$$x \uparrow \hat x \Leftrightarrow x \to \hat x^-$$

x \uparrow \hat x \Leftrightarrow x \to \hat x ^-

$x \uparrow \hat x$ x tends to x hat from below

left continuity does not imply continuity; you can have discontinuity.

$$\lim_{x \uparrow \hat x} f(x) = \lim_{x \to \hat x^-} f(x) = f(\hat x)$$

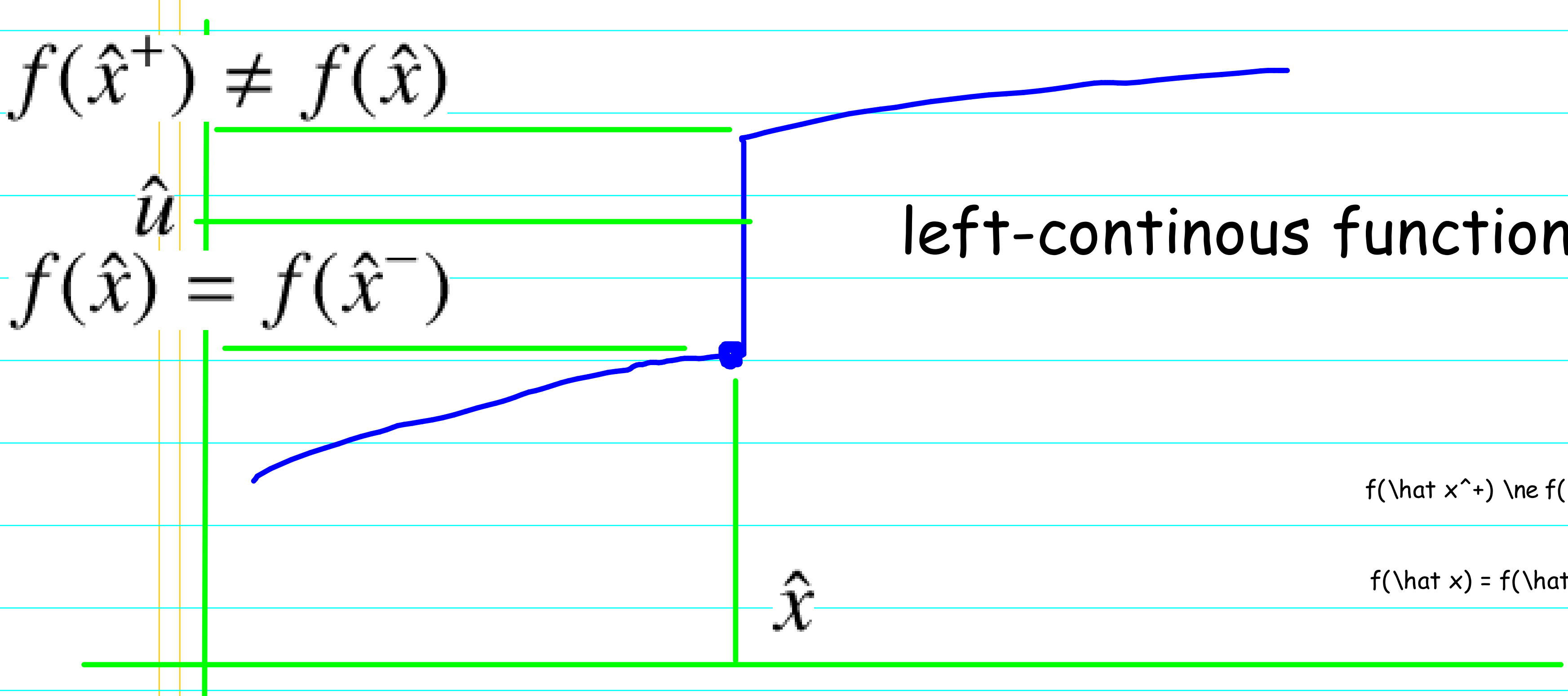
\lim_{x \uparrow \hat x} f(x) = \lim_{x \to \hat x^-} f(x) = f(\hat x)

but it is possible that

$$\lim_{x \downarrow \hat x} f(x) = \lim_{x \to \hat x^+} f(x) \ne f(\hat x)$$

\lim_{x \downarrow \hat x} f(x) = \lim_{x \to \hat x^+} f(x) \ne f(\hat x)

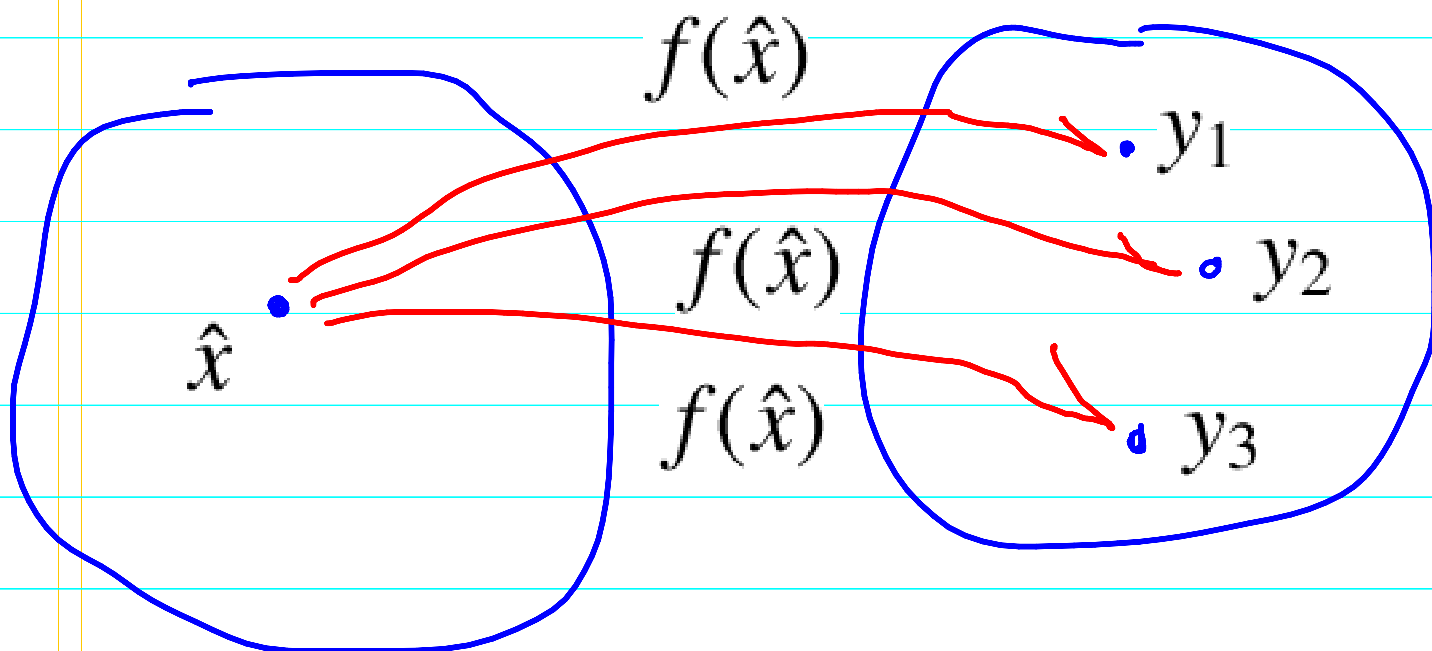
and hence the figure in my hand-written notes.



f(\hat x^+) \ne f(\hat x)

f(\hat x) = f(\hat x^-)

HW: Find the definition of a function, and answer this question: Can a function have multi-values ? i.e., could $f(\hat{x})$ have different values ?



A function is a one-to-one mapping (injective ?)

HW: Draw a figure to give an example of a right-continuous function.

continuity means that you must have both left and right continuity at the same time.

inf = infimum (similar, but not the same as minimum)

take the fig. on p.1: left continuous function

define: $\hat{u} := f(\hat{x}^+)$

\hat u := f(\hat x^+)

$\min\{x \mid f(x) \ge \hat{u} := f(\hat{x}^+)\}$ does not exist; why ?

\min \{ x \ | \ f(x) \ge \hat u := f(\hat x^+) \}

$$\inf\{x \mid f(x) \ge \hat{u} := f(\hat{x}^+)\} = \hat{x}$$

\inf \{ x \ | \ f(x) \ge \hat u := f(\hat x^+) \} = \hat x

$$\hat{x}^+ = \hat{x} + \epsilon$$

\hat x^+ = \hat x + \epsilon

$$\epsilon > 0$$

\epsilon > 0

Consider a sequence $\{\epsilon_i > 0,\ i = 1, 2, \cdots, \infty\}$

\{ \epsilon_i > 0, \ i=1, 2, \cdots, \infty \}

which generates the sequence

$$\{\hat{x}_i^+\} = \{\hat{x} + \epsilon_i,\ i = 1, 2, \cdots, \infty\}$$

\{ \hat x^+_i \} = \{ \hat x + \epsilon_i , \ i=1, 2, \cdots, \infty \}

$\hat u$ needs not be equal to $f(\hat x^+)$

$$\min\{ x \mid f(x) \ge \hat u \cancel{:= f(\hat x^+)} \} := \underset{x > \hat x}{\text{argmin }} f(x)$$

\min \{ x \ | \ f(x) \ge \hat u := f(\hat x^+) \} := \underset{x > \hat x}{\text{argmin }} f(x)

$$\bar x := \underset{x > \hat x}{\text{argmin }} f(x)$$

\bar x := \underset{x > \hat x}{\text{argmin }} f(x)

does not exist for left-continuous function at x hat

inf = infimum = greatest lower bound

Thu Nov 3, 2011 10:08 AM

Refer to the figure on p.1, define $\hat u$ to be anywhere above $f(\hat x)$ but below $f(\hat x^+)$.

$$\min\{ x \mid f(x) \ge \hat u \} := \underset{x}{\text{argmin }} \{ f(x) \ge \hat u \}$$

\min \{ x \ | \ f(x) \ge \hat u \} := \underset{x}{\text{argmin }} \{ f(x) \ge \hat u \}

does not exist. Proof by contradiction: Suppose that x tilde is the minimizer, i.e.,

$$\tilde x = \min\{ x \mid f(x) \ge \hat u \} := \underset{x}{\text{argmin }} \{ f(x) \ge \hat u \}$$

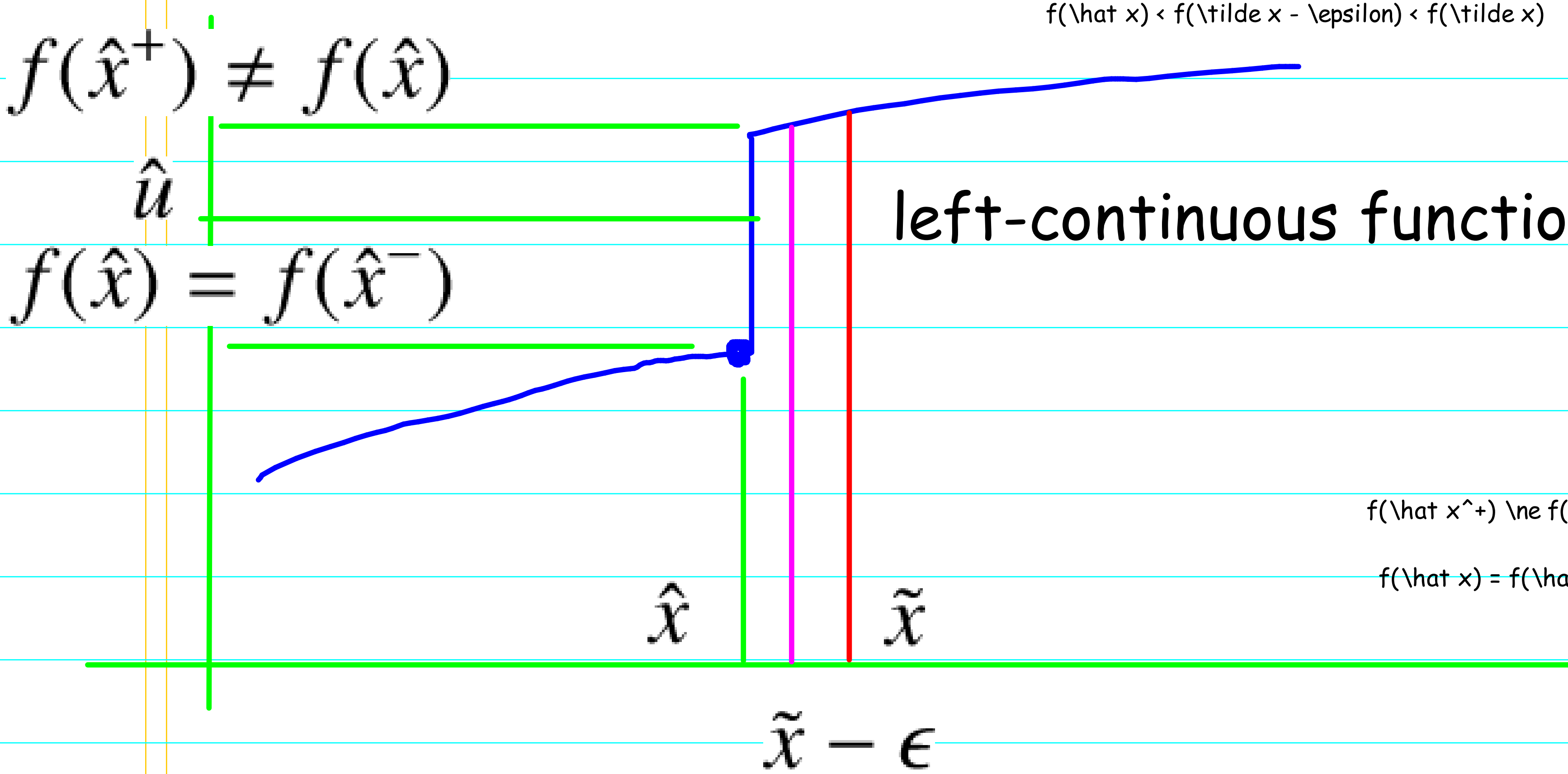
\tilde x = \min \{ x \ | \ f(x) \ge \hat u \} := \underset{x}{\text{argmin }} \{ f(x) \ge \hat u \}

and assume that the function is monotonically increasing, then you can always find $\hat{x} < \tilde{x} - \epsilon < \tilde{x}$

such that $f(\hat{x}) < f(\tilde{x} - \epsilon) < f(\tilde{x})$

\hat x < \tilde x - \epsilon < \tilde x

f(\hat x) < f(\tilde x - \epsilon) < f(\tilde x)



f(\hat x^+) \ne f(\hat x)

f(\hat x) = f(\hat x^-)

and thus $\tilde{x} - \epsilon$ is the new minimum ! and hence the minimum does not exist.

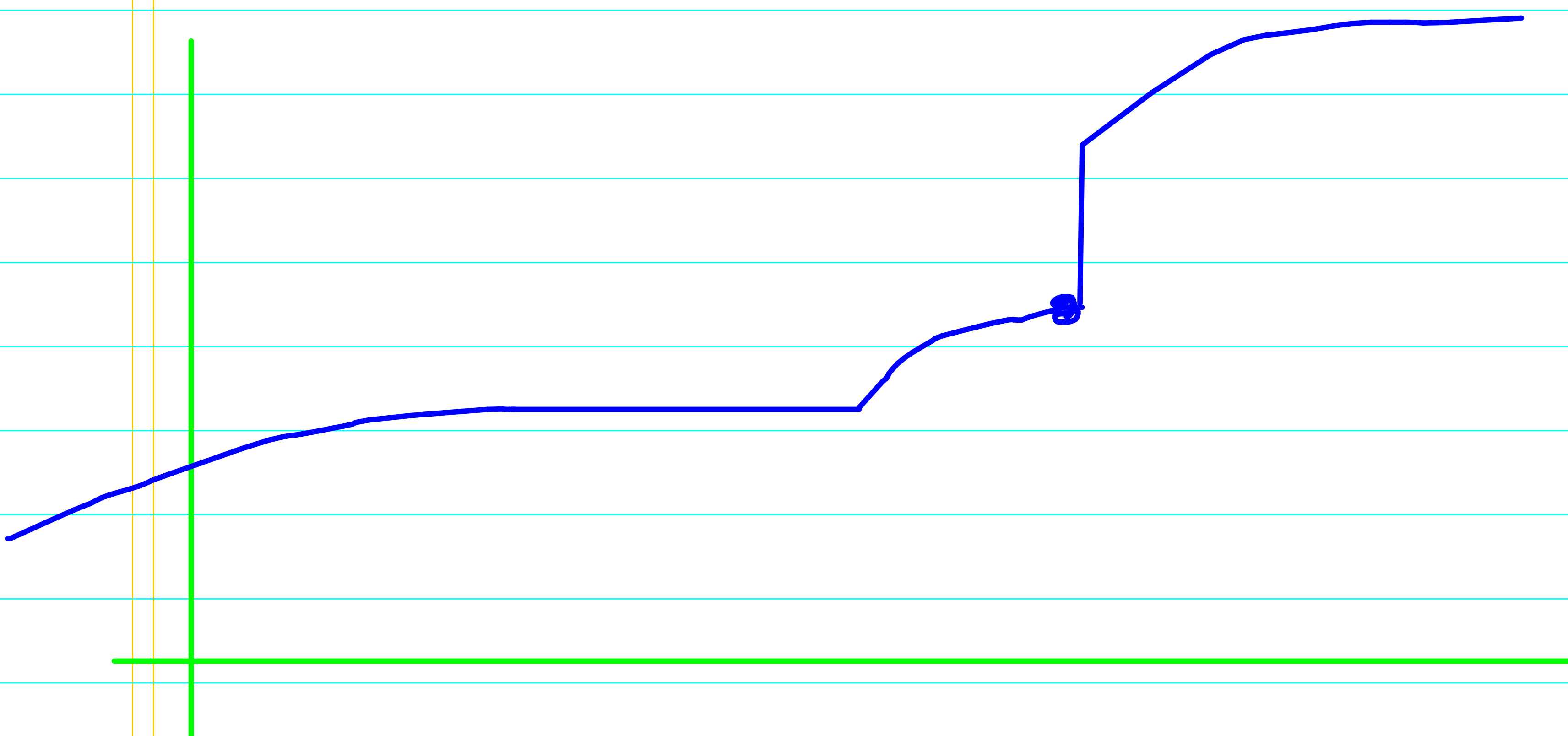
But the infimum (greatest lower bound) exists, and that's

$$\hat{x} = \inf\{x \mid f(x) \geq \hat{u}\} := \underset{x}{\text{arginf }} \{f(x) \geq \hat{u}\}$$

\hat x = \inf \{ x \ | \ f(x) \ge \hat u \} := \underset{x}{\text{arginf }} \{ f(x) \ge \hat u \}

Generalize the above argument (proof by contradiction) to left-continuous functions (not just monotonically increasing functions).

A cdf must be increasing, even though not nec. monotonically increasing; check this statement.



A cdf can be discontinuous, since it can be left-continuous, and therefore discontinuous.

Need to explain Eq.(2.7) in Xiu 2010 p.15.