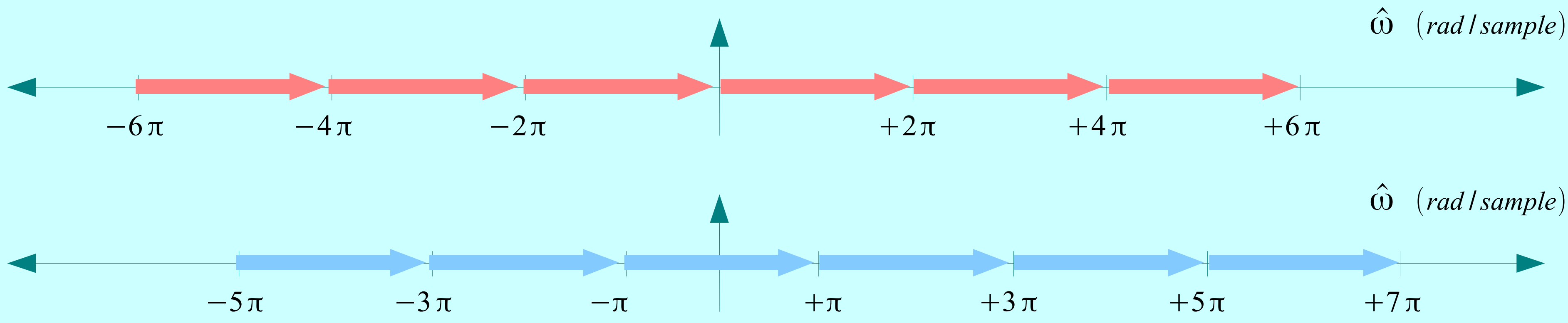
$$0 < 2\pi + \hat{\omega}_2 < \pi$$

Normalized Radian Frequency

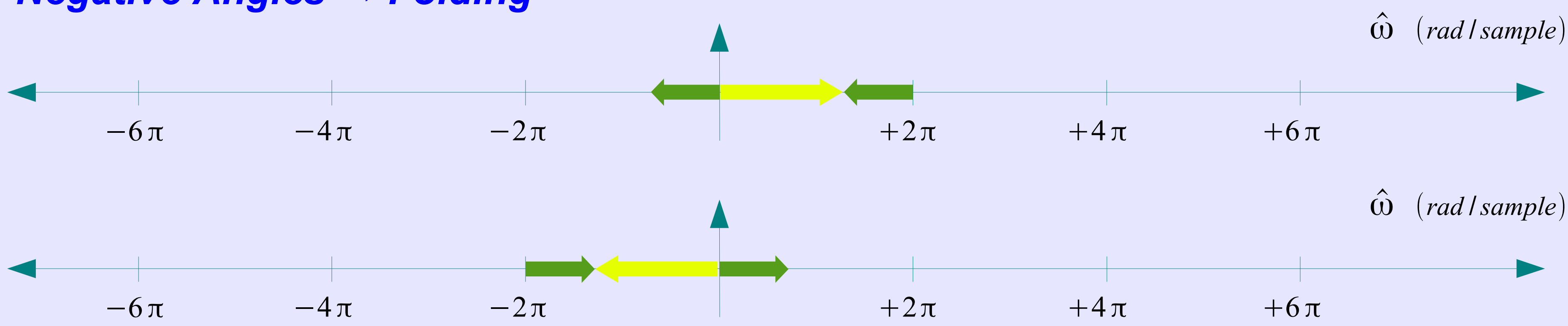


Periodic and Folding

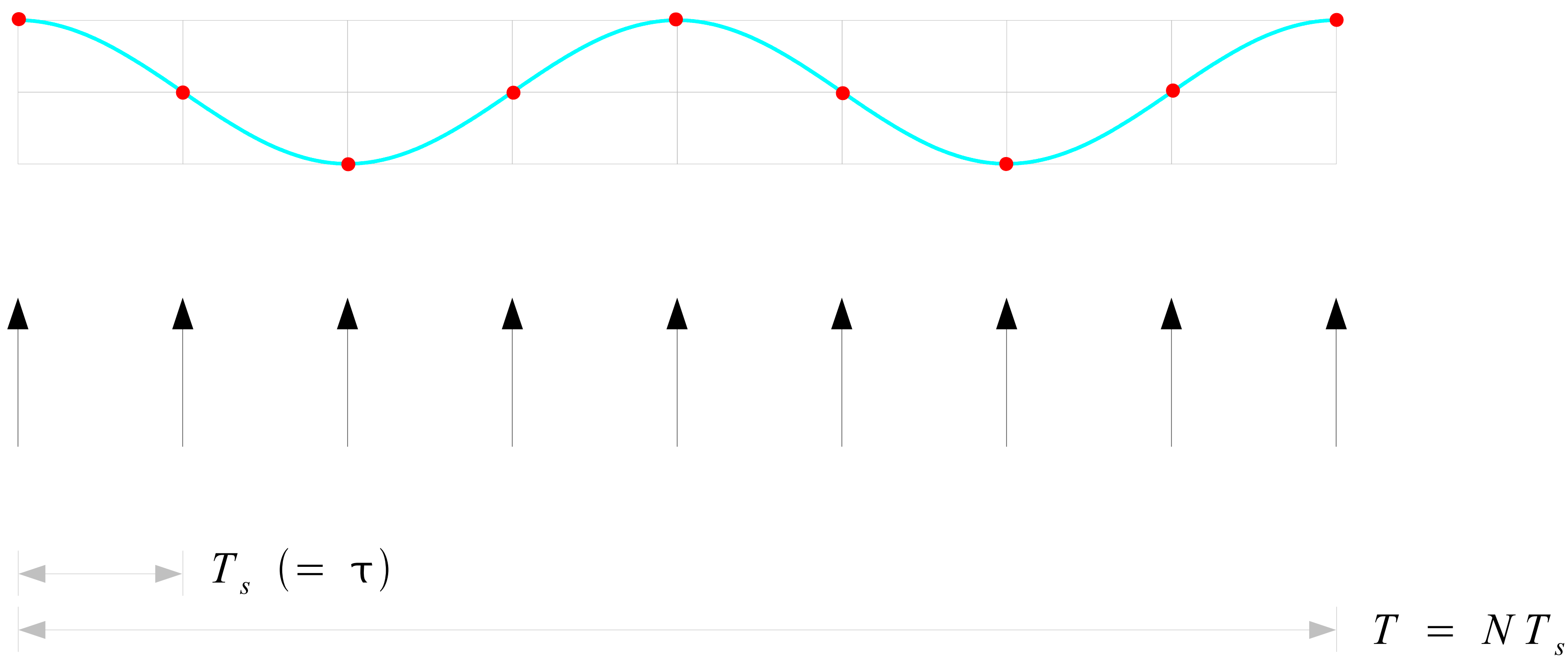
Co-terminal Angles → Periodic



Negative Angles → Folding



Example



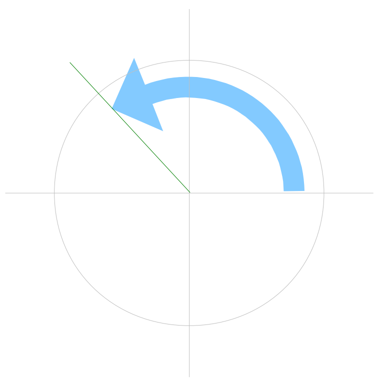
Sampling Time T_s

Sequence Time Length $T = N T_s$

Sampling Frequency $f_s = \dfrac{1}{T_s}$ *(samples per second)*

Sampling

$$\omega_s = 2\pi f_s \quad (rad/sec)$$



$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \quad (rad/sec)$$

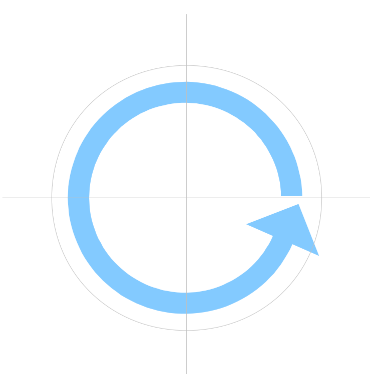
$$f_1 = \frac{f_s}{2} \quad (rad/sec)$$

$$\omega_2 = 2\pi f_2$$

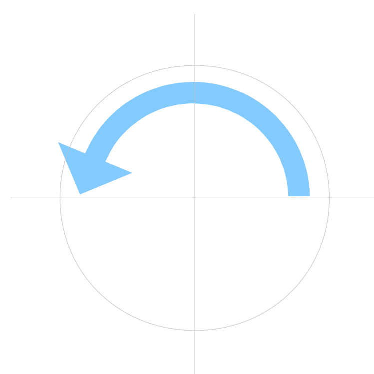
$$\omega_2 = -\frac{\omega_s}{2} \quad (rad/sec)$$

$$f_2 = -\frac{f_s}{2} \quad (rad/sec)$$

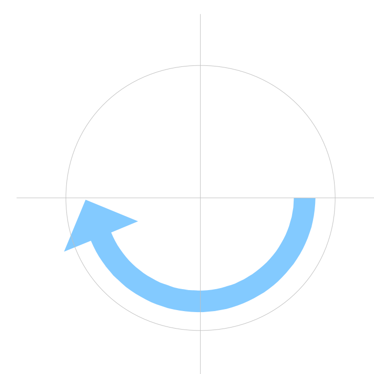
$$2\pi \ (rad) \ / \ T_s \ (sec)$$



$$\pi \ (rad) \ / \ T_s \ (sec)$$



$$-\pi \ (rad) \ / \ T_s \ (sec)$$



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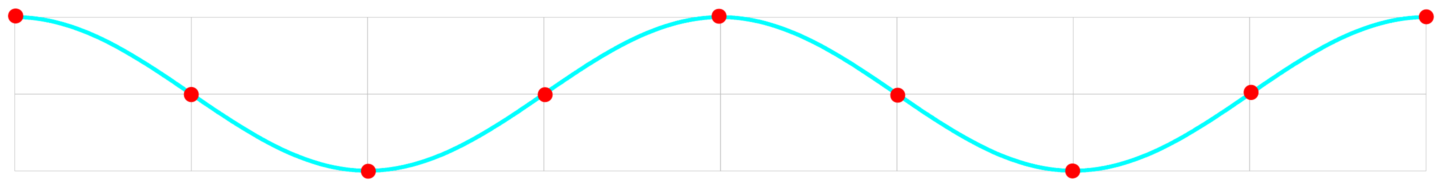
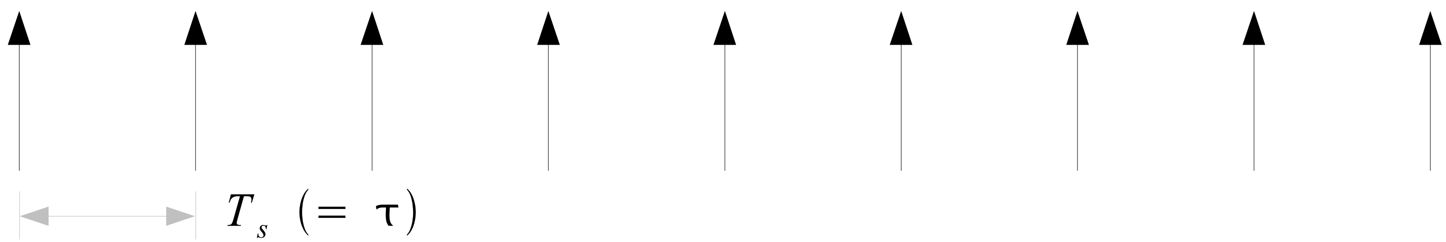
Sampling

$\omega_1 = 2\pi f_1 \ (rad/sec)$

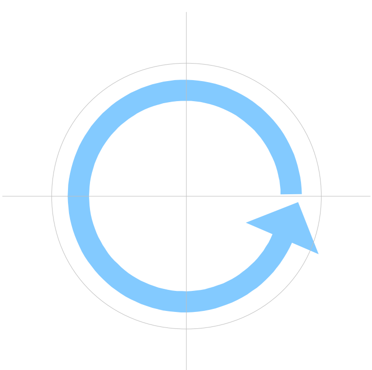


$\omega_s = 2\pi f_s \ (rad/sec)$

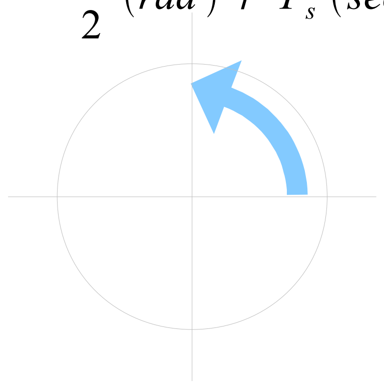


$T_s \ (= \tau)$

$2\pi\ (rad)\ /\ T_s\ (sec)$



$\frac{\pi}{2}\ (rad)\ /\ T_s\ (sec)$



For the period of T_s

Angular displacement $\frac{\pi}{2}\ (rad)$

$$\hat{\omega} = \omega \cdot T_s \ (rad)$$

$$= 2\pi f_1 \cdot T_s \ (rad)$$

$$= 2\pi \frac{f_s}{4} \cdot T_s \ (rad)$$

$$= \frac{\pi}{2} \ (rad)$$

Angular Frequencies in Sampling

continuous-time signals

Signal Frequency

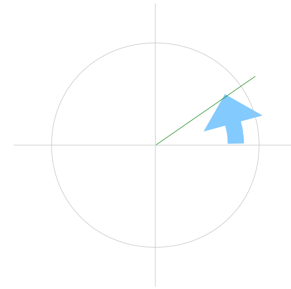
$$f_0 = \frac{1}{T_0}$$

Signal Angular Frequency

$$\omega_0 = 2\pi f_0 \ (rad/sec)$$

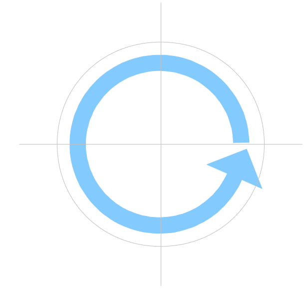
For 1 second

$\omega_0 = 2\pi f_0 \ (rad/sec)$



For 1 revolution

$2\pi \ (rad) \ / \ T_0 \ (sec)$



sampling sequence

Sampling Frequency

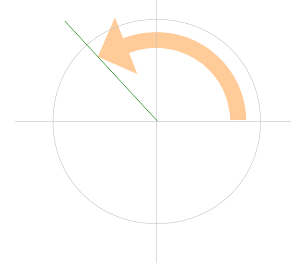
$$f_s = \frac{1}{T_s}$$

Sampling Angular Frequency

$$\omega_s = 2\pi f_s \ (rad/sec)$$

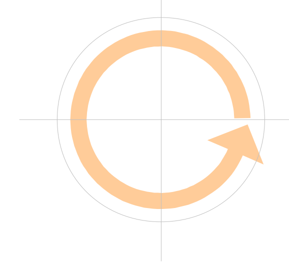
For 1 second

$\omega_s = 2\pi f_s \ (rad/sec)$



For 1 revolution

$2\pi \ (rad) \ / \ T_s \ (sec)$



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann