DFT Sampling (5B)

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Normalized Radian Frequency

$$x(t) = A\cos(\omega t + \phi)$$

$$x[n] = x(nT_s)$$

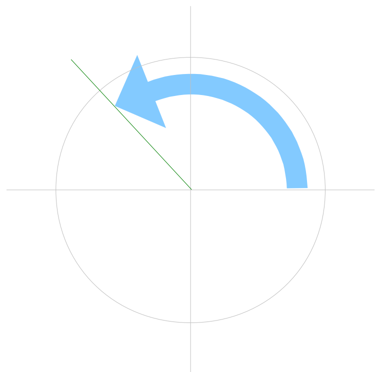
$$= A\cos(\omega \cdot nT_s + \phi)$$

$$= A\cos(\hat{\omega} \cdot T_s + \phi)$$

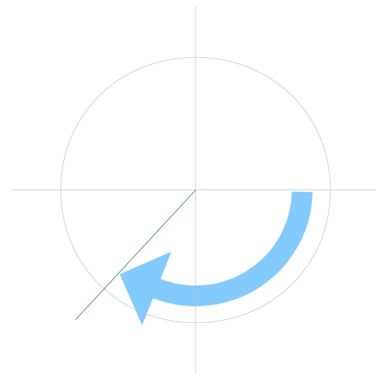
$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s} = \frac{\omega}{f_s}$$

Angular Speed

$+\omega_0 \ (rad/sec)$



$-\omega_0 \ (rad/sec)$

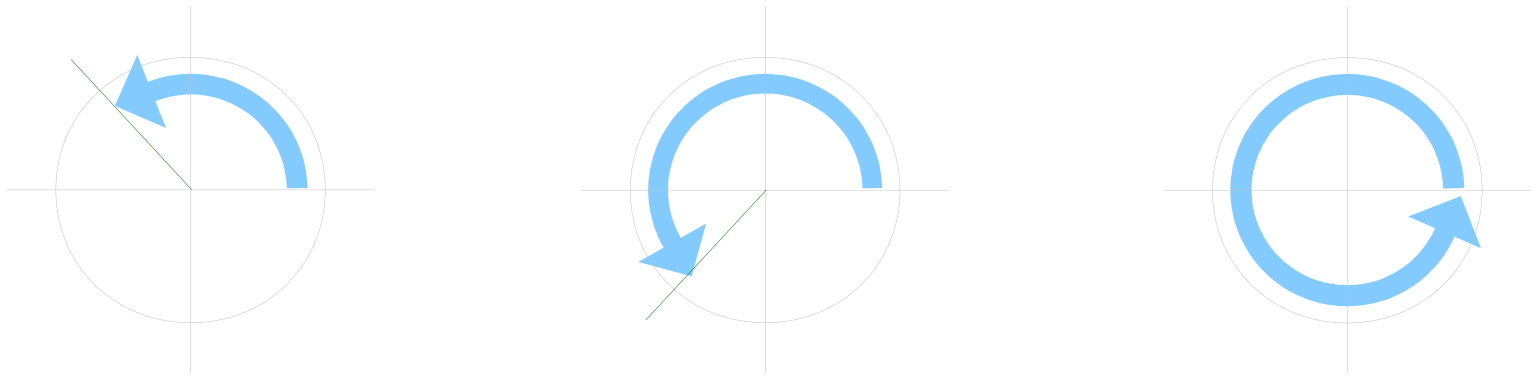


$+\omega_0$ *radians per second*

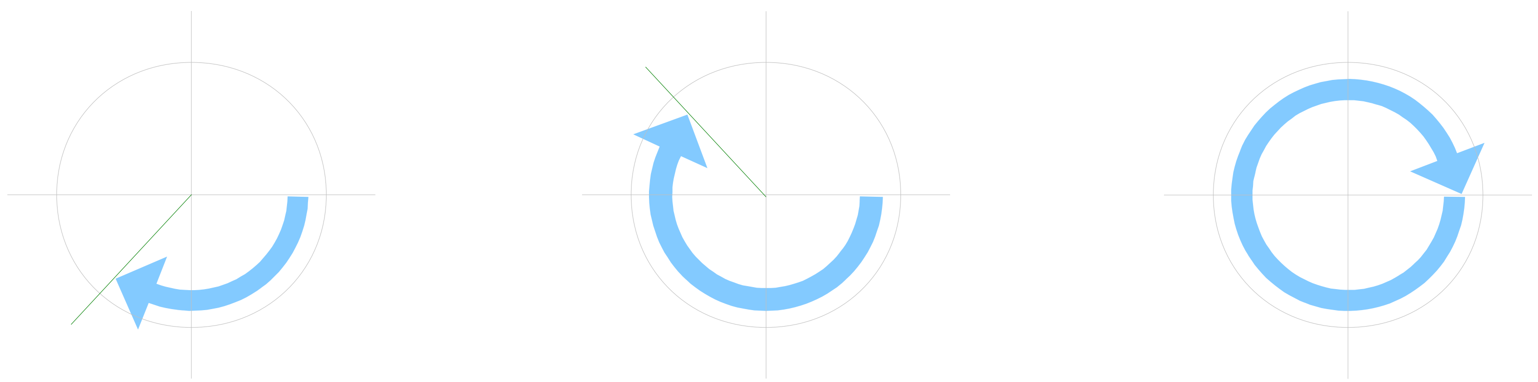
$-\omega_0$ *radians per second*

Angular Speed

$+\omega_0 \;\; (rad/sec)$ $\qquad +2\omega_0 \;\; (rad/sec)$ $\qquad +3\omega_0 \;\; (rad/sec)$



$-\omega_0 \;\; (rad/sec)$ $\qquad -2\omega_0 \;\; (rad/sec)$ $\qquad -3\omega_0 \;\; (rad/sec)$



Angular Speed

$0.01\ sec$	$0.1\ sec$	$1\ sec$	$10\ sec$	$100\ sec$
$100\ Hz$	$10\ Hz$	$1\ Hz$	$0.1\ Hz$	$0.01\ Hz$
200π (rad/sec)	20π (rad/sec)	2π (rad/sec)	0.2π (rad/sec)	0.02π (rad/sec)
628	62.8	6.28	0.628	0.0628

Sampling

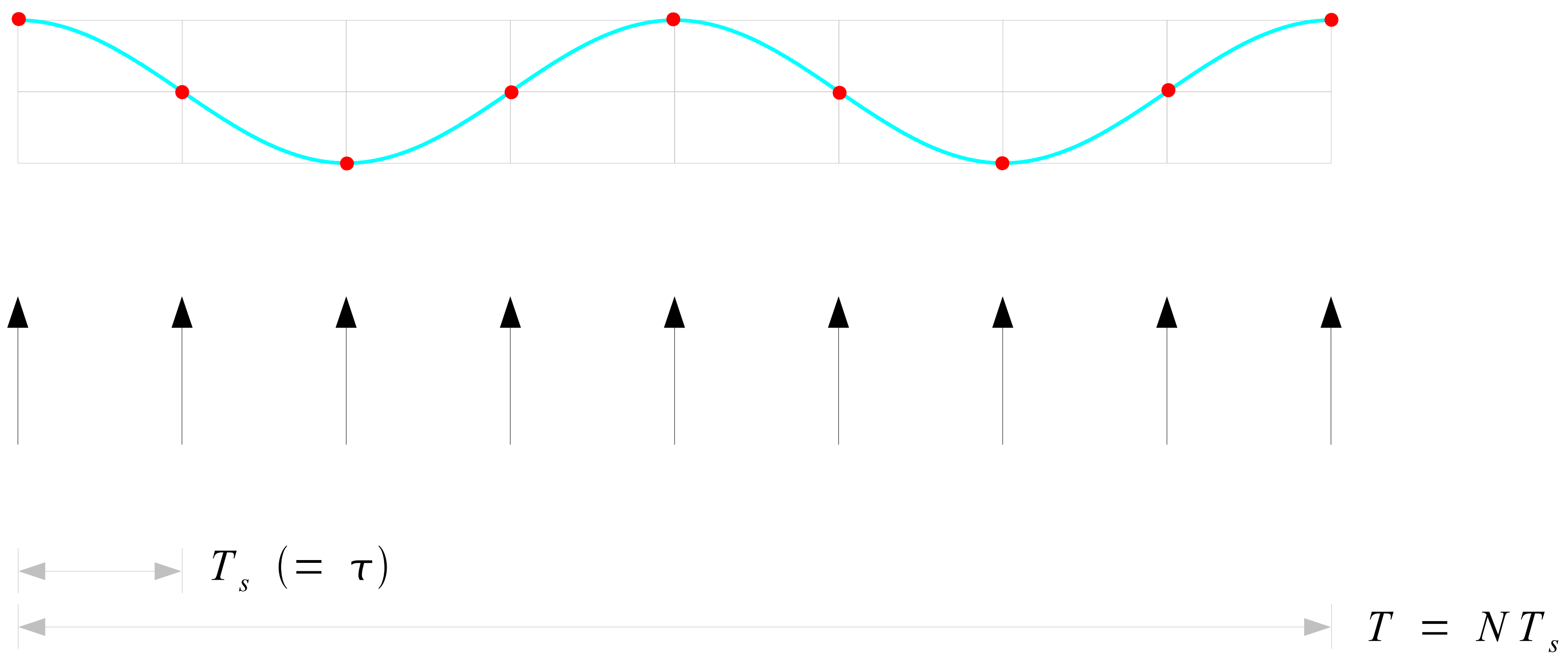
$$\hat{\omega} \quad (rad/sec)$$

$$\hat{\omega} = \omega \cdot T_s \quad (rad)$$ *Dimensionless quantity*

$$x[n] = x(nT_s)$$ *Dimensionless sequence*

Infinite number of continuous time signals → *Sampling* → *The same discrete-time sinusoid*

Sampling

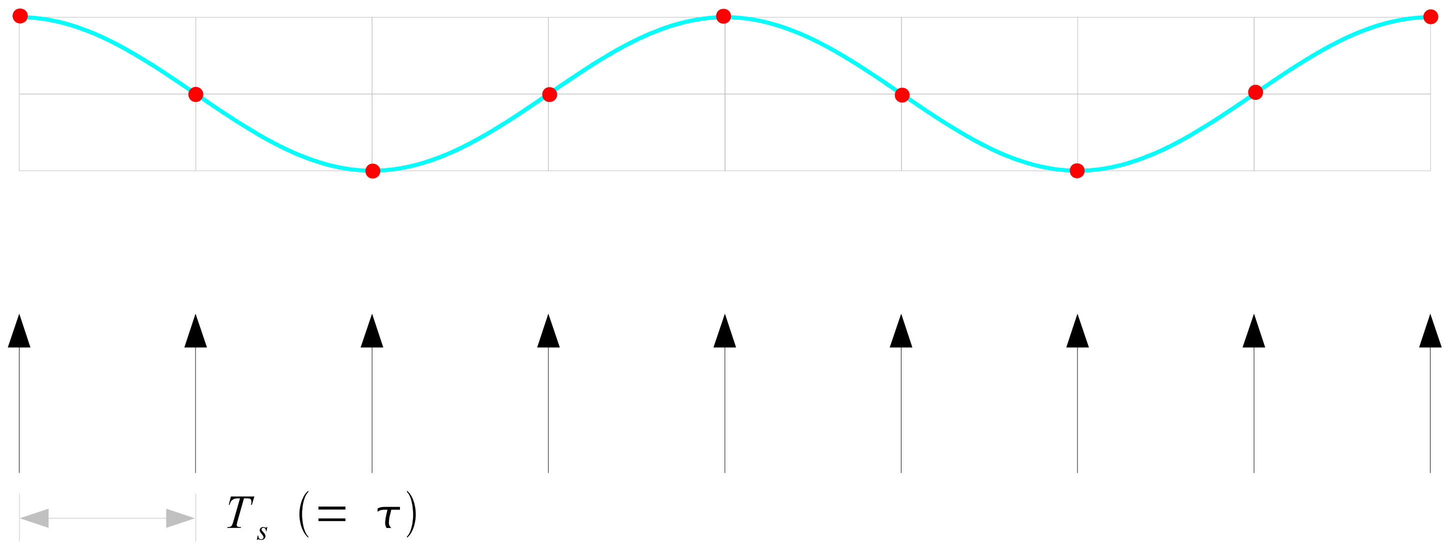


Sampling Time T_s

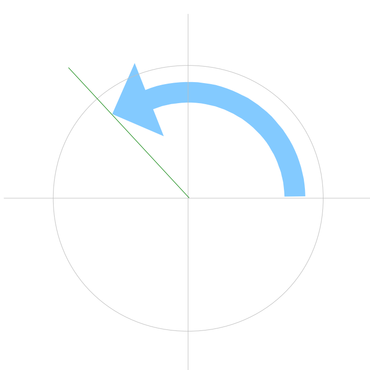
Sequence Time Length $T = N T_s$

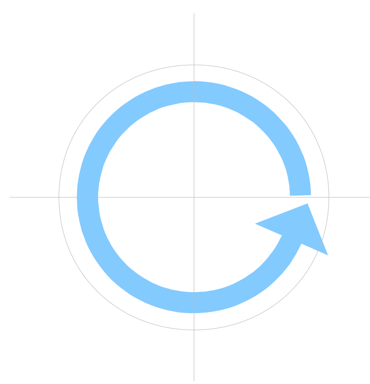
Sampling Frequency $f_s = \dfrac{1}{T_s}$ *(samples per second)*

Sampling



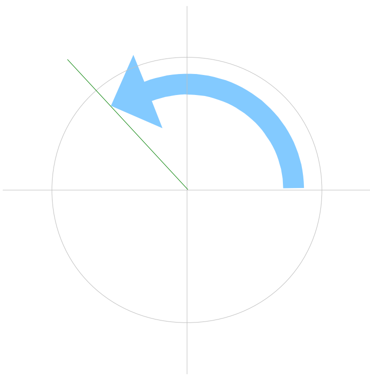
$\omega_s = 2\pi f_s \; (rad/sec)$ $\qquad$ $2\pi \, (rad) \; / \; T_s \, (sec)$





Sampling

$$\omega_s = 2\pi f_s \ (rad/sec)$$



$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \ (rad/sec)$$

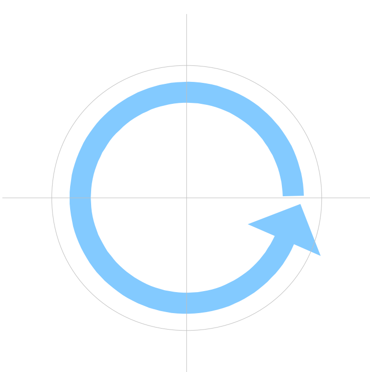
$$f_1 = \frac{f_s}{2} \ (rad/sec)$$

$$\omega_2 = 2\pi f_2$$

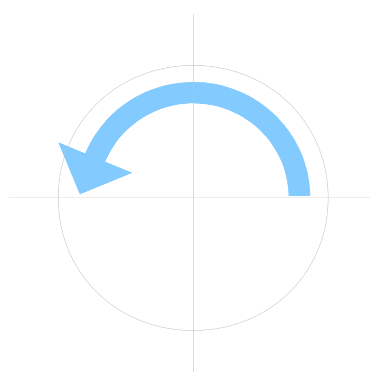
$$\omega_2 = \frac{\omega_s}{4} \ (rad/sec)$$

$$f_2 = \frac{f_s}{4} \ (rad/sec)$$

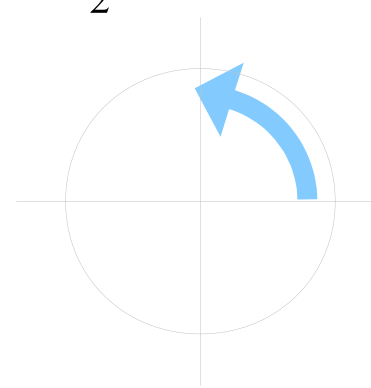
$$2\pi \ (rad) \ / \ T_s \ (sec)$$



$$\pi \ (rad) \ / \ T_s \ (sec)$$

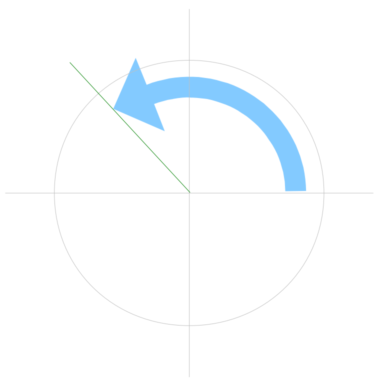


$$\frac{\pi}{2} \ (rad) \ / \ T_s \ (sec)$$



Sampling

$$\omega_s = 2\pi f_s \ (rad/sec)$$



$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \ (rad/sec)$$

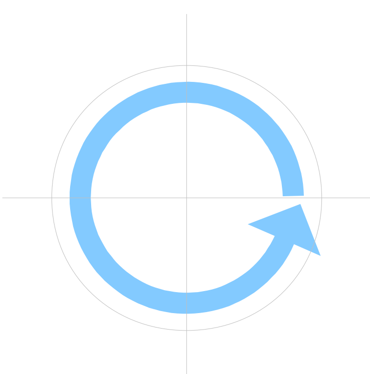
$$f_1 = \frac{f_s}{2} \ (rad/sec)$$

$$\omega_2 = 2\pi f_2$$

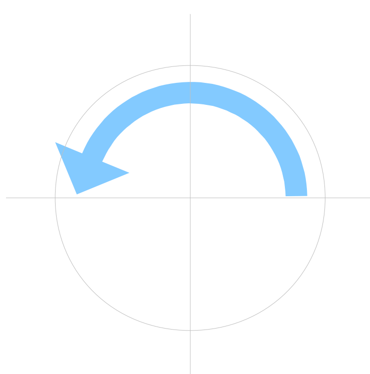
$$\omega_2 = -\frac{\omega_s}{2} \ (rad/sec)$$

$$f_2 = -\frac{f_s}{2} \ (rad/sec)$$

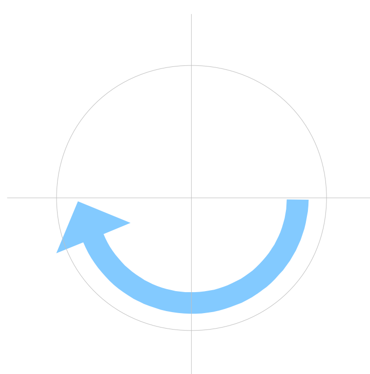
$$2\pi \ (rad) \ / \ T_s \ (sec)$$



$$\pi \ (rad) \ / \ T_s \ (sec)$$

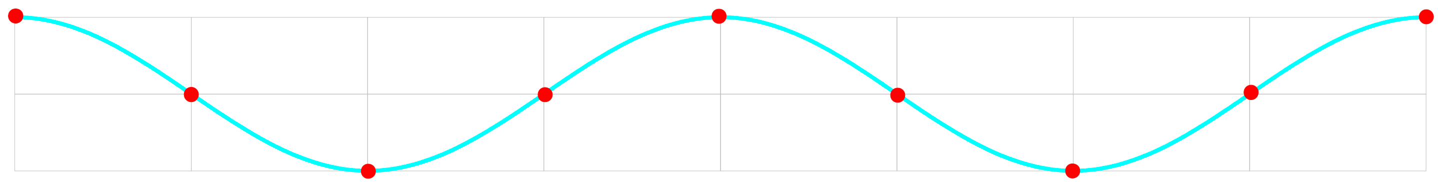


$$-\pi \ (rad) \ / \ T_s \ (sec)$$

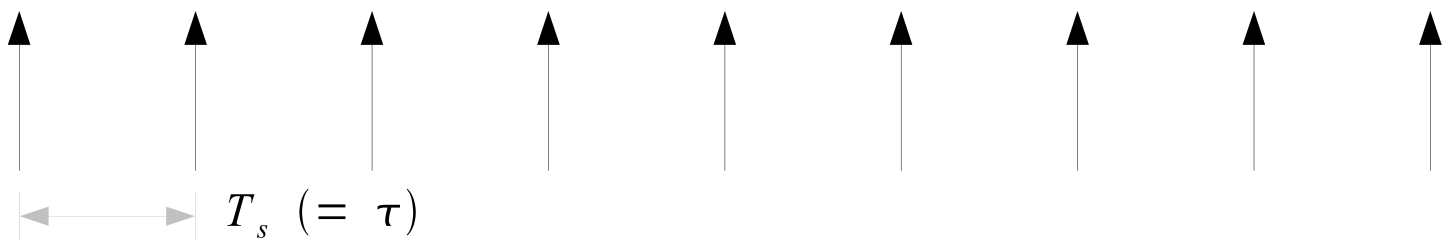


Sampling

$\omega_1 = 2\pi f_1 \;\; (rad/sec)$

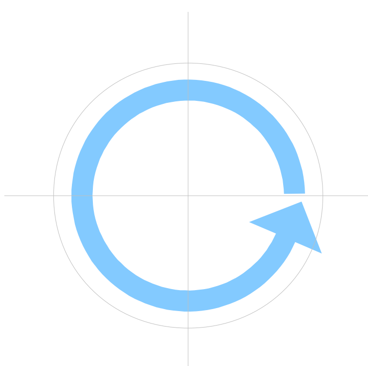


$\omega_s = 2\pi f_s \;\; (rad/sec)$

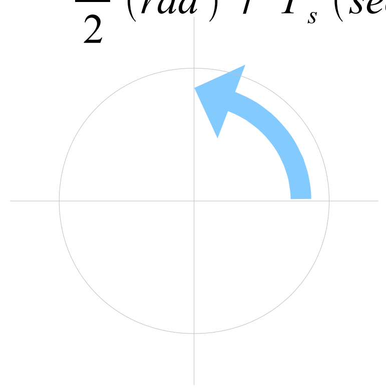


$T_s \;\; (= \tau)$

$2\pi \; (rad) \;\; / \;\; T_s \; (sec)$



$\frac{\pi}{2} \; (rad) \;\; / \;\; T_s \; (sec)$



For the period of T_s

Angular displacement $\frac{\pi}{2} \; (rad)$

$$\hat{\omega} = \omega \cdot T_s \;\; (rad)$$

$$= 2\pi f_1 \cdot T_s \;\; (rad)$$

$$= 2\pi \frac{f_s}{4} \cdot T_s \;\; (rad)$$

$$= \frac{\pi}{2} \;\; (rad)$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann