

Double Integrals (5A)

- Double Integral
- Double Integrals in Polar Coordinates
- Green's Theorem

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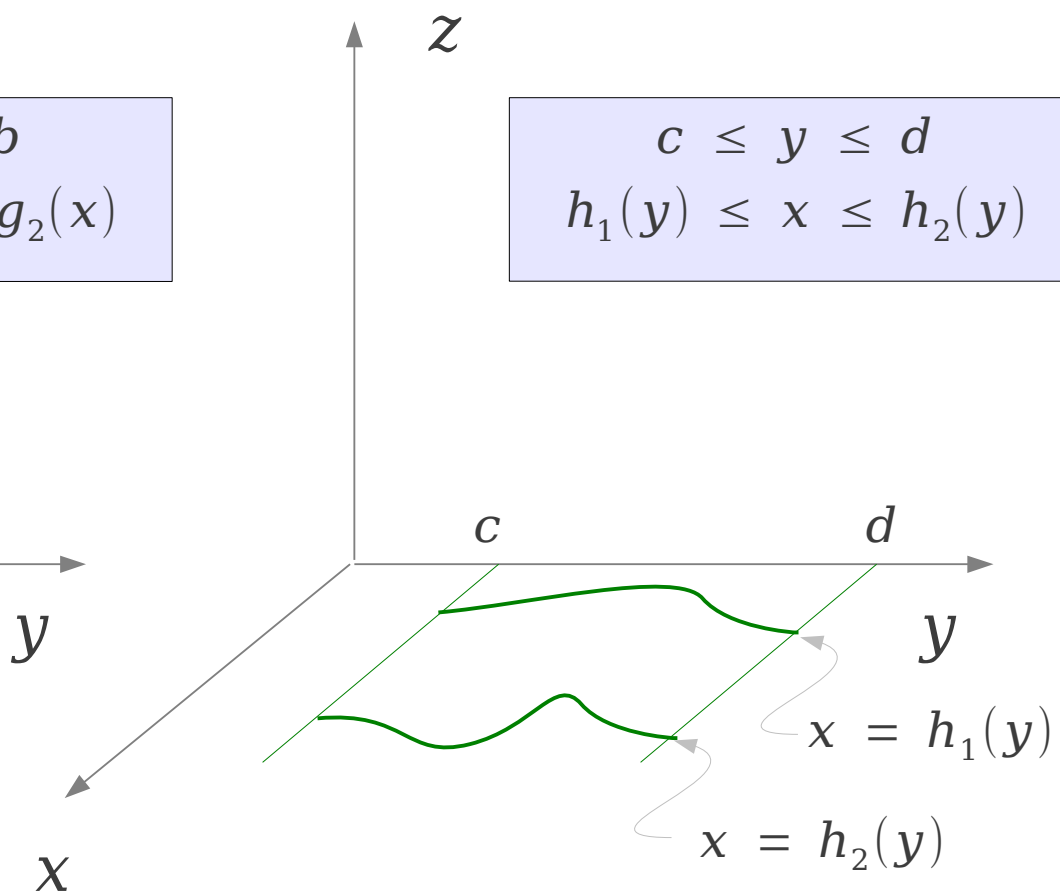
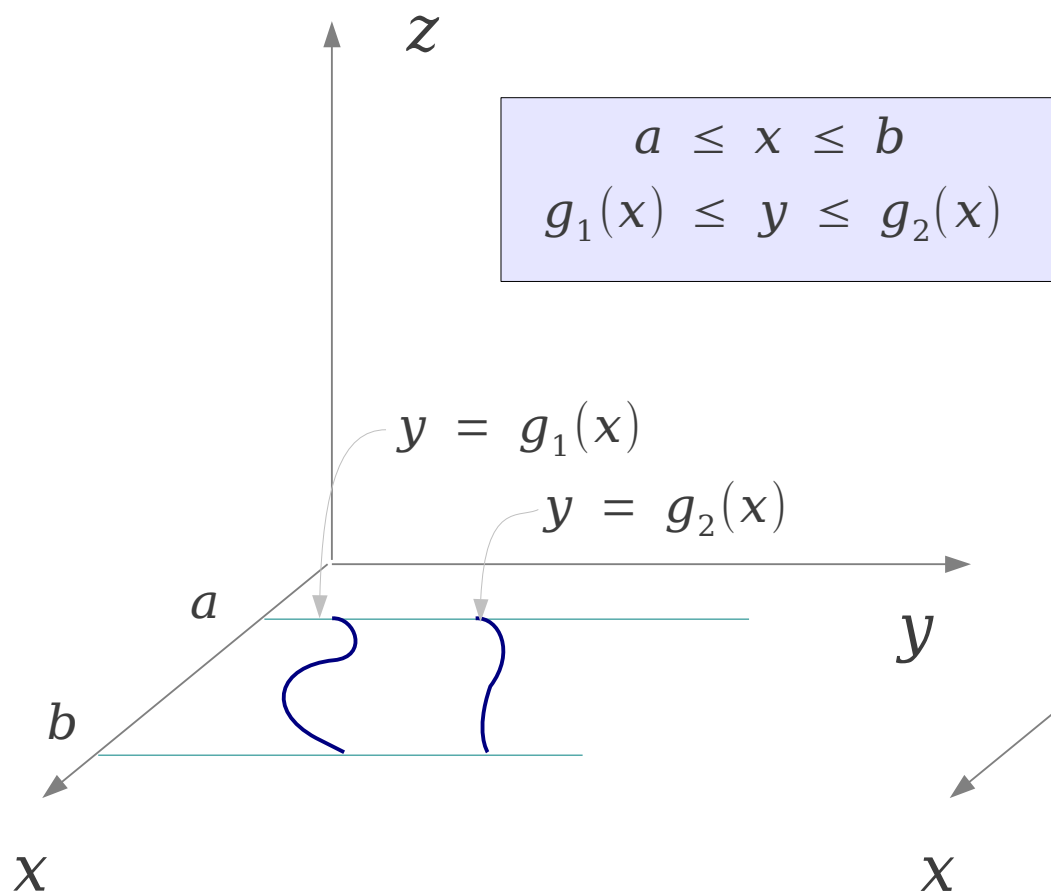
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Area and Volume

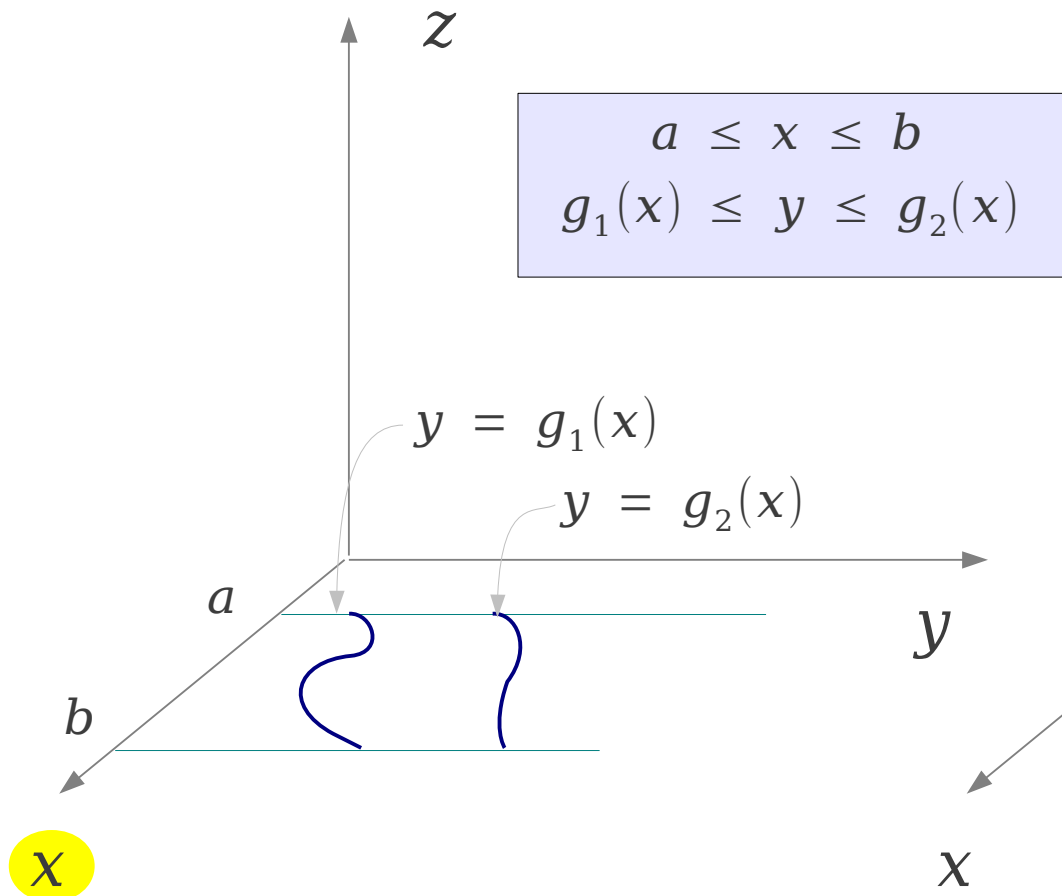
$$A = \iint_R dA$$

$$V = \iint_R f(x, y) dA$$

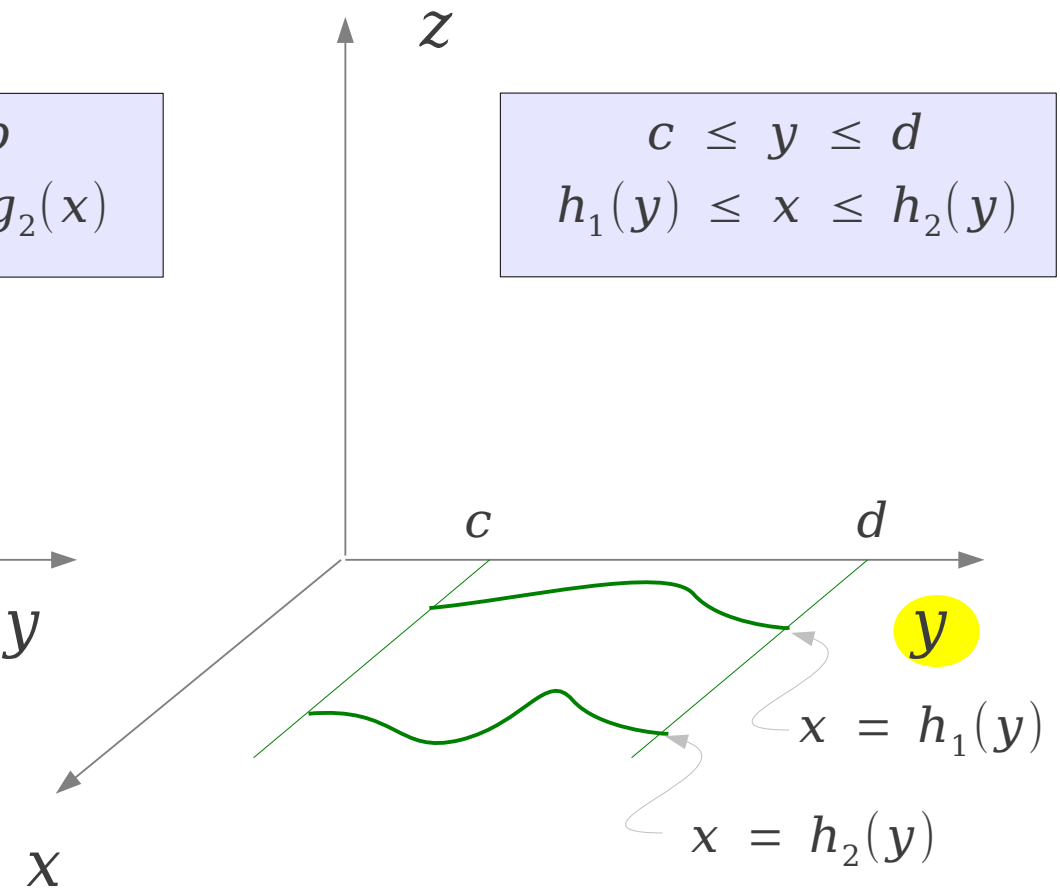
Type I and Type II



Fubini's Theorem

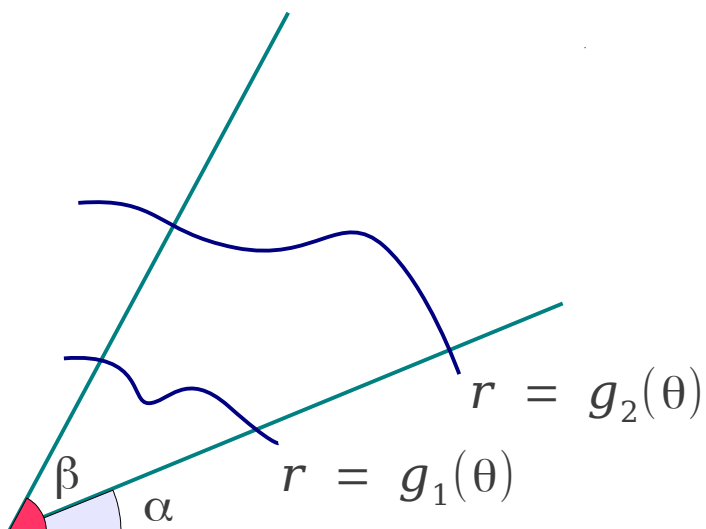


$$\begin{aligned}
 & \iint_R f(x, y) \, dA \\
 &= \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, dy \, dx
 \end{aligned}$$

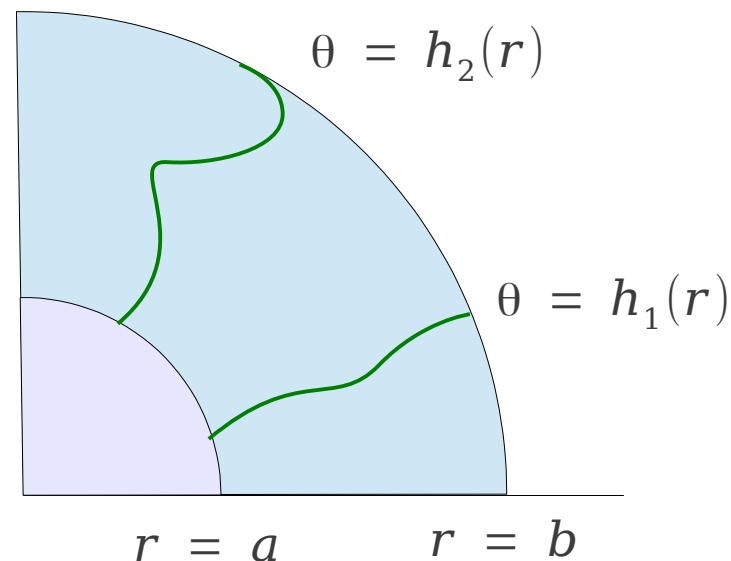


$$\begin{aligned}
 & \iint_R f(x, y) \, dA \\
 &= \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) \, dx \, dy
 \end{aligned}$$

Type A and Type B



$$\iint_R f(r, \theta) dA \\ = \int_{\alpha}^{\beta} \int_{g_1(\theta)}^{g_2(\theta)} f(r, \theta) r dr d\theta$$

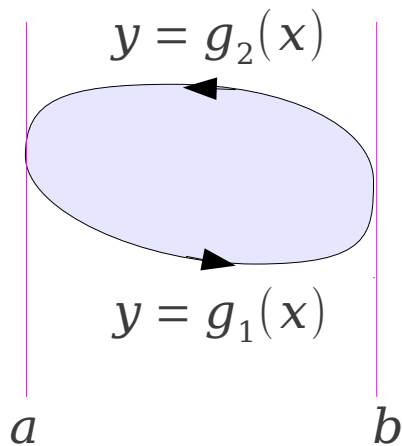


$$\iint_R f(r, \theta) dA \\ = \int_a^b \int_{h_1(r)}^{h_2(r)} f(r, \theta) d\theta r dr$$

Green's Theorem in the Plane (1)

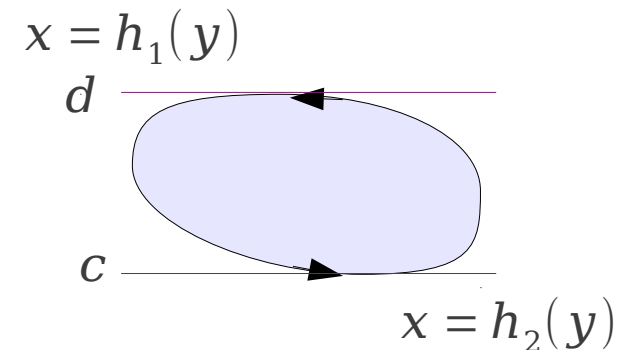
C: a piecewise c simple closed curve
 bounding a simply connected region **R**

$$\underbrace{\oint_C P dx + Q dy}_{\text{Line Integral}} = \underbrace{\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA}_{\text{Double Integral}}$$



$$\iint_R -\frac{\partial P}{\partial y} dA$$

$$\iint_R \frac{\partial Q}{\partial x} dA$$

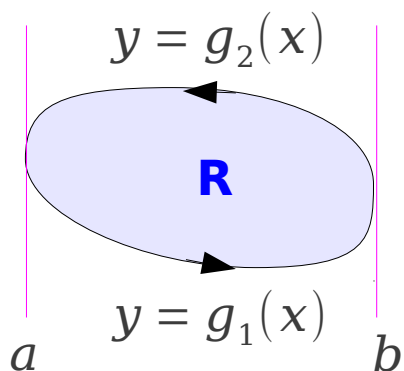


Green's Theorem in the Plane (2)

C: a piecewise c simple closed curve

R: a simply connected bounding region

$$\oint_C P dx + Q dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$



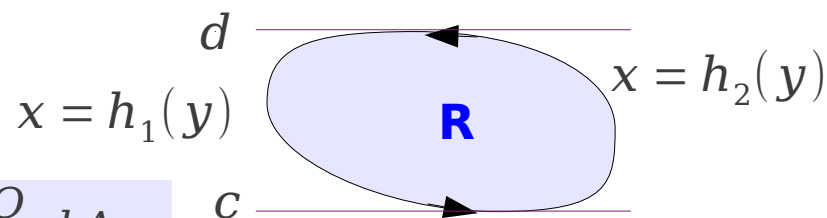
$$\iint_R -\frac{\partial P}{\partial y} dA$$

$$= -\int_a^b \int_{g_1(x)}^{g_2(x)} \frac{\partial P}{\partial y} dy dx$$

$$= -\int_a^b [P(x, g_2(x)) - P(x, g_1(x))] dx$$

$$= \int_a^b P(x, g_1(x)) dx - \int_a^b P(x, g_2(x)) dx$$

$$= \oint_C P dx$$



$$\iint_R \frac{\partial Q}{\partial x} dA$$

$$= -\int_c^d \int_{h_1(y)}^{h_2(y)} \frac{\partial Q}{\partial x} dx dy$$

$$= -\int_c^d [Q(h_2(y), y) - Q(h_1(y), y)] dy$$

$$= \int_c^d Q(h_1(y), y) dy - \int_c^d Q(h_2(y), y) dy$$

$$= \oint_C Q dy$$

2-Divergence

Flux across rectangle boundary

$$\approx \left(\frac{\partial M}{\partial x} \Delta x \right) \Delta y + \left(\frac{\partial N}{\partial y} \Delta y \right) \Delta x = \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) \Delta x \Delta y$$

Flux density

$$= \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right)$$

Divergence of \mathbf{F}

Flux Density

References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
- [4] D.G. Zill, “Advanced Engineering Mathematics”