DFT Matrix Examples (DFT.2.A)

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  - DFT Matrix in Exponential Terms
  - DFT Matrix in Cosine and Sine Terms
  - DFT Matrix in Real and Imaginary Terms
  - DFT Real and Imaginary Phase Factors
  - DFT Real and Imaginary Phase Factors Symmetry
- N=8 IDFT Matrix
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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Longleftrightarrow \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

$$W_N \triangleq e^{-j(2\pi/N)}$$

$$W_N^{nk} \triangleq e^{-j(2\pi/N)nk}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn} \quad \Longleftrightarrow \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{-kn}$$

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N=8 DFT

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

N=8 DFT Matrix in Exponential Terms

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad\qquad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix}
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

N=8 DFT Matrix in Cosine and Sine Terms

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} = \cos\left(\frac{\pi}{4}\cdot k\cdot n\right) - j\sin\left(\frac{\pi}{4}\cdot k\cdot n\right)$$

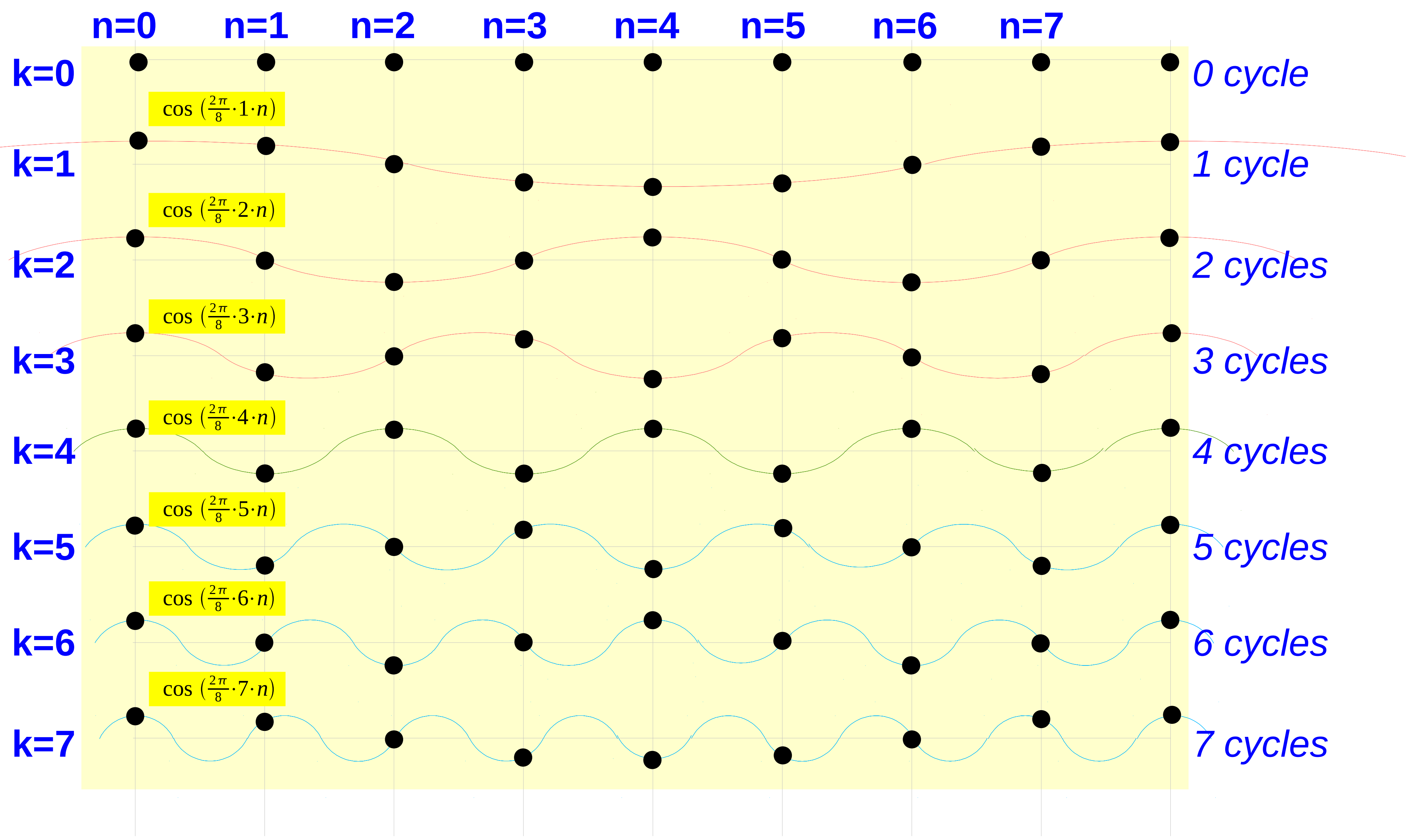
$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$
$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 1$ $-j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 2$ $-j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 3$ $-j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 5$ $-j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 6$ $-j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 7$ $-j\sin(\pi/4)\cdot 7$
$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 2$ $-j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 6$ $-j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 2$ $-j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 6$ $-j\sin(\pi/4)\cdot 6$
$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 3$ $-j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 6$ $-j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 1$ $-j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 7$ $-j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 2$ $-j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 5$ $-j\sin(\pi/4)\cdot 5$
$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$
$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 5$ $-j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 2$ $-j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 7$ $-j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 1$ $-j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 6$ $-j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 3$ $-j\sin(\pi/4)\cdot 3$
$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 6$ $-j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 2$ $-j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 6$ $-j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 2$ $-j\sin(\pi/4)\cdot 2$
$\cos(\pi/4)\cdot 0$ $-j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 7$ $-j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 6$ $-j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 5$ $-j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 4$ $-j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 3$ $-j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 2$ $-j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 1$ $-j\sin(\pi/4)\cdot 1$

N=8 DFT Matrix Real and Imaginary Terms

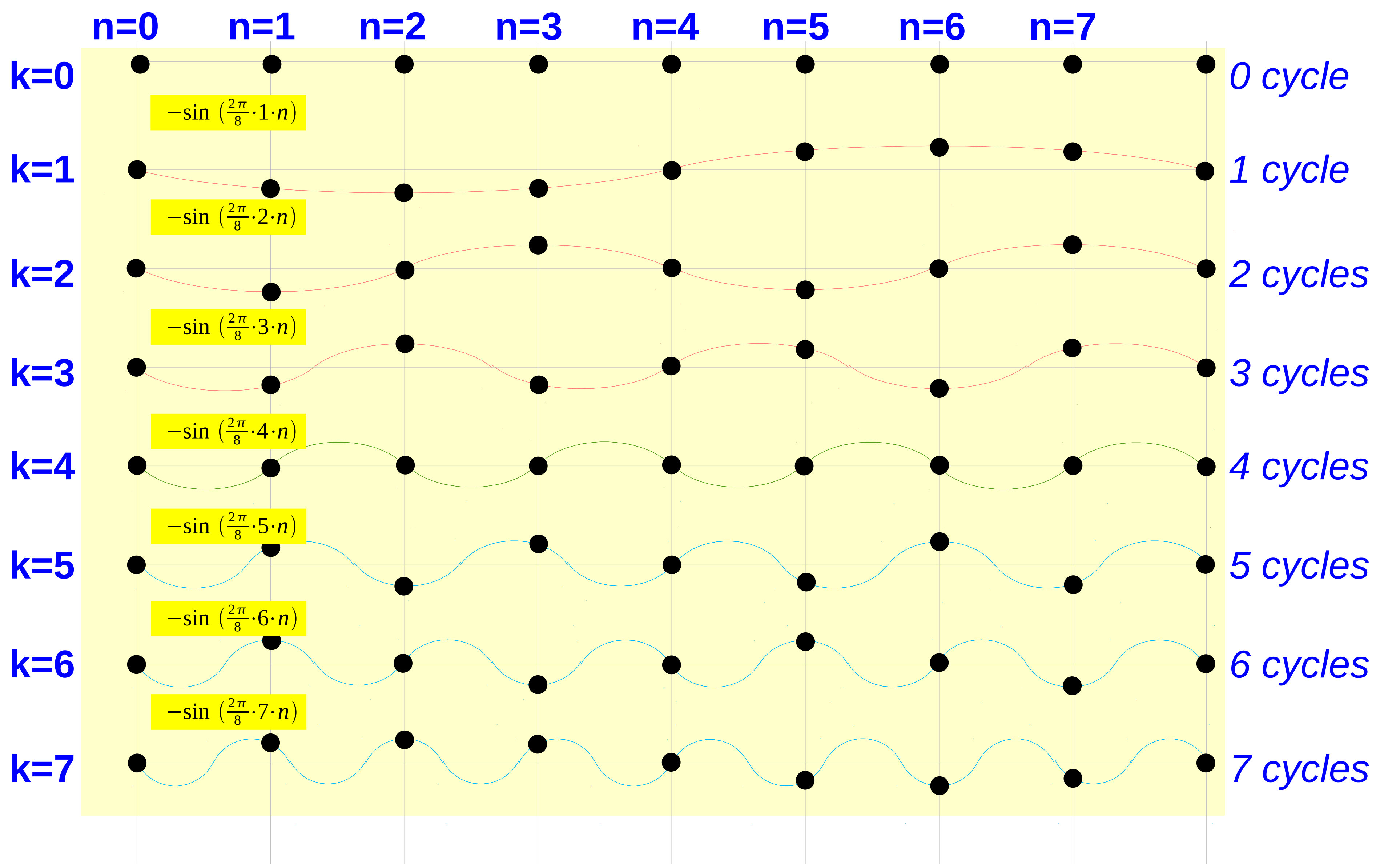
$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} = \cos\left(\frac{\pi}{4}\cdot k\cdot n\right) - j\,\sin\left(\frac{\pi}{4}\cdot k\cdot n\right)$$

$$\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & +j & +\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\
0 & -j & -1 & +j & 0 & -j & -1 & +j \\
0 & -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & +j & +\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -1 & +\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -j & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\
0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\
0 & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -j & +\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -1 & +\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & +j & -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \\
0 & +j & -1 & -j & 0 & +j & -1 & -j \\
0 & +\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & +j & -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -1 & -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -j & +\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}
\end{matrix}$$

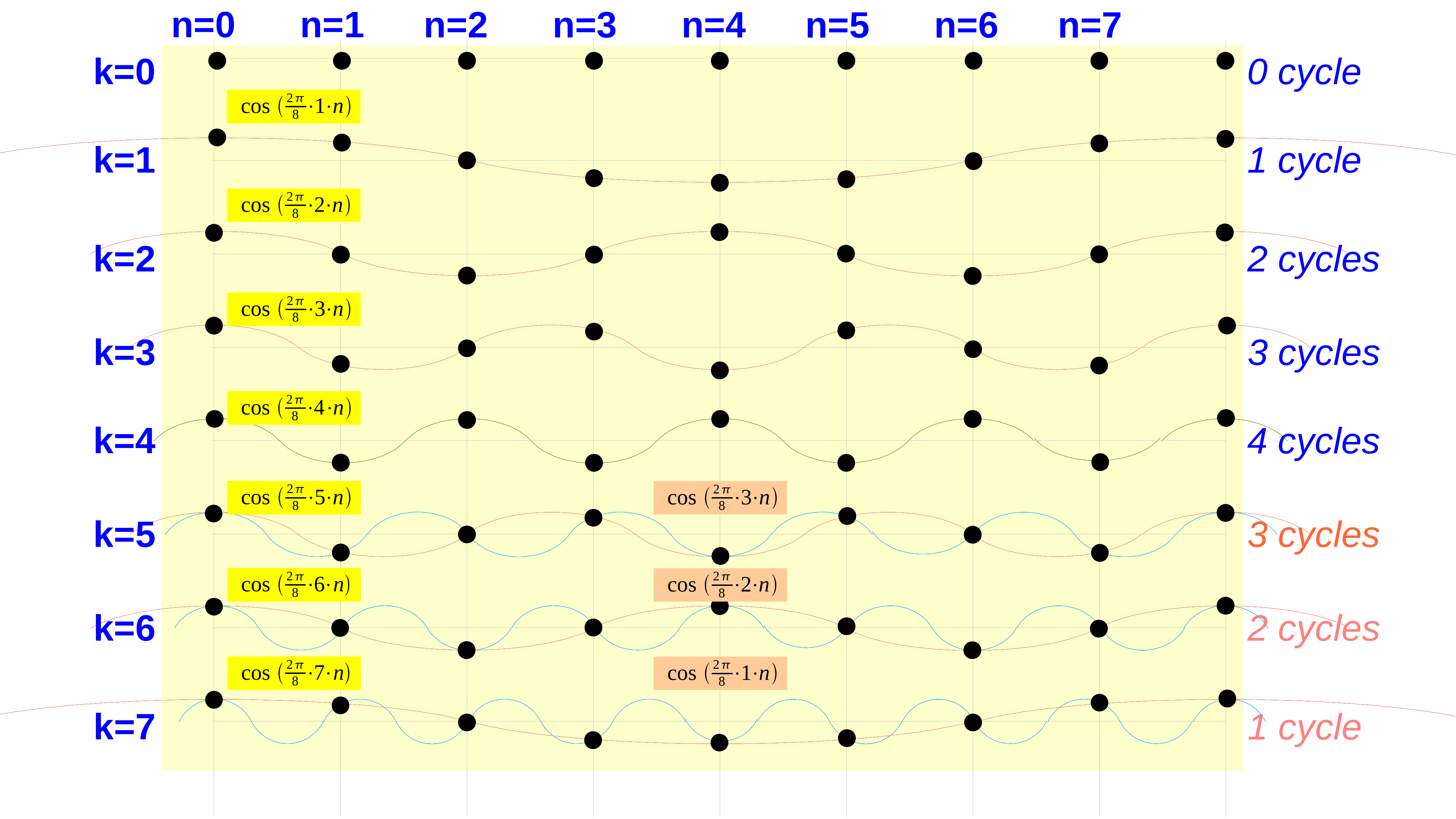
N=8 DFT Real Phase Factors



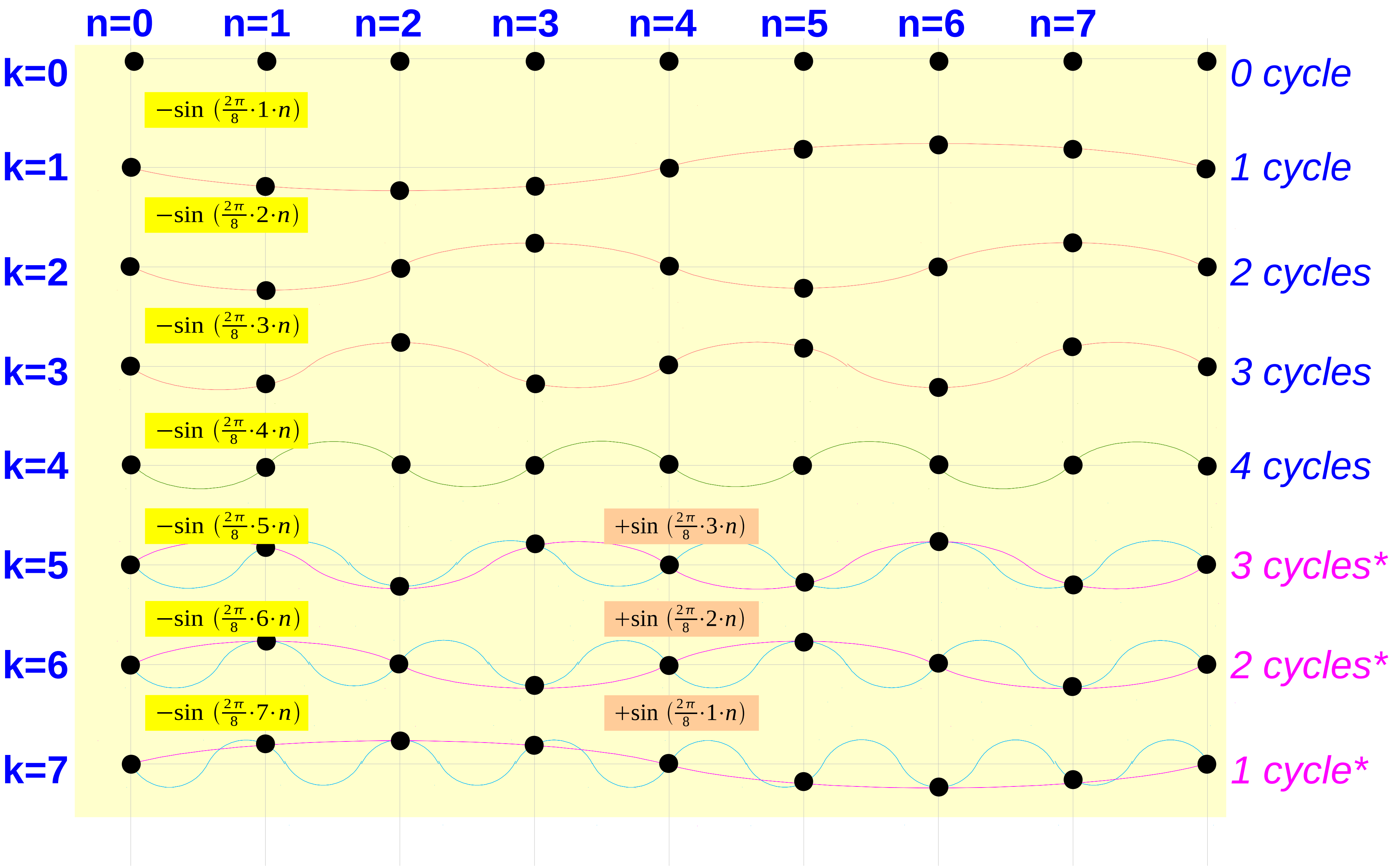
N=8 DFT Imaginary Phase Factors



N=8 DFT Real Phase Factor Symmetry



N=8 DFT Imaginary Phase Factor Symmetry



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N=8 IDFT Matrix

$$x[n] = \frac{1}{N}\sum_{k=0}^{7} W_8^{-kn}\, X[k] \qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} \\ W_8^{0} & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7} \\ W_8^{0} & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^{-8} & W_8^{-10} & W_8^{-12} & W_8^{-14} \\ W_8^{0} & W_8^{-3} & W_8^{-6} & W_8^{-9} & W_8^{-12} & W_8^{-15} & W_8^{-18} & W_8^{-21} \\ W_8^{0} & W_8^{-4} & W_8^{-8} & W_8^{-12} & W_8^{-16} & W_8^{-20} & W_8^{-24} & W_8^{-28} \\ W_8^{0} & W_8^{-5} & W_8^{-10} & W_8^{-15} & W_8^{-20} & W_8^{-25} & W_8^{-30} & W_8^{-35} \\ W_8^{0} & W_8^{-6} & W_8^{-12} & W_8^{-18} & W_8^{-24} & W_8^{-30} & W_8^{-36} & W_8^{-42} \\ W_8^{0} & W_8^{-7} & W_8^{-14} & W_8^{-21} & W_8^{-28} & W_8^{-35} & W_8^{-42} & W_8^{-49} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix}$$

N=8 IDFT Matrix in Exponential Terms

$$x[n] = \frac{1}{N}\sum_{k=0}^{7} W_8^{-kn}\, X[k] \qquad\qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix}
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 7} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 5} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 3} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

N=8 IDFT Matrix in Cosine and Sine Terms

$$W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn} = \cos\left(\frac{\pi}{4}\cdot k\cdot n\right) + j\sin\left(\frac{\pi}{4}\cdot k\cdot n\right)$$

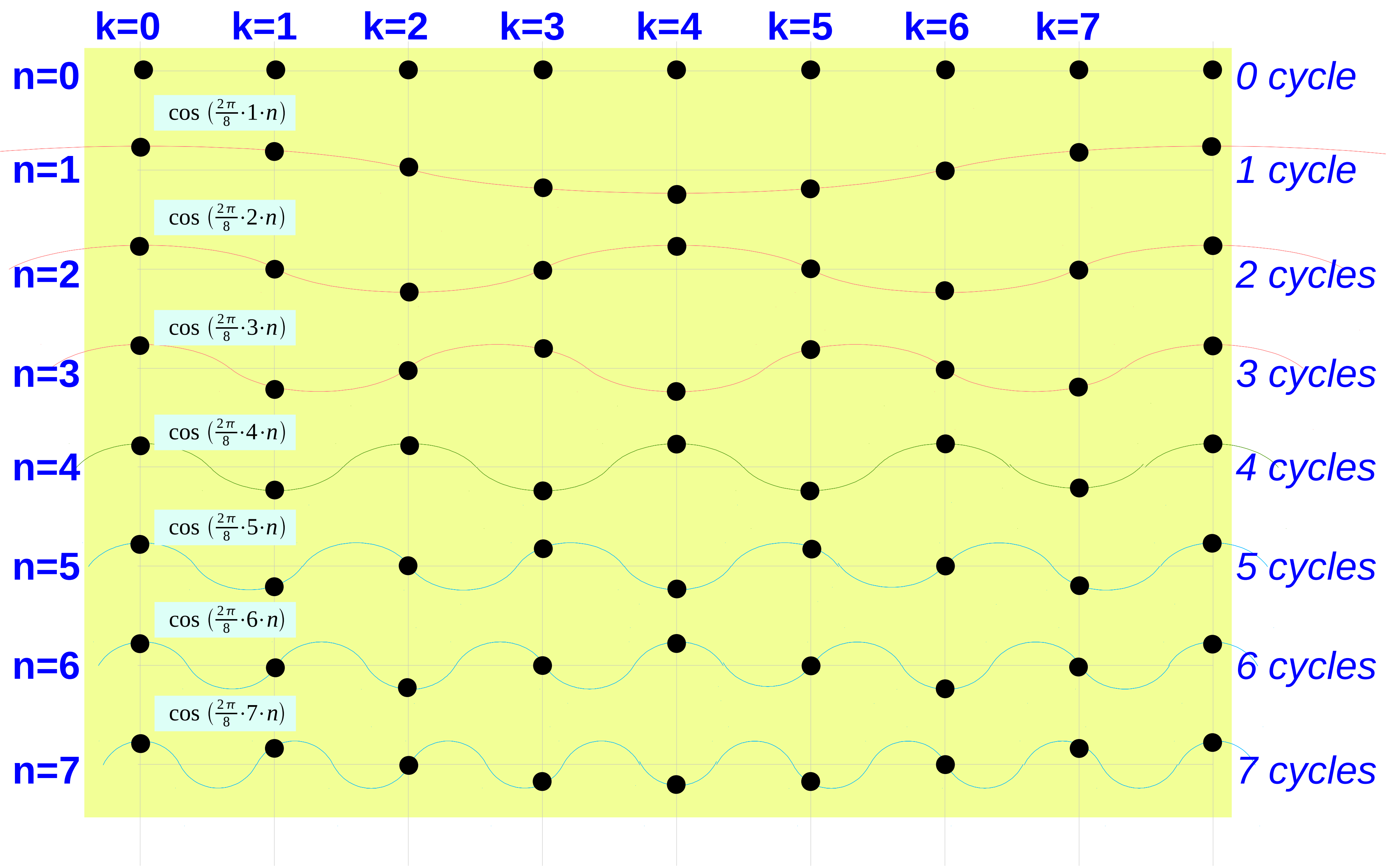
$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$
$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 1$ $+j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 2$ $+j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 3$ $+j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 5$ $+j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 6$ $+j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 7$ $+j\sin(\pi/4)\cdot 7$
$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 2$ $+j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 6$ $+j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 2$ $+j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 6$ $+j\sin(\pi/4)\cdot 6$
$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 3$ $+j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 6$ $+j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 1$ $+j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 7$ $+j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 2$ $+j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 5$ $+j\sin(\pi/4)\cdot 5$
$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$
$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 5$ $+j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 2$ $+j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 7$ $+j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 1$ $+j\sin(\pi/4)\cdot 1$	$\cos(\pi/4)\cdot 6$ $+j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 3$ $+j\sin(\pi/4)\cdot 3$
$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 6$ $+j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 2$ $+j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 6$ $+j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 2$ $+j\sin(\pi/4)\cdot 2$
$\cos(\pi/4)\cdot 0$ $+j\sin(\pi/4)\cdot 0$	$\cos(\pi/4)\cdot 7$ $+j\sin(\pi/4)\cdot 7$	$\cos(\pi/4)\cdot 6$ $+j\sin(\pi/4)\cdot 6$	$\cos(\pi/4)\cdot 5$ $+j\sin(\pi/4)\cdot 5$	$\cos(\pi/4)\cdot 4$ $+j\sin(\pi/4)\cdot 4$	$\cos(\pi/4)\cdot 3$ $+j\sin(\pi/4)\cdot 3$	$\cos(\pi/4)\cdot 2$ $+j\sin(\pi/4)\cdot 2$	$\cos(\pi/4)\cdot 1$ $+j\sin(\pi/4)\cdot 1$

N=8 IDFT Matrix in Real and Imaginary Terms

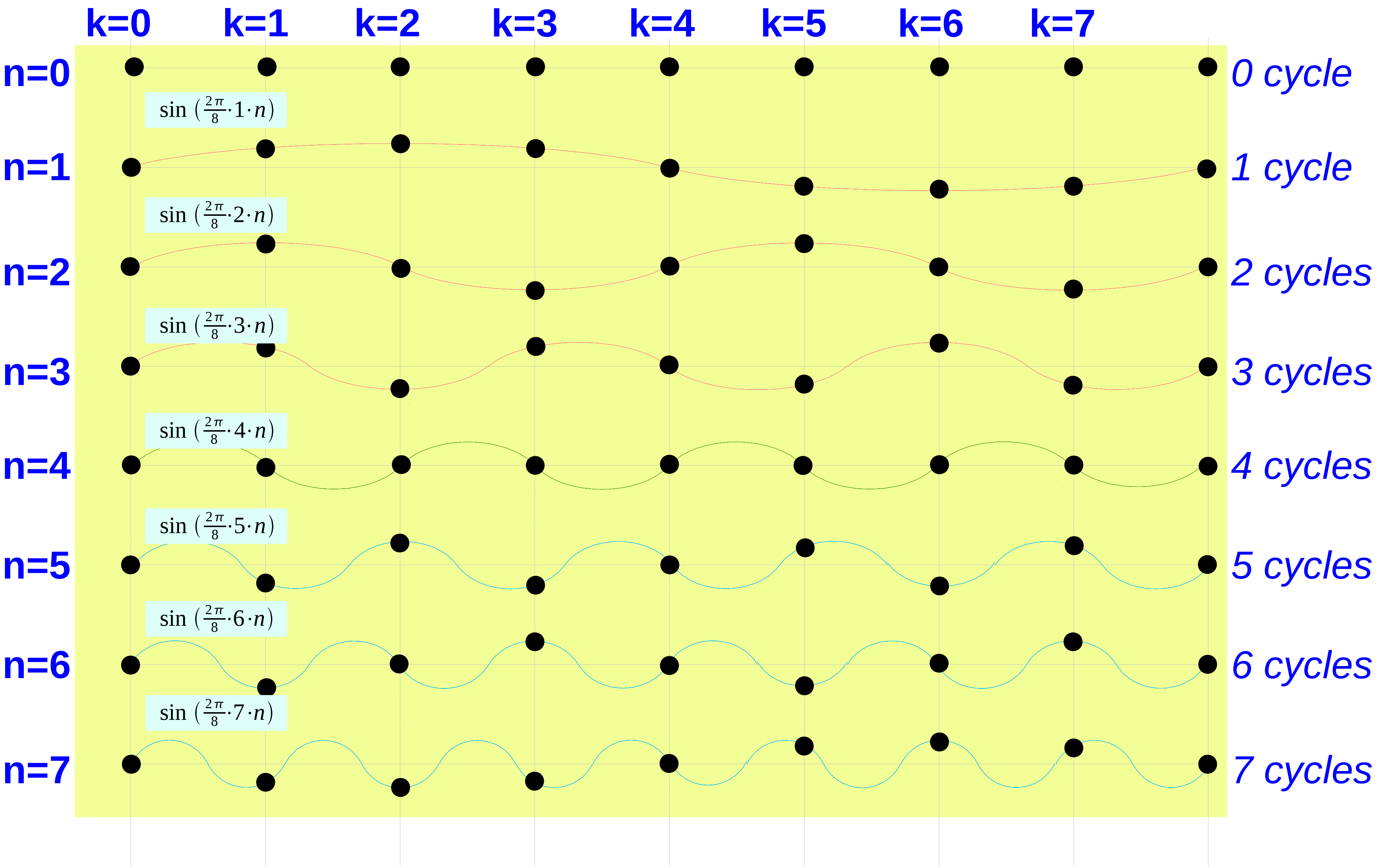
$$W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn} = \cos\left(\frac{\pi}{4}\cdot k\cdot n\right) + j\sin\left(\frac{\pi}{4}\cdot k\cdot n\right)$$

0	0	0	0	0	0	0	0
0	$+\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$	$+j$	$-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}$	$-j$	$+\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}$
0	$+j$	-1	$-j$	0	$+j$	-1	$-j$
0	$-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$	$-j$	$+\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$	-1	$+\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}$	$+j$	$-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}$
0	-1	0	-1	0	-1	0	-1
0	$-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}$	$+j$	$+\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}$	-1	$+\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$	$-j$	$-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$
0	$-j$	-1	$+j$	0	$-j$	-1	$+j$
0	$+\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}$	$-j$	$-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$	$+j$	$+\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$

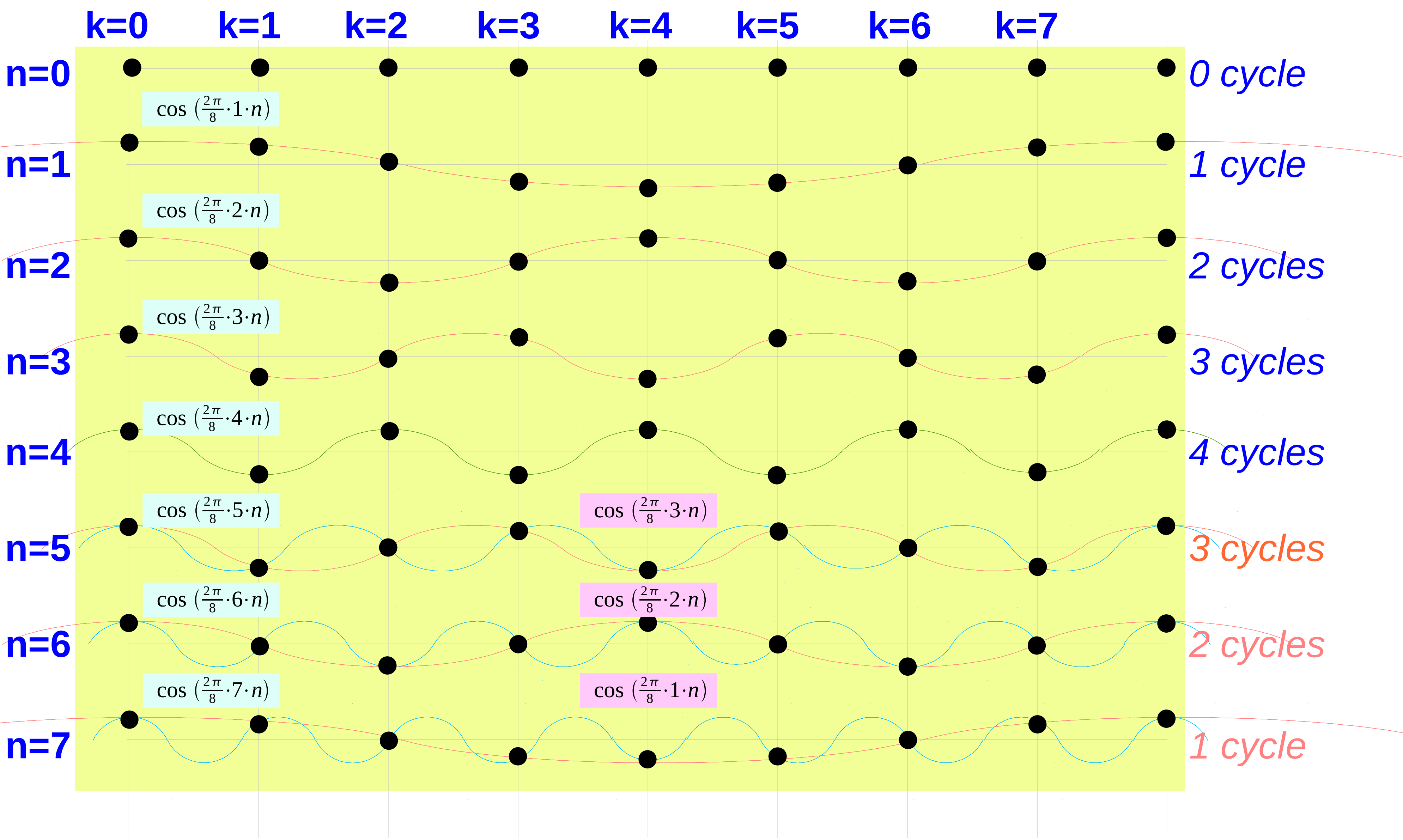
N=8 IDFT Real Phase Factors



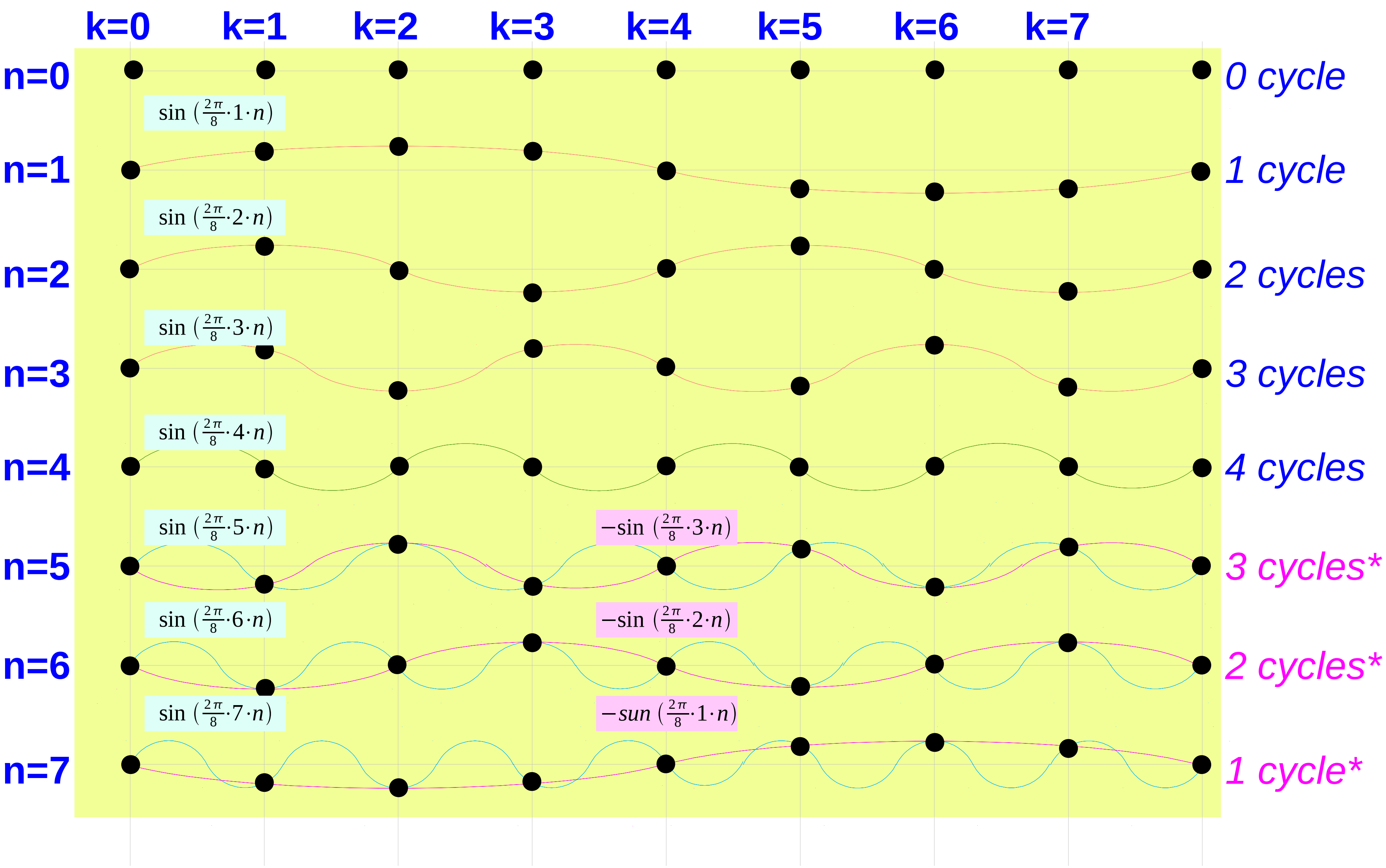
N=8 IDFT Imaginary Phase Factors



N=8 IDFT Real Phase Factor Symmetry



N=8 IDFT Imaginary Phase Factor Symmetry



References

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