Mtg 22: Fri, 8 Oct 10

p.19-1: $h(x,y) = x^m y^n$ (1)

where $m, n \in \mathbb{R}$ to be determined.

For appl on p.19-1, Eq.(1): Find (m,n) st:

$$(x^m y^n)\left[\sqrt{x}\, y'' + 2xy' + 3y\right] = 0 \quad (2)$$

is exact. $\Rightarrow$ See Fϕ9.

Result = 1st integral ϕ

$$\phi(x,y,p) = x\underbrace{p}_{} + (2x^{3/2} - 1)y \overset{(3)}{=} k$$

$p = y'$

Fϕ9: Solve L1-ODE-VC (3) for $y(x)$ (IFM).

Math. struct. of ϕ for a class of "exact" L2-ODE-VC

$$F(x,y,y',y'') = \frac{d\phi(x,y,\underbrace{y'}_{p=y'})}{dx}$$

$$= \phi_x + \phi_y p + \phi_p \underbrace{y''}_{p'} \quad (1)$$

Consider case:

$$F = \underbrace{\underbrace{R(x)}_{a_0(x)}}_{\phi_x} y + \underbrace{Q(x)}_{a_1(x) = \phi_y} y' + \underbrace{P(x)}_{a_2(x) \equiv \phi_p} y'' \quad (2)$$

Ans:

$$(3) \quad \boxed{\phi(x,y,p) = P(x)\,p + T(x)\,y + k}$$

(3) p. 22-1 $\Rightarrow$ $\begin{cases} P(x) = x \\ T(x) = 2x^{3/2} - 1 \end{cases}$

See FØ9 for derivation of (3)

Appl: Select ϕ satisfying (3)

$P(x) = \cos x \qquad k = 1$

$T(x) = x^2$

$$\phi = (\cos x)\,p + x^2 y + 1 \quad \text{(1)} \quad \underline{22\text{-}3}$$

(1) p. 22-2 $\Rightarrow$ $\phi_x = -(\sin x)\,p + 2xy$ (2)

$$\phi_y = x^2 \quad (3)$$

L2-ODE-VC $\quad \phi_p = \cos x \quad (4)$

$$F = (\cos x)\,y'' + (x^2 - \sin x)\,y' + 2xy = 0 \quad (5)$$

HW: show (5) is "exact";
find ϕ; solve for $y(x)$. ///

Exact Nn-ODE's

Nonlinear, nth order

$$F(x, y^{(0)}, y^{(1)}, y^{(2)}, \ldots, y^{(n)}) = 0$$

$$\underbrace{y^{(0)}}_{y}, \quad \underbrace{y^{(1)}}_{y'}, \quad \underbrace{y^{(2)}}_{y''}, \quad \underbrace{y^{(n)}}_{= \frac{d^n}{dx^n} y}$$

Exactness cond. 1:

$$F = \frac{d}{dx}\phi\left(x, y^{(0)}, y^{(1)}, \ldots, y^{(n-1)}\right) \qquad (1)$$

$$= \phi_x + \phi_{y^{(0)}} y^{(1)} + \phi_{y^{(1)}} y^{(2)} + \cdots + \phi_{y^{(n-1)}} y^{(n)}$$

$$\left(x, y^{(0)}, y^{(1)}, \ldots, y^{(n-1)}\right)$$

Exactness cond. 2:

$i = 0, 1, 2, \ldots, n$

$$f_i := \frac{\partial F}{\partial y^{(i)}} \qquad (2)$$

$$f_0 - \frac{d f_1}{dx} + \frac{d^2}{dx^2} f_2 - \cdots + (-1)^n \frac{d^n}{dx^n} f_n = 0 \qquad (3)$$

Case $n = 1$: N1-ODE

Cond 2: (exactness) (3) $\Rightarrow f_0 - \dfrac{d f_1}{dx} = 0$

$$\Leftrightarrow \quad \phi_{xy} = \phi_{yx}$$

See F&G for details