Double Integrals (5A)

- Double Integral
- Double Integrals in Polar Coordinates
- Green's Theorem

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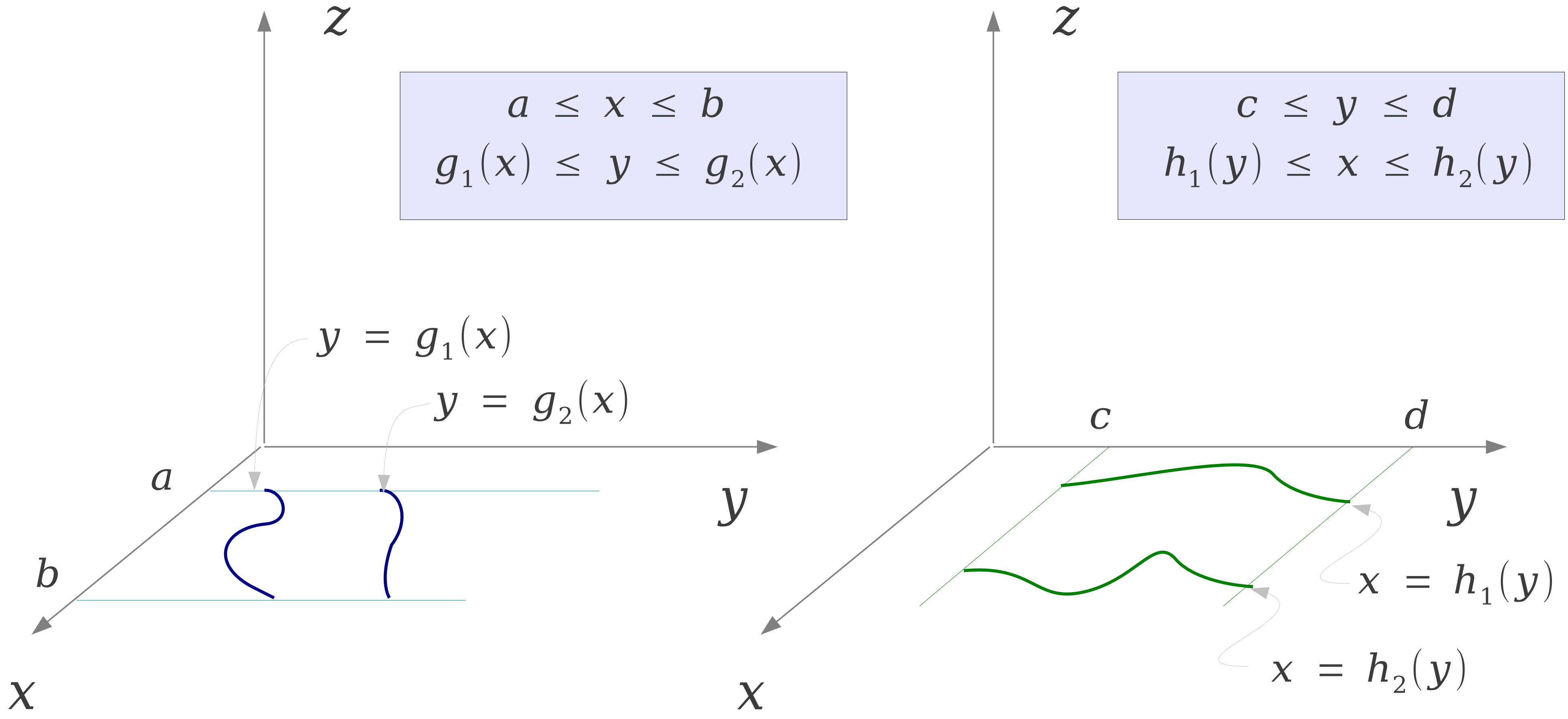
Area and Volume

$$A = \iint_R dA$$

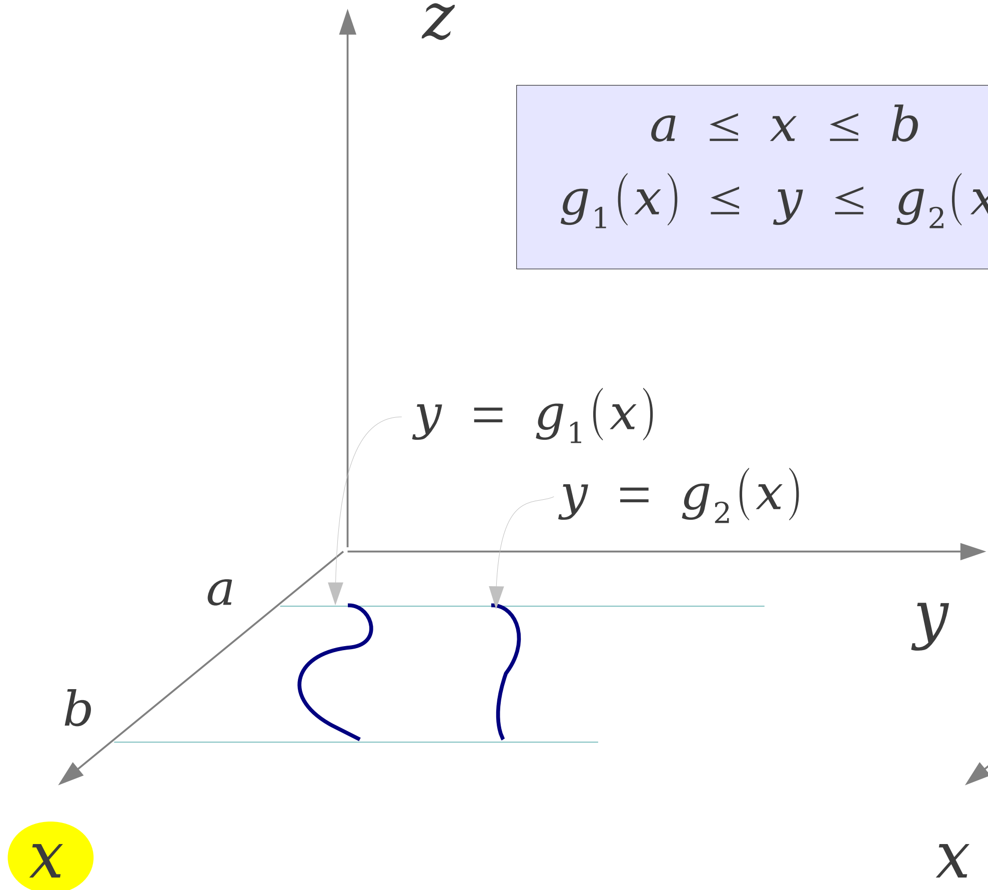
$$V = \iint_R f(x, y) dA$$

Type I and Type II



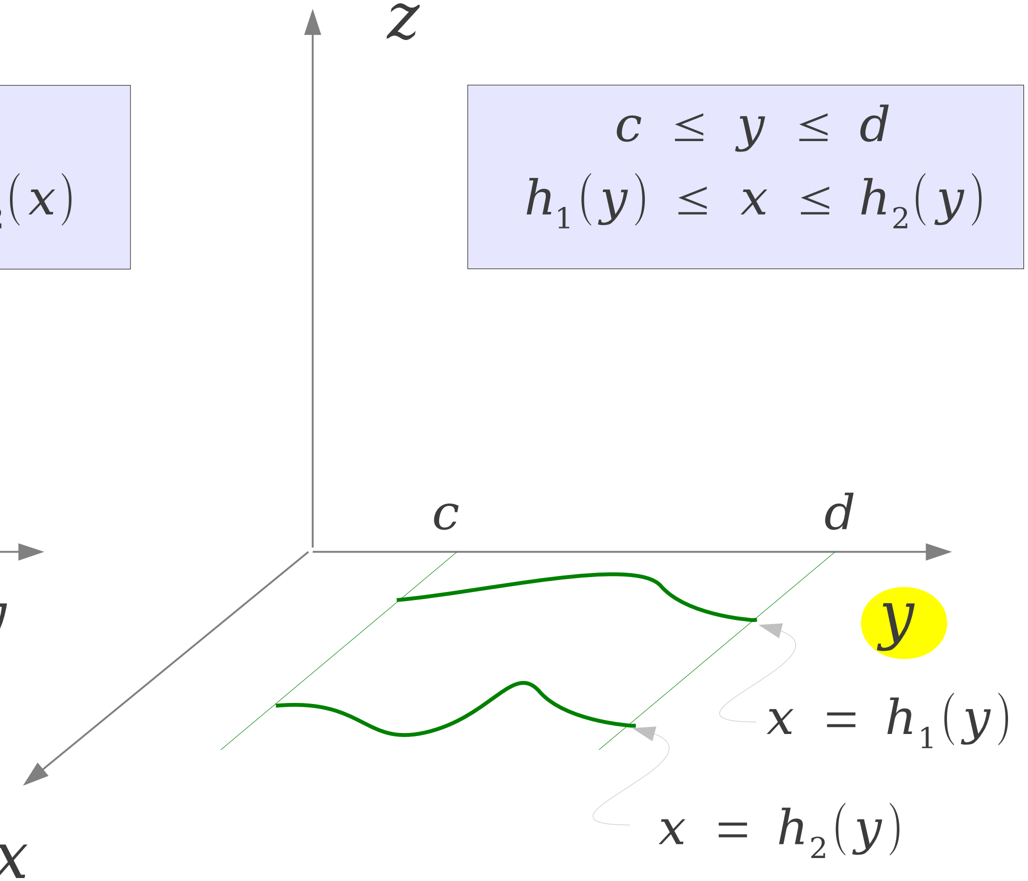
Fubini's Theorem

$$a \le x \le b$$
$$g_1(x) \le y \le g_2(x)$$



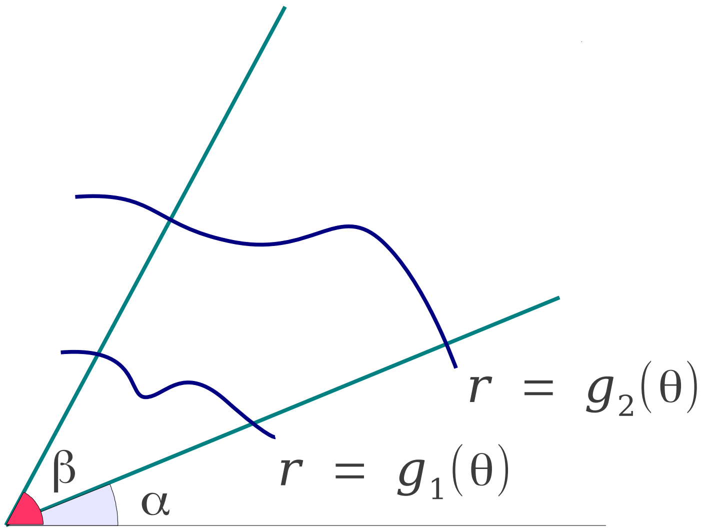
$$\iint_R f(x,y)\,dA$$
$$= \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

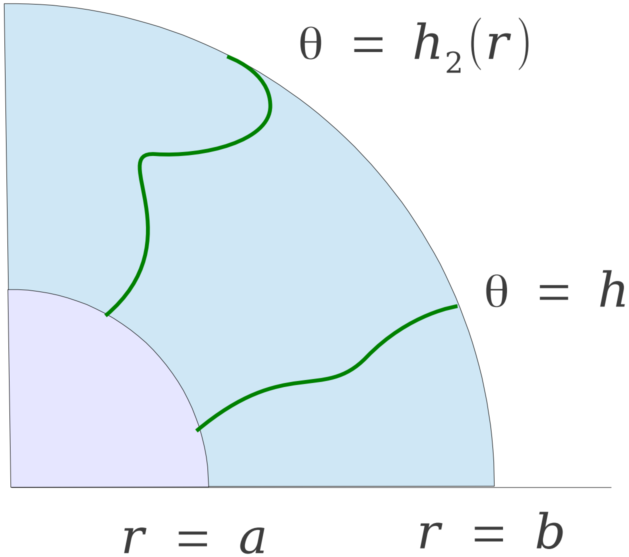
$$c \le y \le d$$
$$h_1(y) \le x \le h_2(y)$$



$$\iint_R f(x,y)\,dA$$
$$= \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

Type A and Type B





$$\iint_R f(r,\theta)\, dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(r,\theta)\, r\, dr\, d\theta$$

$$\iint_R f(r,\theta)\, dA = \int_a^b \int_{h_1(r)}^{h_2(r)} f(r,\theta)\, d\theta\, r\, dr$$

Work using an Arc Length Parameter s

$$W = \boldsymbol{F} \cdot \boldsymbol{d}$$

A force field $\boldsymbol{F}(x, y) = P(x, y)\boldsymbol{i} + Q(x, y)\boldsymbol{j}$

A smooth curve $C : x = f(t), \; y = g(t), \; a \le t \le b$

Work done by **F** along C $\quad W = \int_c \boldsymbol{F}(x, y) \cdot d\boldsymbol{r}$

$$= \int_C P(x, y)\, dx + Q(x, y)\, dy$$

Unit Tangent Vector

$$\frac{d\boldsymbol{r}}{dt} = \frac{d\boldsymbol{r}}{ds}\frac{ds}{dt} \qquad d\boldsymbol{r} = \frac{d\boldsymbol{r}}{ds} ds \qquad d\boldsymbol{r} = \boldsymbol{T}\, ds$$

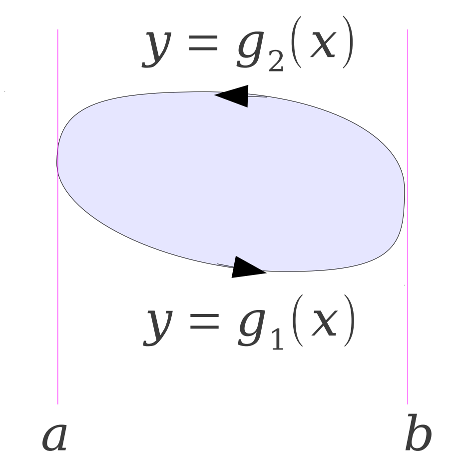
$$W = \int_c \boldsymbol{F} \cdot d\boldsymbol{r} = \int_c \boldsymbol{F} \cdot \boldsymbol{T}\, ds$$

Green's Theorem in the Plane (1)

C: a piecewise c simple closed curve
bounding a simply connected region **R**

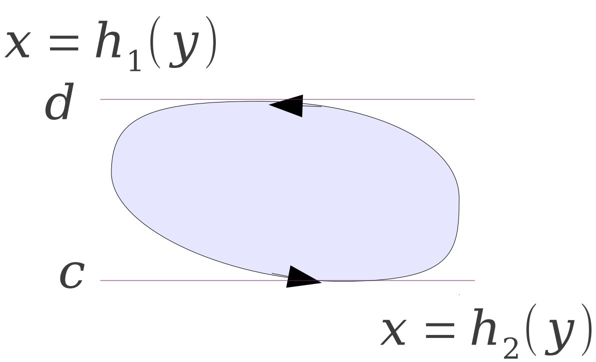
$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

Line Integral — Double Integral



$$\iint_R -\frac{\partial P}{\partial y}\,dA$$

$$\iint_R \frac{\partial Q}{\partial x}\,dA$$

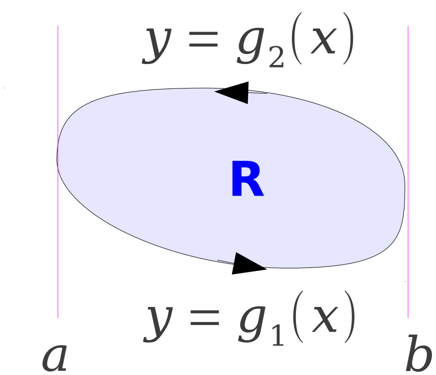


Green's Theorem in the Plane (2)

C: a piecewise c simple closed curve

R: a simply connected bounding region

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$



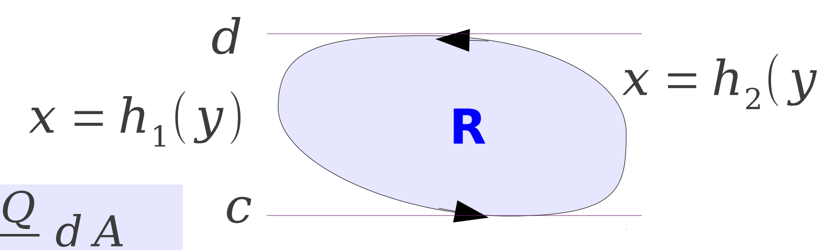
$$\iint_R -\frac{\partial P}{\partial y}\,dA$$

$$= -\int_a^b \int_{g_1(x)}^{g_2(x)} \frac{\partial P}{\partial y}\,dy\,dx$$

$$= -\int_a^b \left[P(x, g_2(x)) - P(x, g_1(x))\right] dx$$

$$= \int_a^b P(x, g_1(x))\,dx - \int_a^b P(x, g_2(x))\,dx$$

$$= \oint_C P\,dx$$



$$\iint_R \frac{\partial Q}{\partial x}\,dA$$

$$= -\int_c^d \int_{h_1(y)}^{h_2(y)} \frac{\partial Q}{\partial x}\,dx\,dy$$

$$= -\int_c^d \left[Q(h_2(y), y) - Q(h_1(y), y)\right] dy$$

$$= \int_c^d Q(h_1(y), y)\,dx - \int_a^b P(h_2(y), y)\,dx$$

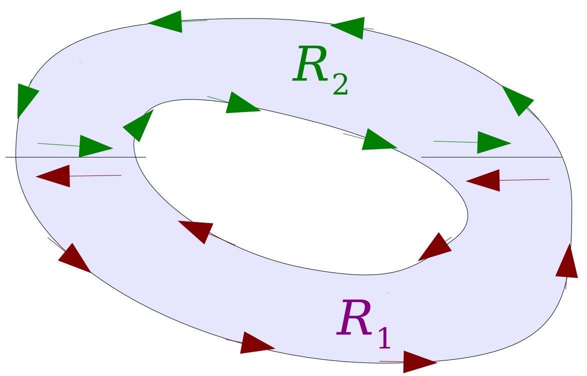
$$= \oint_C Q\,dy$$

Region with Holes

C: a piecewise c simple closed curve
bounding a simply connected region **R**

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Line Integral | Double Integral



$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$= \iint_{R_1} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA + \iint_{R_2} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$= \oint_{C_1} P\,dx + Q\,dy + \oint_{C_2} P\,dx + Q\,dy$$

$$= \oint_C P\,dx + Q\,dy$$

2-Divergence

Flux across rectangle boundary

$$\approx \left(\frac{\partial M}{\partial x}\Delta x\right)\Delta y + \left(\frac{\partial N}{\partial y}\Delta y\right)\Delta x \qquad = \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)\Delta x \Delta y$$

Flux density

$$= \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)$$

Divergence of **F**

Flux Density

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[4] D.G. Zill, “Advanced Engineering Mathematics”