

DFT Matrix Properties (3A)

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Symmetric Matrices

DFT

$$A = \left[\begin{array}{c} \text{Cyan shaded square} \\ \text{Dark green diagonal line} \end{array} \right]$$

$$A = A^T$$

ID
F

$$B = \left[\begin{array}{c} \text{Green shaded square} \\ \text{Dark green diagonal line} \end{array} \right]$$

$$B = B^T$$

Conjugate Transpose Matrices

$$A = B^*$$

DFT

$$A = \begin{bmatrix} \text{cyan triangle} \\ \text{green diagonal} \end{bmatrix} = \begin{bmatrix} \text{green triangle} \\ \text{cyan diagonal} \end{bmatrix}^*$$

$$B = A^*$$

IDFT

$$B = \begin{bmatrix} \text{green triangle} \\ \text{cyan diagonal} \end{bmatrix} = \begin{bmatrix} \text{cyan triangle} \\ \text{green diagonal} \end{bmatrix}^*$$

Product AB

DFT

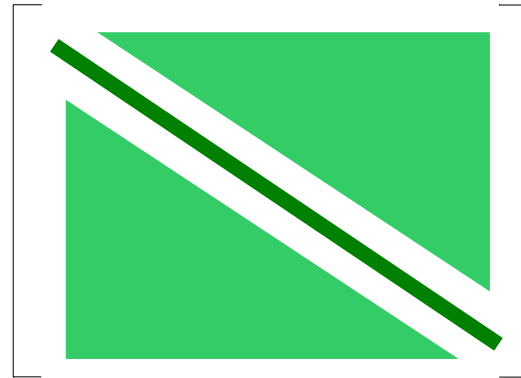
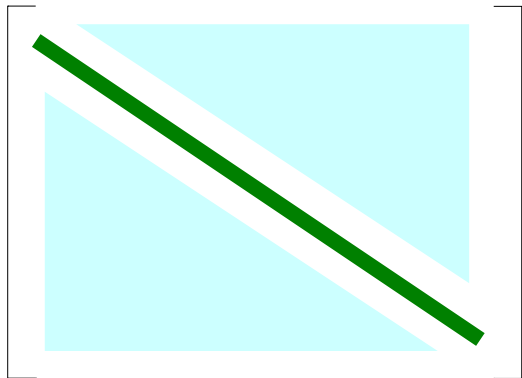
A

IDFT

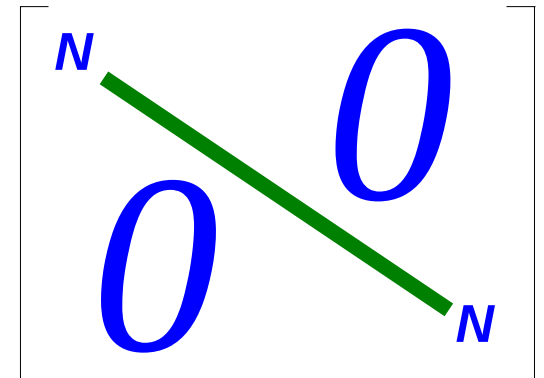
B

=

C



=



Unitary Matrix

$$C = \begin{bmatrix} N & & 0 \\ & \ddots & \\ 0 & & N \end{bmatrix}$$

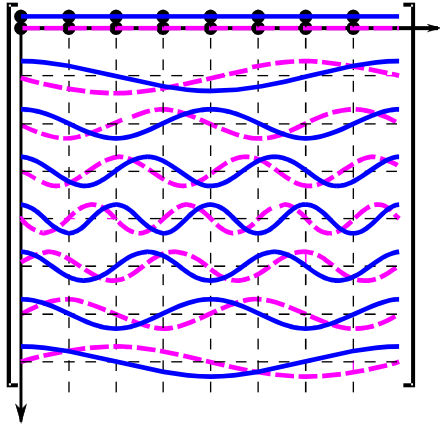
$$C = A \cdot B = A \cdot A^* = N I$$

$$U \cdot U^* = I$$

Unitary Matrix

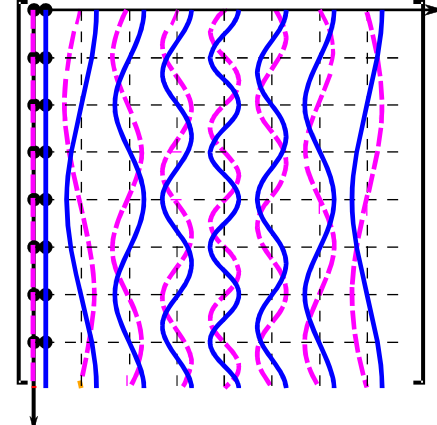
Symmetric Matrices

DFT Matrix in the row-wise view

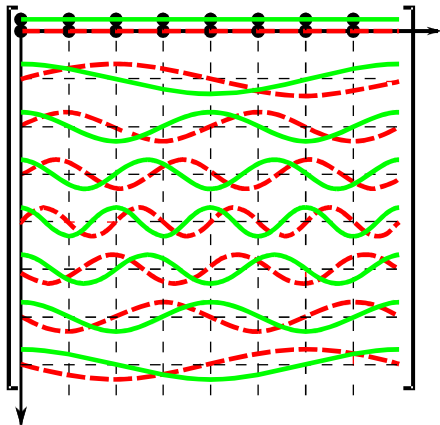


$$\mathbf{A} = \mathbf{A}^T$$

DFT Matrix in the column-wise view

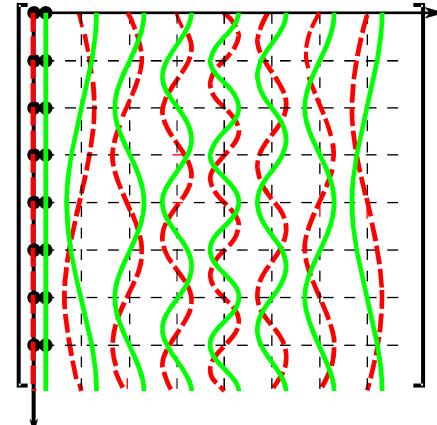


IDFT Matrix in the row-wise view



$$\mathbf{B} = \mathbf{B}^T$$

IDFT Matrix in the column-wise view



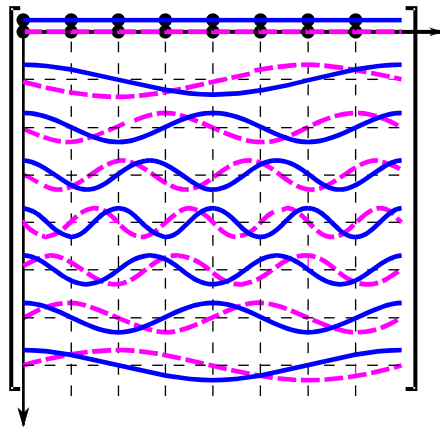
Conjugate Transpose Matrices

$$A = B^*$$

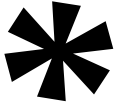
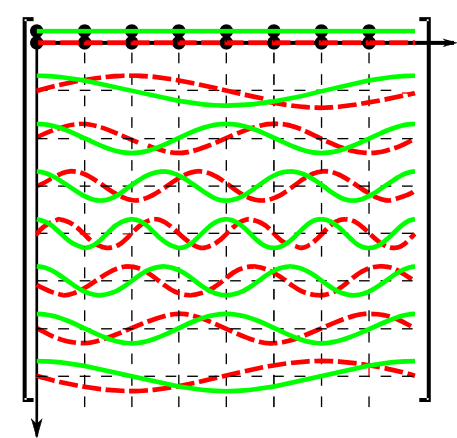
— Real - - Imaginary

DFT

$$A =$$



=

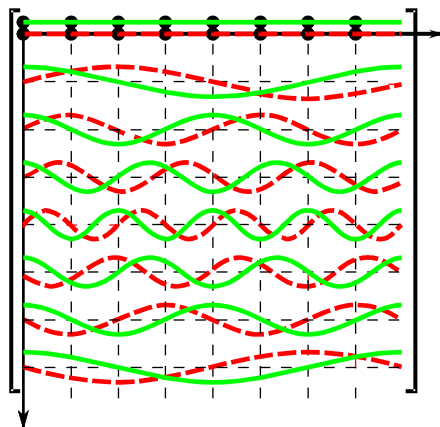


$$B = A^*$$

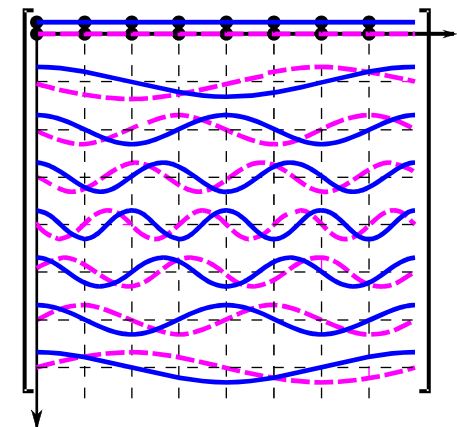
— Real - - Imaginary

IDFT

$$B =$$



=



Product AB

DFT

IDFT

A

\cdot

B

$=$

C

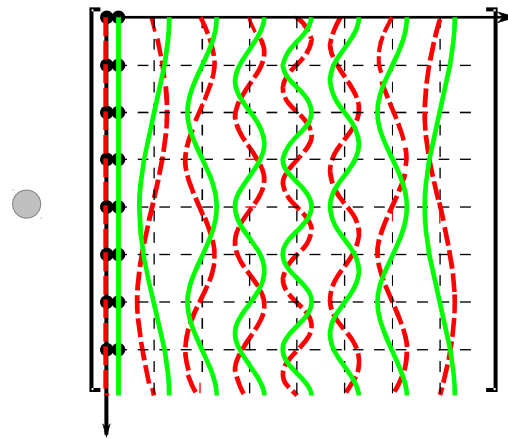
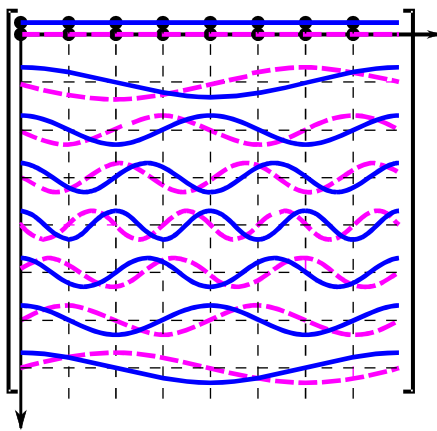
A

\cdot

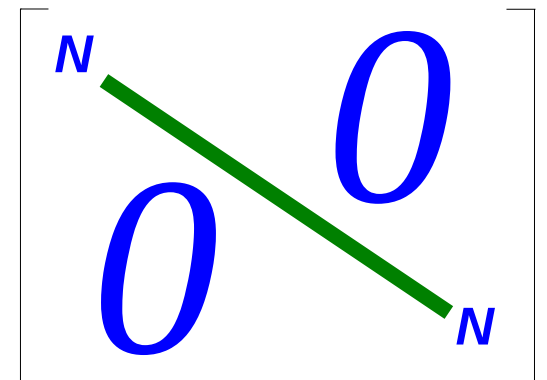
A^*

$=$

C



$=$



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A “graphical interpretation” of the DFT and FFT, by Steve Mann