DFT Matrix Examples (DFT.2.A)

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5/11/11
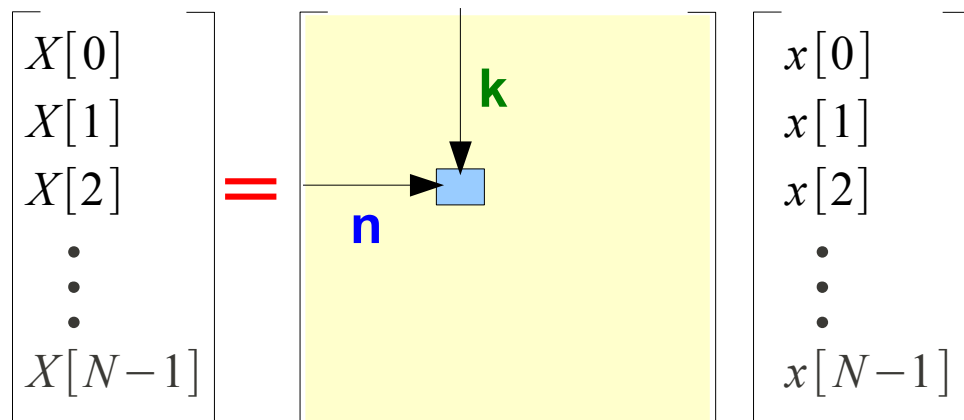
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DFT Matrix Elements

$$X[k] \;=\; \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \qquad\qquad X[k] \;=\; \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = \begin{bmatrix} \\ n \quad k \\ \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix}$$

$$e^{-j\left(\frac{2\pi}{N}\right)kn} \;=\; e^{-j\left(\frac{2\pi}{N}\right)(kn \; mod \; N)}$$
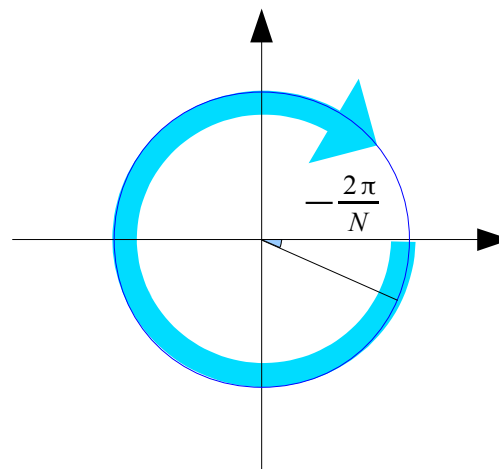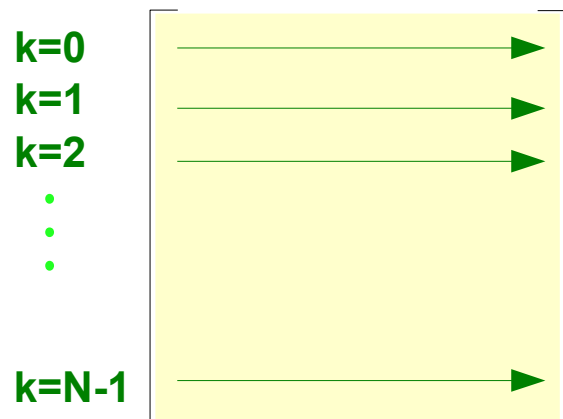
$$= \; \cos\left(-\frac{2\pi}{N}kn\right) \; + \; j\,\sin\left(-\frac{2\pi}{N}kn\right)$$

N multiples of the smallest angle $\left(-\dfrac{2\pi}{N}\right)$

$$\left\{ -\frac{2\pi}{N}\cdot 0, \quad -\frac{2\pi}{N}\cdot 1, \quad \cdots \quad , \quad -\frac{2\pi}{N}\cdot(N-1) \right\}$$
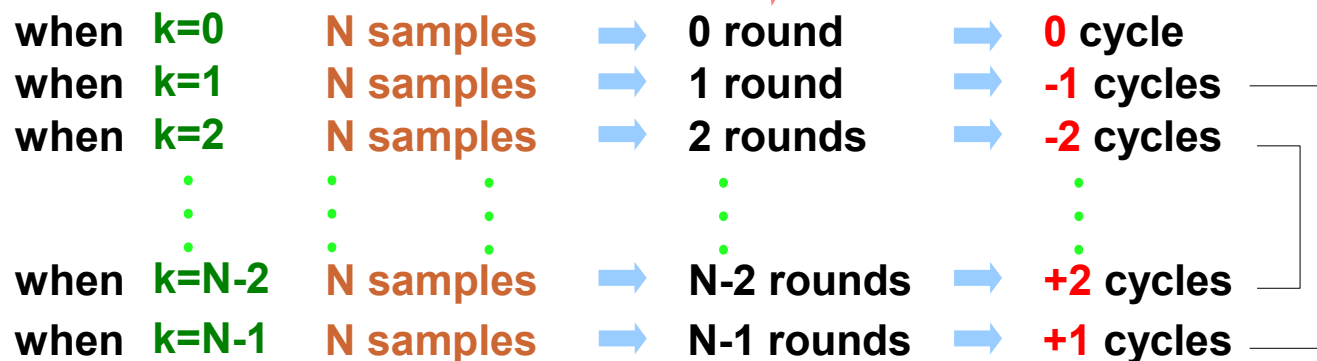
Rows of a DFT Matrix

$$e^{-j\left(\frac{2\pi}{N}\right)kn} \quad \in \quad \left\{ e^{-j\left(\frac{2\pi}{N}\right)\cdot 0}, \quad e^{-j\left(\frac{2\pi}{N}\right)\cdot 1}, \quad e^{-j\left(\frac{2\pi}{N}\right)\cdot 2}, \quad \cdots \quad e^{-j\left(\frac{2\pi}{N}\right)(N-1)} \right\}$$

k=0
k=1
k=2
.
.
.
k=N-1

$-\dfrac{2\pi}{N}$

Negative Angles

Negative Frequency

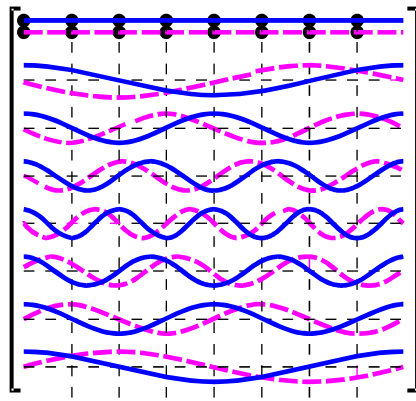|						
when	k=0	N samples	➡	0 round	➡	**0** cycle
when	k=1	N samples	➡	1 round	➡	**-1** cycles
when	k=2	N samples	➡	2 rounds	➡	**-2** cycles
	.	.		.		.
when	k=N-2	N samples	➡	N-2 rounds	➡	**+2** cycles
when	k=N-1	N samples	➡	N-1 rounds	➡	**+1** cycles

complex conjugate

Graphical Representation of a DFT Matrix

$$X[k] = \sum_{n=0}^{7} W_8^{kn} \, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

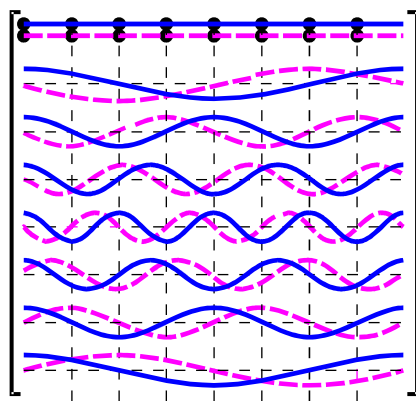$$\begin{pmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{pmatrix} = \begin{bmatrix} \end{bmatrix} \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

$$Re\left[e^{-j\frac{2\pi}{8}kn} \right] = \cos\left(-\frac{2\pi}{8}kn \right)$$

$$Im\left[e^{-j\frac{2\pi}{8}kn} \right] = \sin\left(-\frac{2\pi}{8}kn \right)$$

0 cycle
-1 cycles
-2 cycles
-3 cycles
-4 cycles
+3 cycles
+2 cycles
+1 cycles

complex conjugate

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003