Uncertainty

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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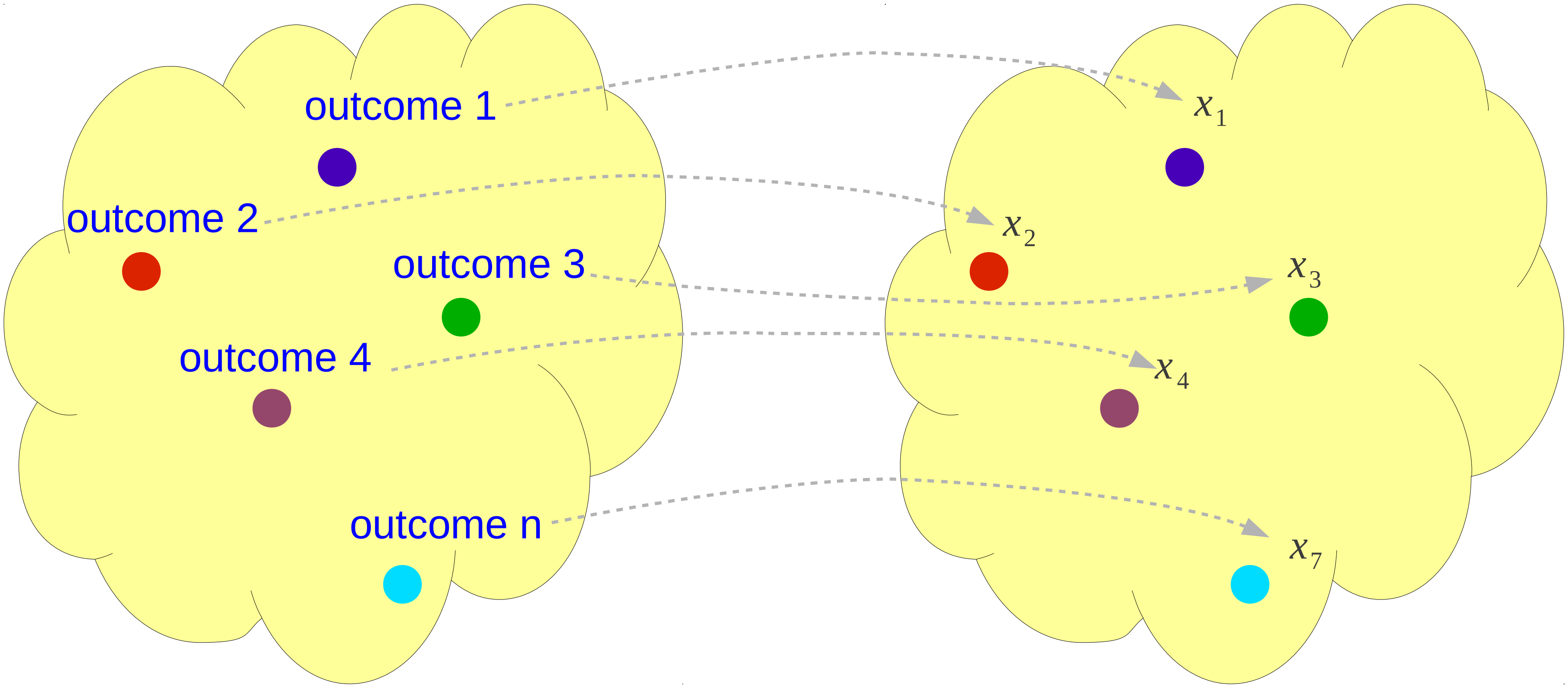
Random Variable



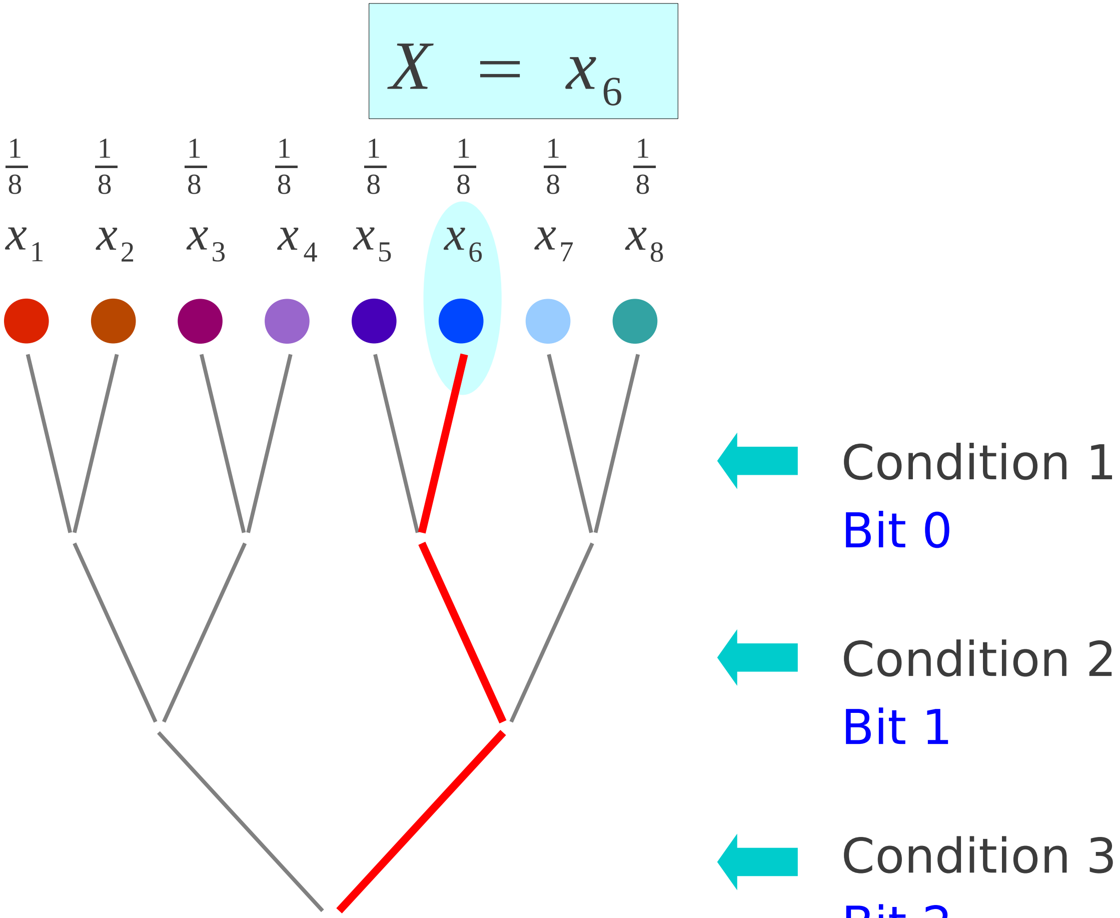
State Space

$$\Omega = \{outcome_1, \ outcome_2, \ \cdots \ , outcome_n\}$$

Random Variable

$$X = x_i \qquad i = 1, \ 2, \ \cdots, \ n$$

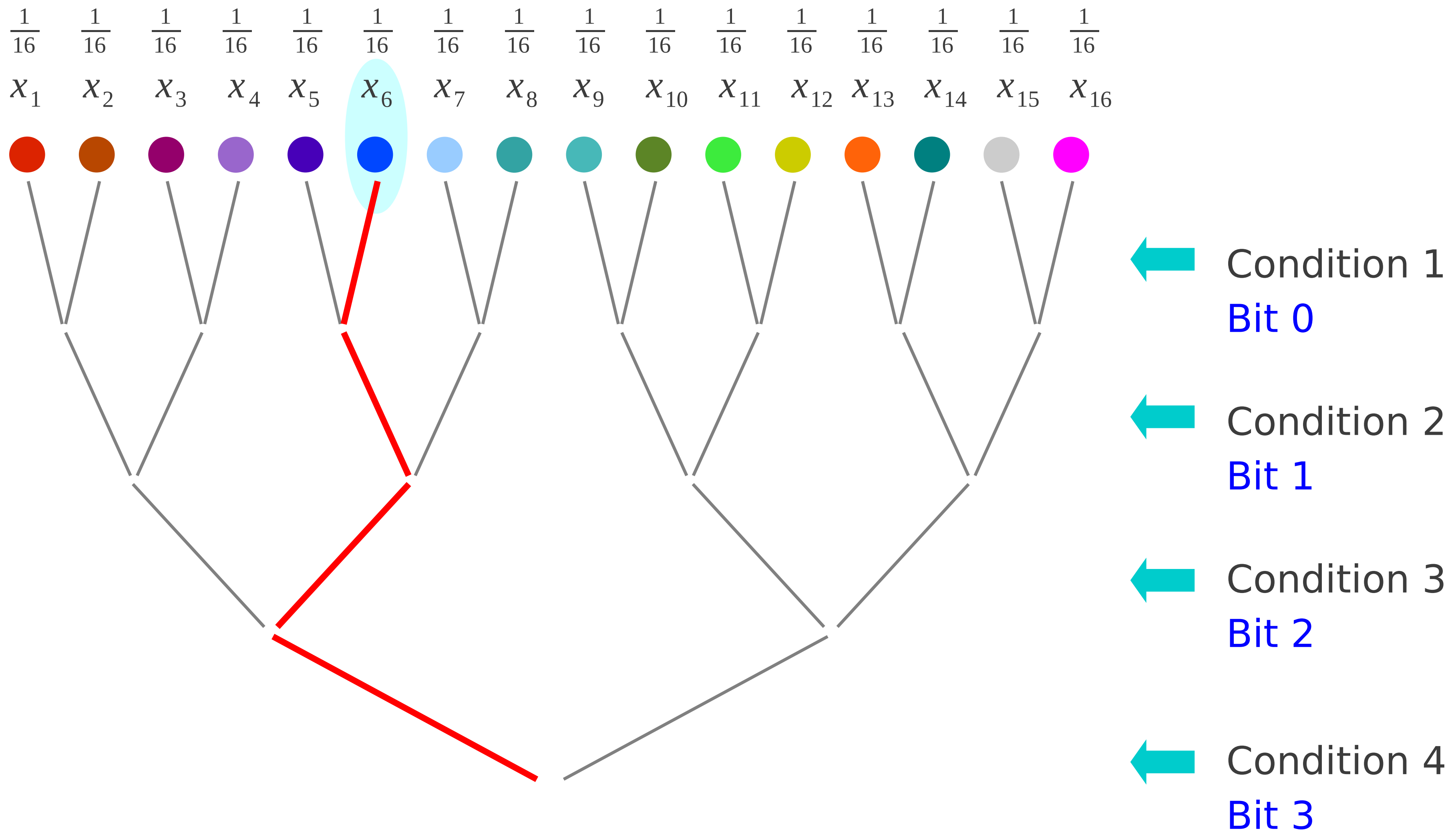
Event



$X = x_6$
$\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$
x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8
Condition 1
Bit 0
Condition 2
Bit 1
Condition 3
Bit 2

Event

$$X = x_6$$



1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16
x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 x16
Condition 1
Bit 0
Condition 2
Bit 1
Condition 3
Bit 2
Condition 4
Bit 3

Self-Information

$$I(x_i) = \log\left(\frac{1}{P(x_i)}\right) = -\log P(x_i)$$

Unit = bits $\log_2$

Unit = nats $\log_e$

Probability of the event $X = x_i$

Self-information ⇧

Probability ⇩

A Priori and a Posteriori

Two types of knowledge, justification, or arguments

A Priori - “from the earlier”

independent of experience

“All bachelors are unmarried”

A Posteriori - “from the later”

Dependent on experience or empirical evidence

“Some bachelors are happy”

Bayes' Rule (1)

| means "given"
H : Hypothesis
E : Evidence

The **prior probability**
the probability of **H**
before **E** is observed.

The **likelihood**
of observing **E** given **H**

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

The **posterior probability**
the probability of **H** given **E**,
i.e., after **E** is observed.

the **marginal likelihood**
or "model evidence"
the same for all possible hypotheses

Bayes' Rule (2)

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

P(H), the **prior probability** –
the probability of **H** before **E** is observed.
This indicates one's *preconceived beliefs* about how likely different hypotheses are, absent evidence regarding the instance under study.

P(H|E), the **posterior probability** –
the probability of **H** given **E**, i.e., after **E** is observed.
the probability of a hypothesis given the observed evidence

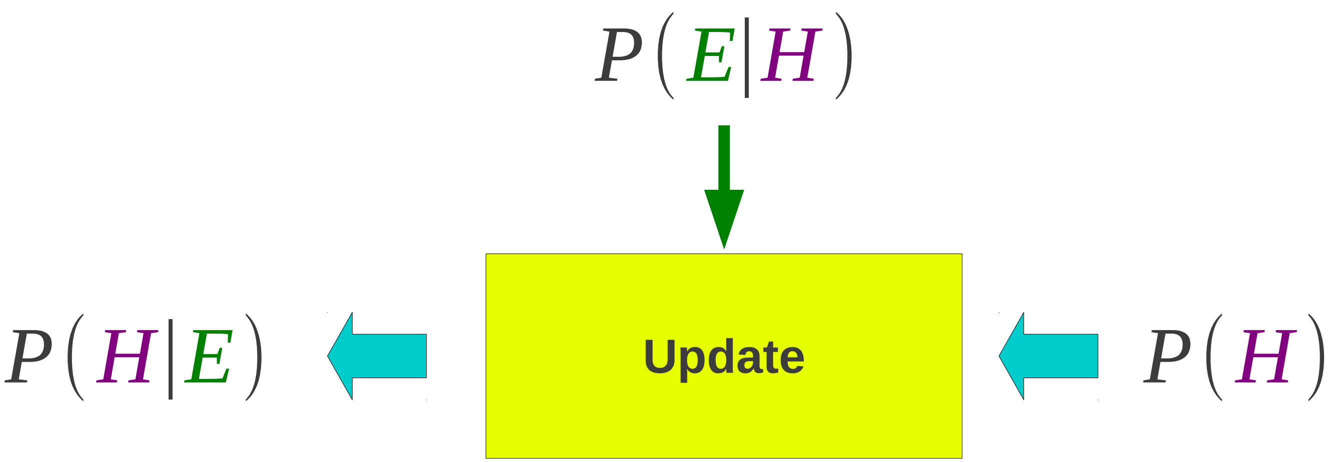
P(E|H), the probability of observing **E** given **H**, is also known as the **likelihood**.
It indicates the compatibility of the evidence with the given hypothesis.

P(E), the **marginal likelihood** or "model evidence". This factor is the same for all possible hypotheses being considered. This means that this factor does not enter into determining the relative probabilities of different hypotheses.

Bayes' Rule (3)

| means “given”
H : Hypothesis
E : Evidence

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$



Example

If the evidence doesn't match up with a hypothesis, one should reject the hypothesis.
But if a hypothesis is extremely unlikely a priori, one should also reject it,
even if the evidence does appear to match up.

Three hypotheses about the nature of a newborn baby of a friend, including:

- H1: the baby is a brown-haired boy
- H2: the baby is a blond-haired girl.
- H3: the baby is a dog.

Consider two scenarios:
I'm presented with evidence in the form of a picture of a blond-haired baby girl.
I find this evidence supports H2 and opposes H1 and H3.

I'm presented with evidence in the form of a picture of a baby dog.
I don't find this evidence supports H3,
since my prior belief in this hypothesis (that a human can give birth to a dog) is extremely small.

Bayes' rule
a principled way of combining new evidence with prior beliefs, through the application of Bayes' rule.
can be applied iteratively: after observing some evidence, the resulting posterior probability can then be treated as a prior probability, and a new posterior probability computed from new evidence.
Bayesian updating.

Maintain Magnetic Field

Suppose there are two full bowls of cookies.
Bowl #1 has 10 chocolate chip and 30 plain cookies,
while bowl #2 has 20 of each.

When picking a bowl at random, and then picking a cookie at random.
No reason to treat one bowl differently from another, likewise for the cookies.
The drawn cookie turns out to be a plain one.
How probable is it from bowl #1?

more than a half, since there are more plain cookies in bowl #1.

The precise answer

Let H1 correspond to bowl #1, and H2 to bowl #2. P(H1)=P(H2)=0.5.

The event E is the observation of a plain cookie.
From the contents of the bowls, P(E|H1) = 30/40 = 0.75 and P(E|H2) = 20/40 = 0.5.

Bayes' formula then yields

$$P(H1|E) = \frac{P(E|H1)\cdot P(H1)}{P(E|H1)P(H1) + P(E|H2)P(H2)} = \frac{0.75 \times 0.5}{0.75 \times 0.5 + 0.5 \times 0.5} = 0.6$$

Posterior Probability

Conditional probability that is assigned
after the relevant **evidence** is taken into account

Probability of the parameters
given the **evidence**

θ

$p(\theta|X)$

$p(X|\theta)$

Likelihood function that is the probability of an evidence
given the parameter

Maintain Magnetic Field

Storing Magnetic Energy

Dissipate Magnetic Energy

Pulse

Pulse

References

[1] http://en.wikipedia.org/
[2] R Bose, Information Theory Coding and Cryptography, 2003