

DFT Sampling (5B)

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Normalized Radian Frequency

$$x(t) = A \cos(\omega t + \phi)$$

$$x[n] = x(nT_s)$$

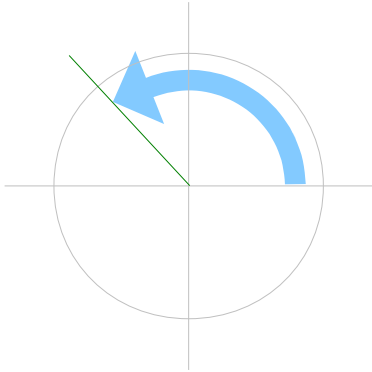
$$= A \cos(\omega \cdot nT_s + \phi)$$

$$= A \cos(\hat{\omega} \cdot T_s + \phi)$$

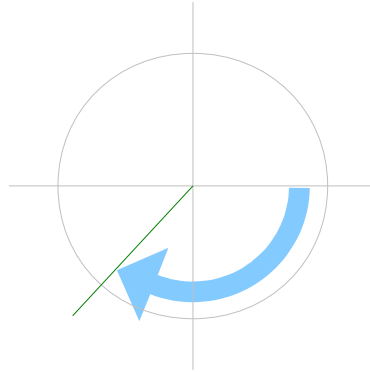
$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s} = \frac{\omega}{f_s}$$

Angular Speed

$+\omega_0$ (*rad/sec*)



$-\omega_0$ (*rad/sec*)

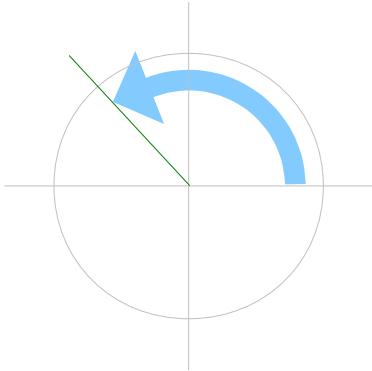


$+\omega_0$ *radians*
per second

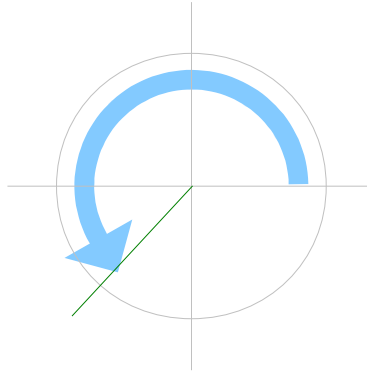
$-\omega_0$ *radians*
per second

Angular Speed

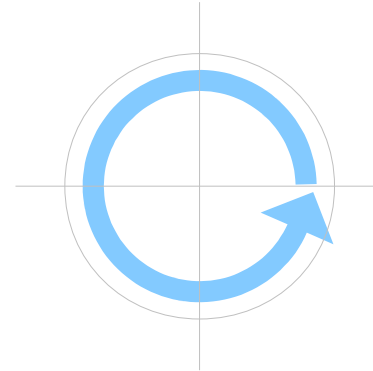
$+\omega_0$ (rad/sec)



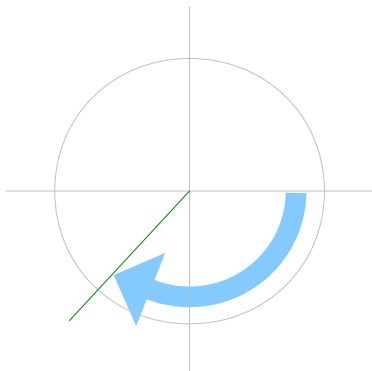
$+2\omega_0$ (rad/sec)



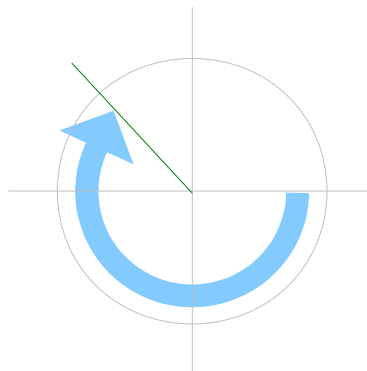
$+3\omega_0$ (rad/sec)



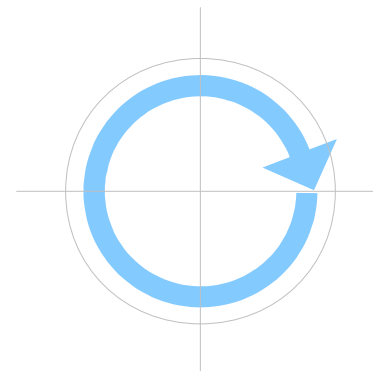
$-\omega_0$ (rad/sec)



$-2\omega_0$ (rad/sec)



$-3\omega_0$ (rad/sec)



Angular Speed

<i>0.01 sec</i>	<i>0.1 sec</i>	<i>1 sec</i>	<i>10 sec</i>	<i>100 sec</i>
<i>100 Hz</i>	<i>10 Hz</i>	<i>1 Hz</i>	<i>0.1 Hz</i>	<i>0.01 Hz</i>
200π <i>(rad/sec)</i>	20π <i>(rad/sec)</i>	2π <i>(rad/sec)</i>	0.2π <i>(rad/sec)</i>	0.02π <i>(rad/sec)</i>
<i>628</i>	<i>62.8</i>	<i>6.28</i>	<i>0.628</i>	<i>0.0628</i>

Sampling

$$\hat{\omega} \text{ (rad/sec)}$$

$$\hat{\omega} = \omega \cdot T_s \text{ (rad)} \quad \textit{Dimensionless quantity}$$

$$x[n] = x(nT_s) \quad \textit{Dimensionless sequence}$$

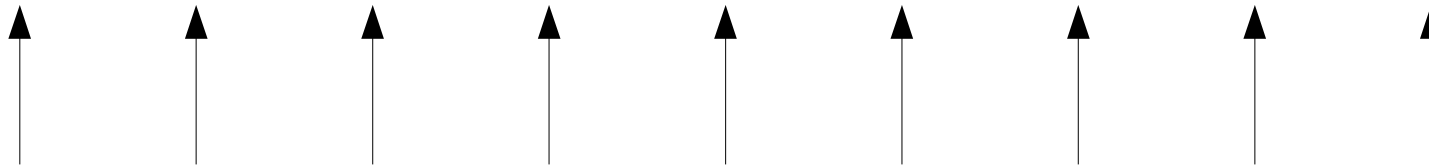
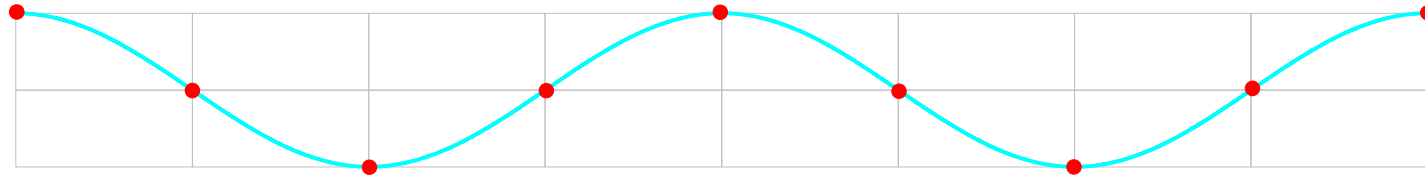
*Infinite number of
continuous time signals*

Sampling



*The same discrete-time
sinusoid*

Sampling



$T_s (= \tau)$

$T = NT_s$

Sampling Time

$$T_s$$

Sequence Time Length

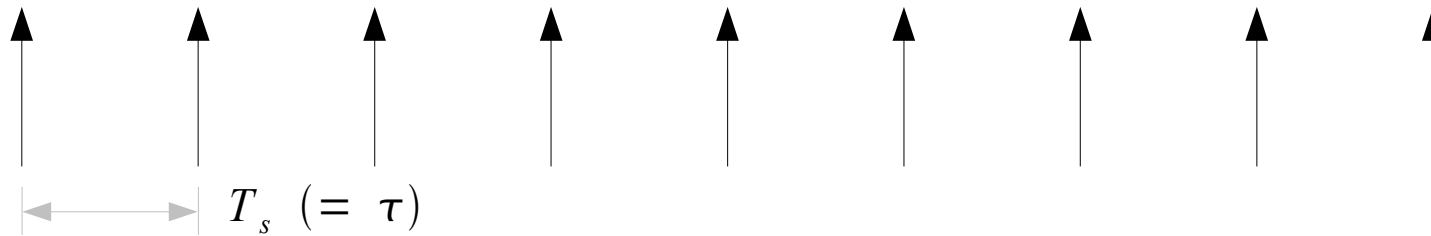
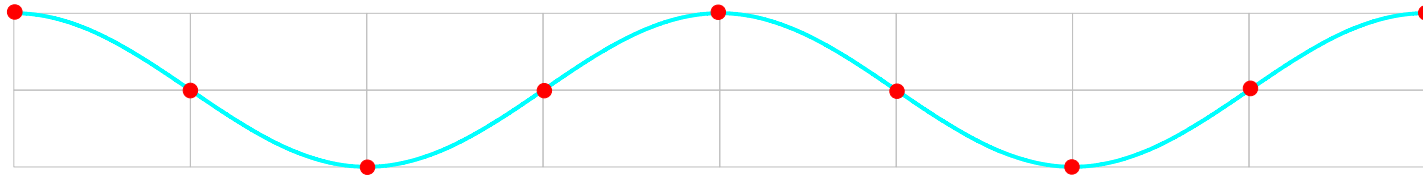
$$T = NT_s$$

Sampling Frequency

$$f_s = \frac{1}{T_s}$$

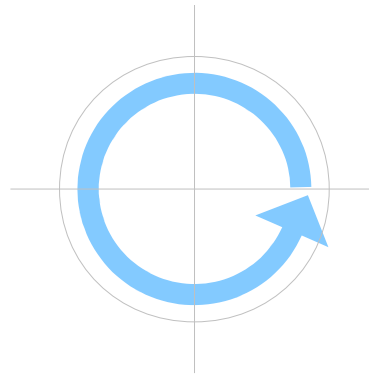
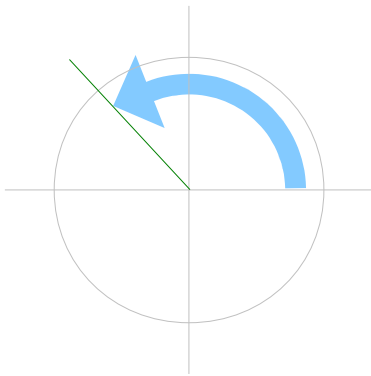
(samples per second)

Sampling



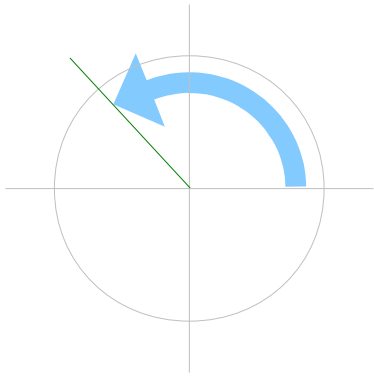
$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$

$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$



Sampling

$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$



$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \text{ (rad/sec)}$$

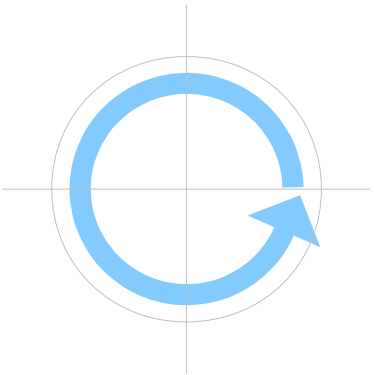
$$f_1 = \frac{f_s}{2} \text{ (rad/sec)}$$

$$\omega_2 = 2\pi f_2$$

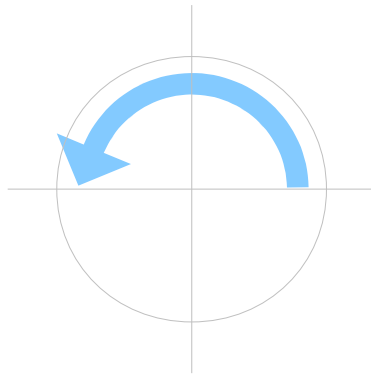
$$\omega_2 = \frac{\omega_s}{4} \text{ (rad/sec)}$$

$$f_2 = \frac{f_s}{4} \text{ (rad/sec)}$$

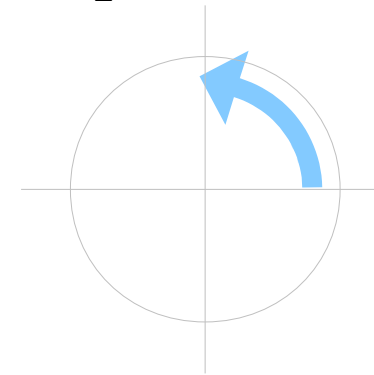
$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$



$$\pi \text{ (rad)} / T_s \text{ (sec)}$$

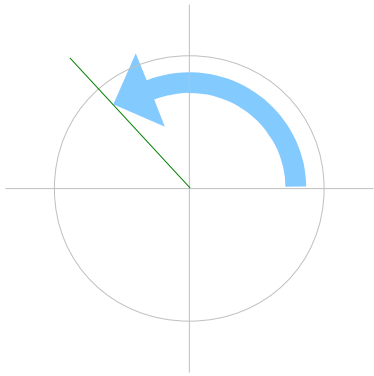


$$\frac{\pi}{2} \text{ (rad)} / T_s \text{ (sec)}$$

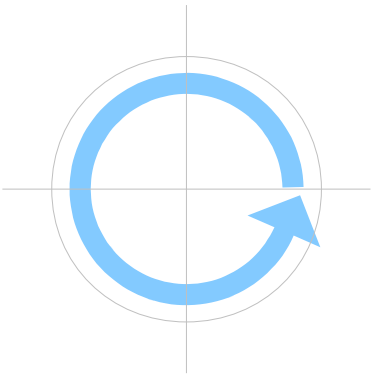


Sampling

$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$



$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$

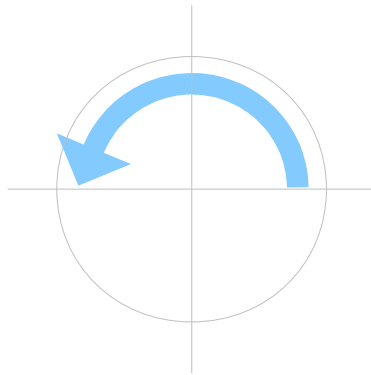


$$\omega_1 = 2\pi f_1$$

$$\omega_1 = \frac{\omega_s}{2} \text{ (rad/sec)}$$

$$f_1 = \frac{f_s}{2} \text{ (rad/sec)}$$

$$\pi \text{ (rad)} / T_s \text{ (sec)}$$

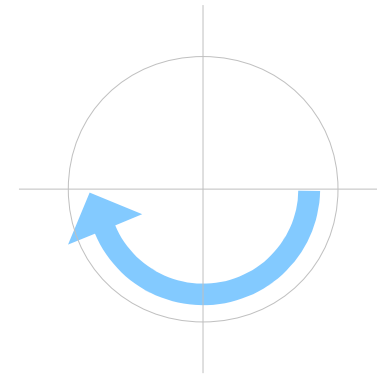


$$\omega_2 = 2\pi f_2$$

$$\omega_2 = -\frac{\omega_s}{2} \text{ (rad/sec)}$$

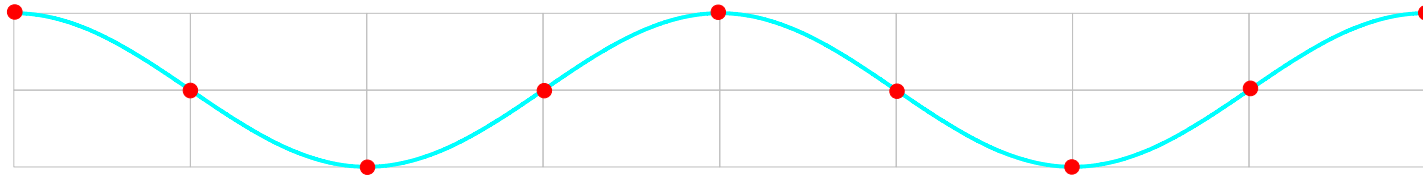
$$f_2 = -\frac{f_s}{2} \text{ (rad/sec)}$$

$$-\pi \text{ (rad)} / T_s \text{ (sec)}$$

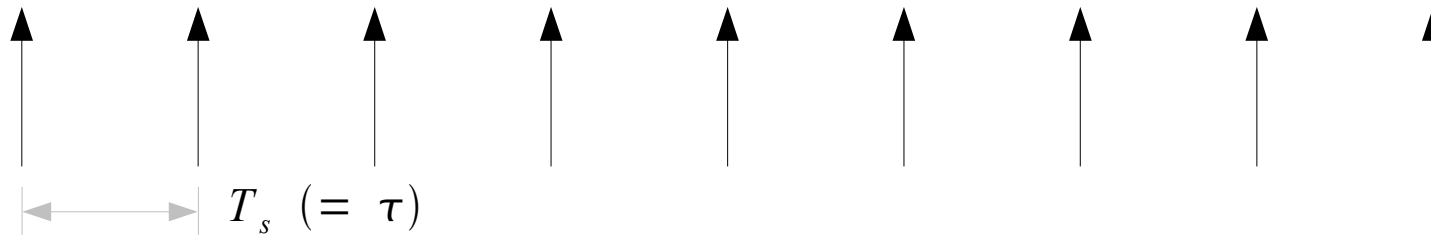


Sampling

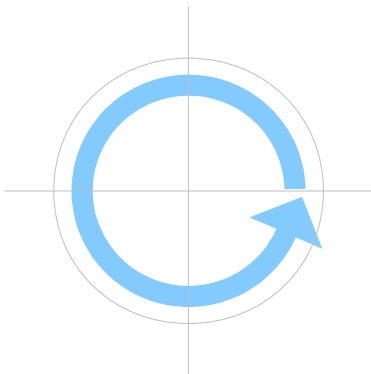
$$\omega_1 = 2\pi f_1 \text{ (rad/sec)}$$



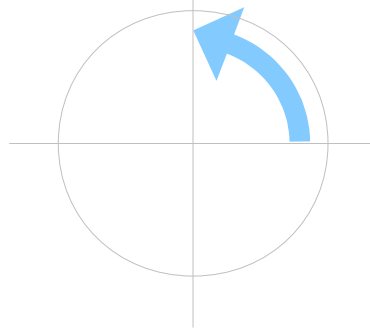
$$\omega_s = 2\pi f_s \text{ (rad/sec)}$$



$$2\pi \text{ (rad)} / T_s \text{ (sec)}$$



$$\frac{\pi}{2} \text{ (rad)} / T_s \text{ (sec)}$$



For the period of T_s
Angular displacement $\frac{\pi}{2}$ (rad)

$$\begin{aligned} \hat{\omega} &= \omega \cdot T_s \text{ (rad)} \\ &= 2\pi f_1 \cdot T_s \text{ (rad)} \\ &= 2\pi \frac{f_s}{4} \cdot T_s \text{ (rad)} \\ &= \frac{\pi}{2} \text{ (rad)} \end{aligned}$$

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A “graphical interpretation” of the DFT and FFT, by Steve Mann