DFT Matrix Examples (DFT.2.A)

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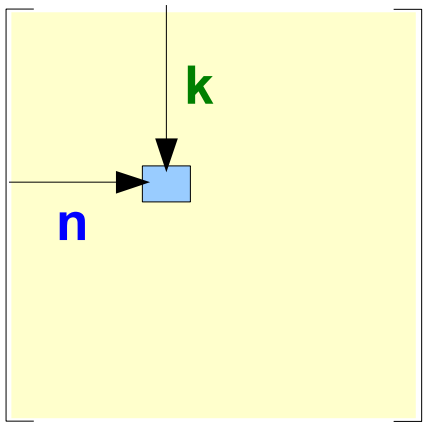
DFT Matrix Elements

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \qquad X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix}$$



$$e^{-j\left(\frac{2\pi}{N}\right)kn} = e^{-j\left(\frac{2\pi}{N}\right)(kn \mod N)}$$

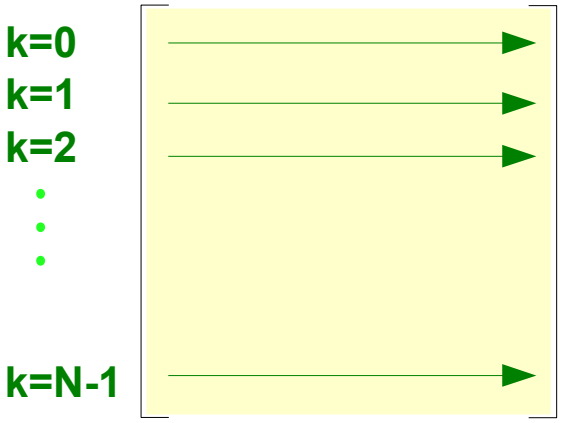
$$= \cos\left(-\frac{2\pi}{N}kn\right) + j\sin\left(-\frac{2\pi}{N}kn\right)$$

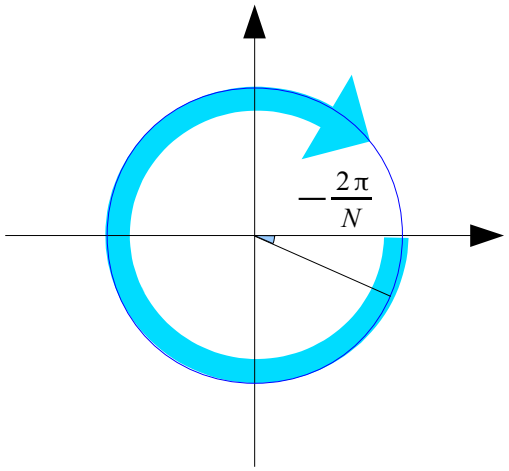
N multiples of the smallest angle $\left(-\frac{2\pi}{N}\right)$

$$\left\{-\frac{2\pi}{N}\cdot 0,\quad -\frac{2\pi}{N}\cdot 1,\quad \cdots,\quad -\frac{2\pi}{N}\cdot(N-1)\right\}$$

Rows of a DFT Matrix

$$e^{-j\left(\frac{2\pi}{N}\right)kn} \in \left\{ e^{-j\left(\frac{2\pi}{N}\right)\cdot 0},\ e^{-j\left(\frac{2\pi}{N}\right)\cdot 1},\ e^{-j\left(\frac{2\pi}{N}\right)\cdot 2},\ \cdots\ e^{-j\left(\frac{2\pi}{N}\right)(N-1)} \right\}$$





Negative Angles

Negative Frequency

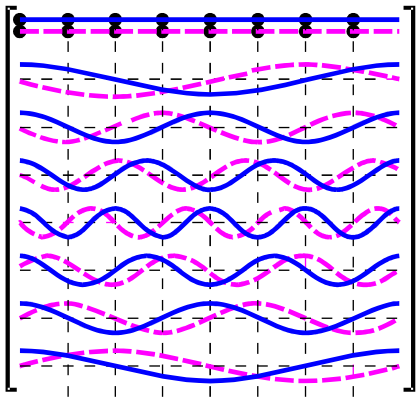
when	k=0	N samples	⇒	0 round	⇒	0 cycle
when	k=1	N samples	⇒	1 round	⇒	-1 cycles
when	k=2	N samples	⇒	2 rounds	⇒	-2 cycles
	⋮	⋮ ⋮		⋮		⋮
when	k=N-2	N samples	⇒	N-2 rounds	⇒	+2 cycles
when	k=N-1	N samples	⇒	N-1 rounds	⇒	+1 cycles

complex conjugate

Graphical Representation of a DFT Matrix

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{pmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{pmatrix} =$$



$$\begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

0 cycle
-1 cycles
-2 cycles
-3 cycles
-4 cycles
+3 cycles
+2 cycles
+1 cycles

complex conjugate

$$Re\left\{ e^{-j\frac{2\pi}{8}kn} \right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$Im\left\{ e^{-j\frac{2\pi}{8}kn} \right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003