# Double Integrals (5A)

- Double Integral
- Double Integrals in Polar Coordinates
- Green's Theorem

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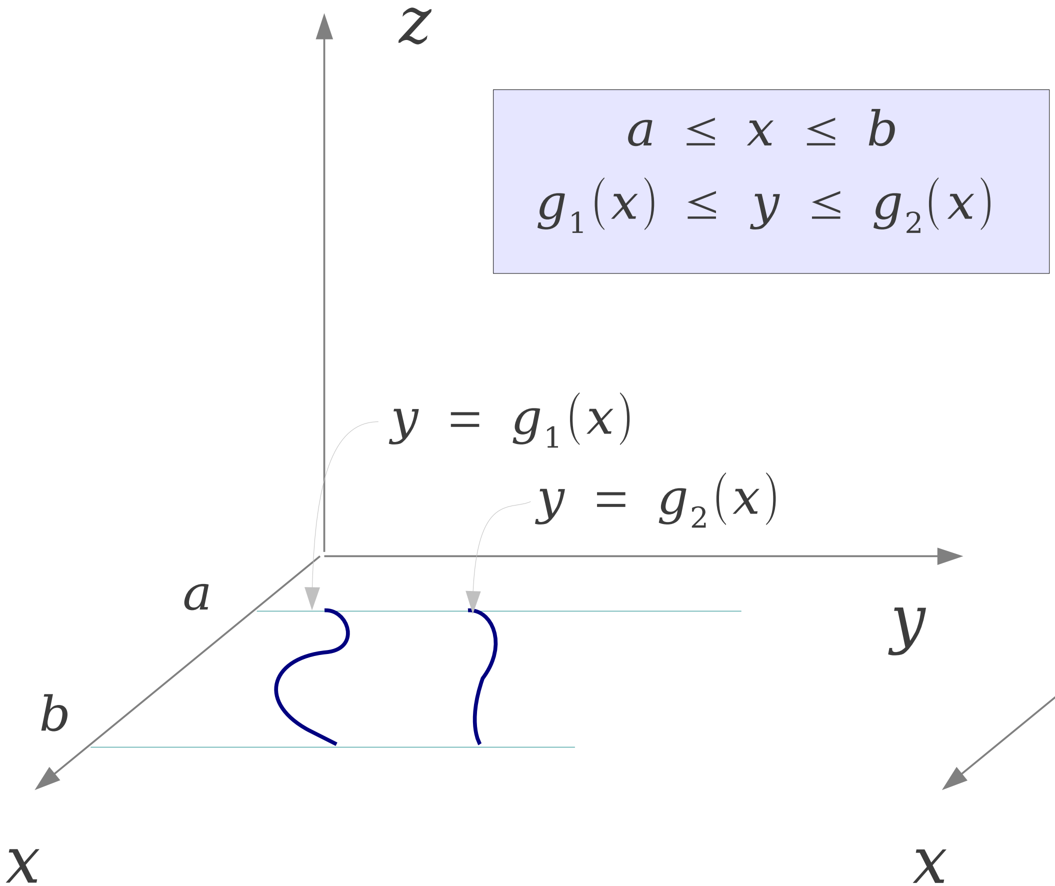
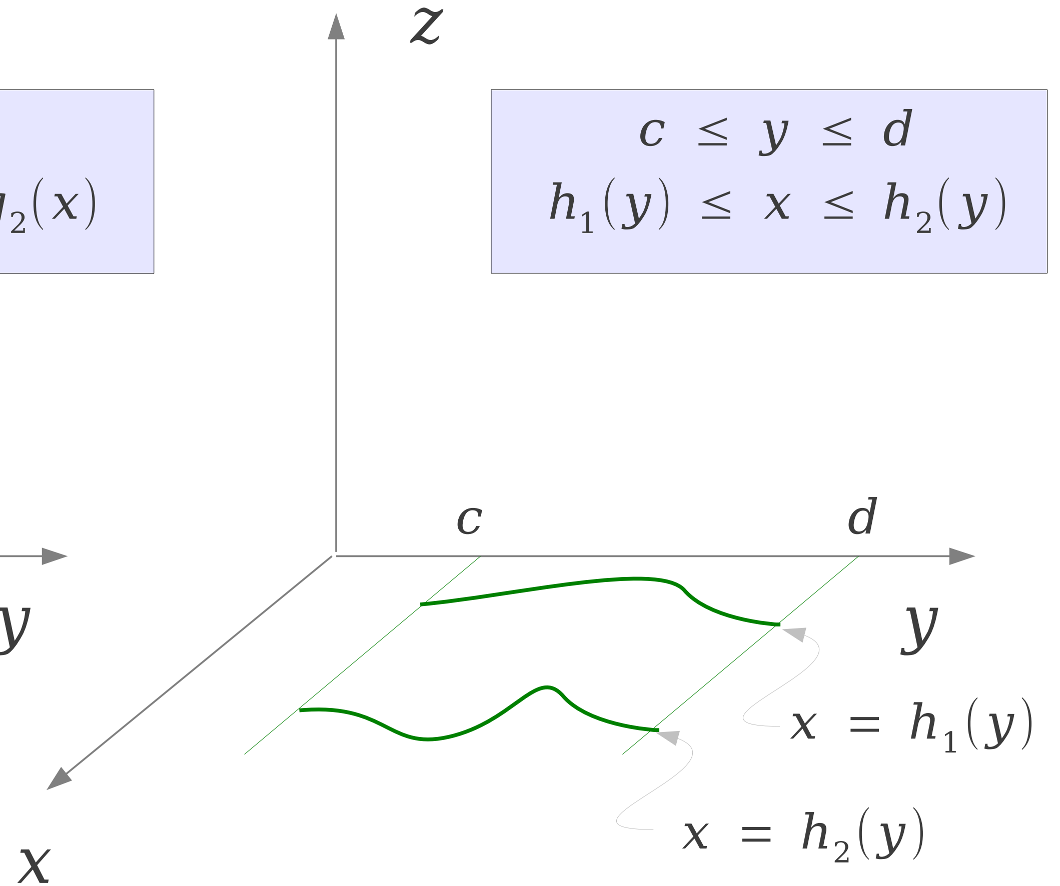
# Area and Volume

$$A = \iint_R dA$$

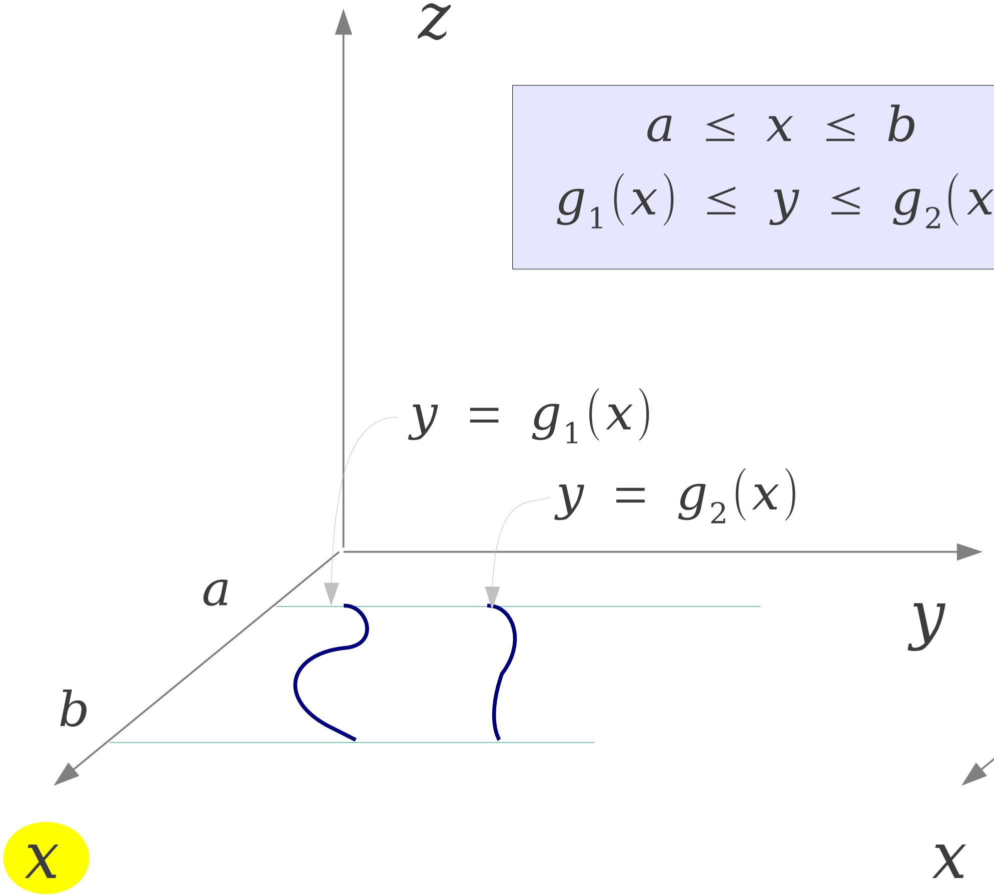
$$V = \iint_R f(x, y) dA$$


$z$
$a \le x \le b$
$g_1(x) \le y \le g_2(x)$
$y = g_1(x)$
$y = g_2(x)$
$a$
$b$
$y$
$x$

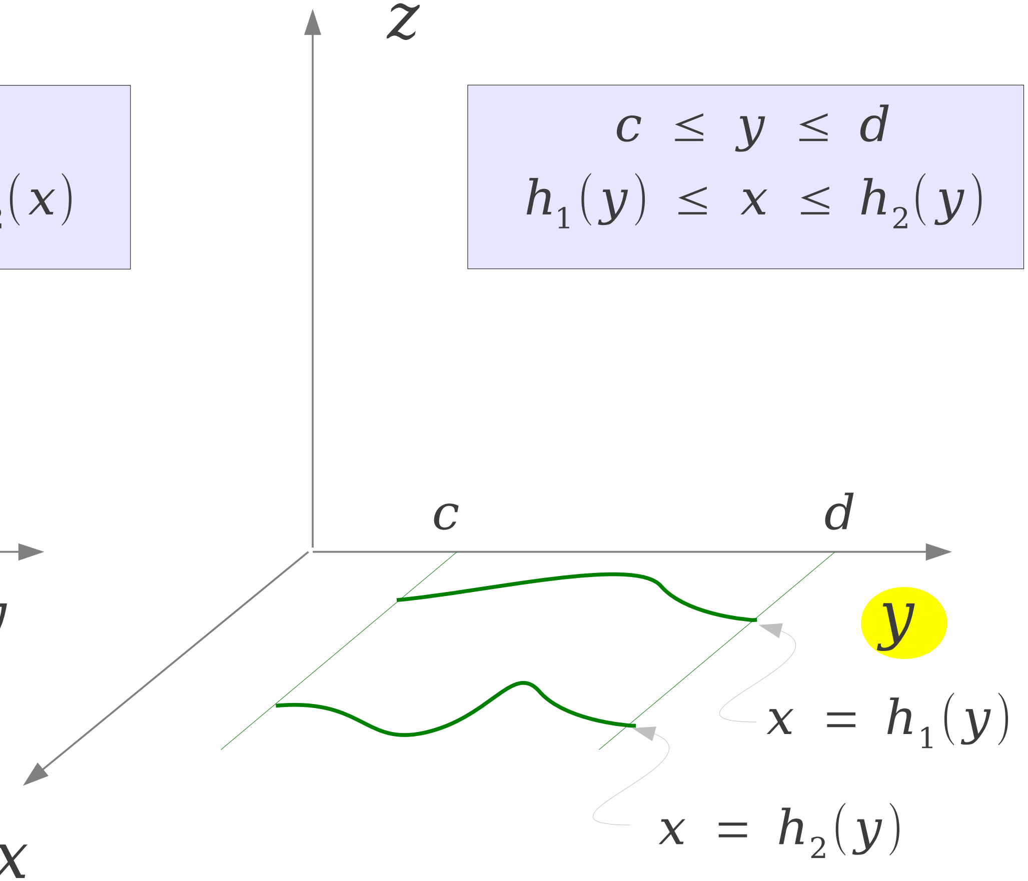

$z$
$c \le y \le d$
$h_1(y) \le x \le h_2(y)$
$c$
$d$
$y$
$x = h_1(y)$
$x = h_2(y)$
$x$

# Fubini's Theorem

$$a \le x \le b$$
$$g_1(x) \le y \le g_2(x)$$



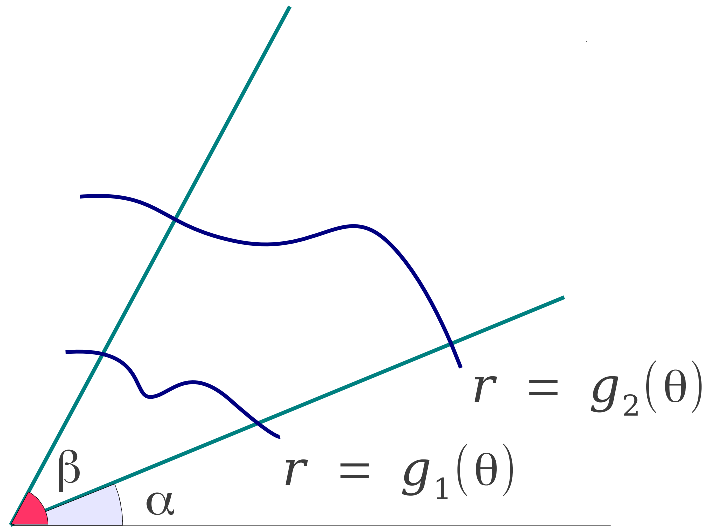

$$c \le y \le d$$
$$h_1(y) \le x \le h_2(y)$$

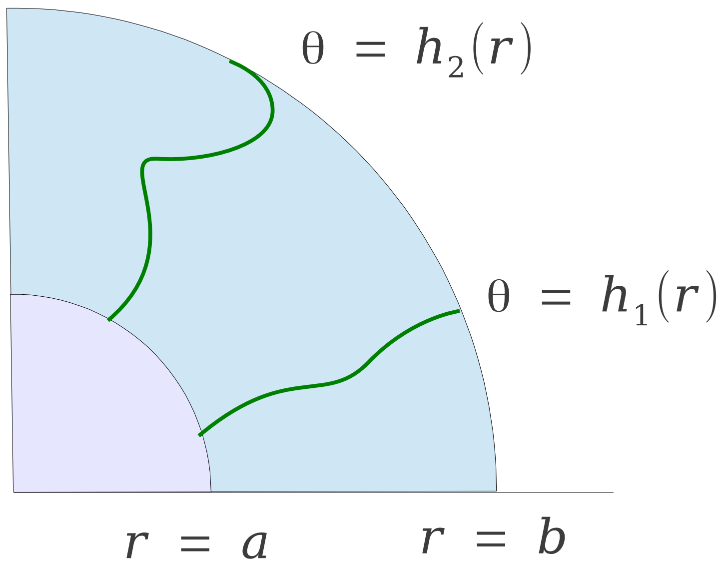



$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

$$\iint_R f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

# Type A and Type B




$$\iint_R f(r,\theta)\, dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(r,\theta)\, r\, dr\, d\theta$$

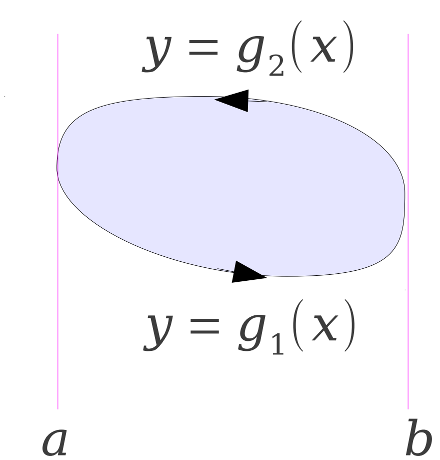
$$\iint_R f(r,\theta)\, dA = \int_a^b \int_{h_1(r)}^{h_2(r)} f(r,\theta)\, d\theta\, r\, dr$$

# Green's Theorem in the Plane (1)

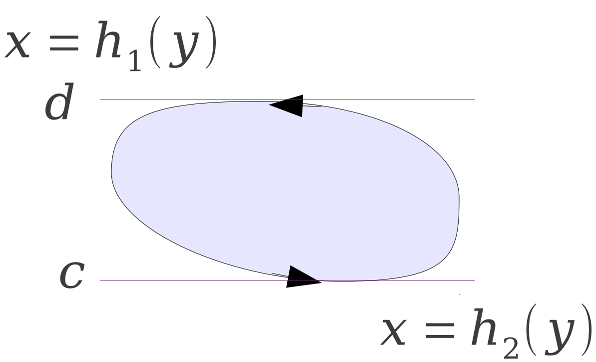
**C**: a piecewise c simple closed curve
bounding a simply connected region **R**

$$\oint_C P\,dx + Q\,dy = \iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Line Integral | Double Integral




$$\iint_R -\frac{\partial P}{\partial y}\, dA \qquad \iint_R \frac{\partial Q}{\partial x}\, dA$$

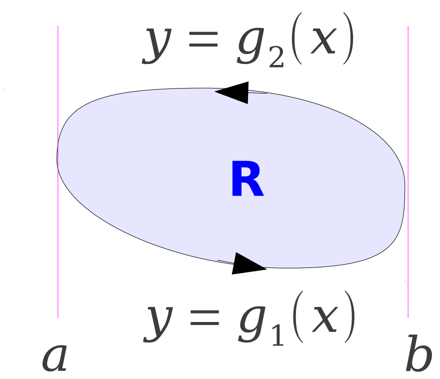


# Green's Theorem in the Plane (2)

**C**: a piecewise c simple closed curve

**R**: a simply connected bounding region

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$



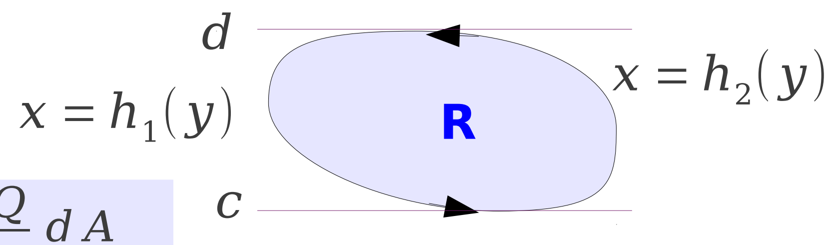

$$\iint_R -\frac{\partial P}{\partial y}\,dA$$

$$= -\int_a^b \int_{g_1(x)}^{g_2(x)} \frac{\partial P}{\partial y}\,dy\,dx$$

$$= -\int_a^b \left[P(x, g_2(x)) - P(x, g_1(x))\right] dx$$

$$= \int_a^b P(x, g_1(x))\,dx - \int_a^b P(x, g_2(x))\,dx$$

$$= \oint_C P\,dx$$




$$\iint_R \frac{\partial Q}{\partial x}\,dA$$

$$= -\int_c^d \int_{h_1(y)}^{h_2(y)} \frac{\partial Q}{\partial x}\,dx\,dy$$

$$= -\int_c^d \left[Q(h_2(y), y) - Q(h_1(y), y)\right] dy$$

$$= \int_c^d Q(h_1(y), y)\,dx - \int_a^b P(h_2(y), y)\,dx$$

$$= \oint_C Q\,dy$$

# 2-Divergence

Flux across rectangle boundary

$$\approx \left(\frac{\partial M}{\partial x}\Delta x\right)\Delta y + \left(\frac{\partial N}{\partial y}\Delta y\right)\Delta x \qquad = \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)\Delta x \Delta y$$

Flux density

$$= \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)$$

Divergence of **F**

Flux Density

## References


[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[4] D.G. Zill, “Advanced Engineering Mathematics”