

Background (1A)

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CORDIC Background

1. CORDIC FAQ, G. R. Griffin, www.dspguru.com/info/faqs/cordic2.htm

Complex Multiplication

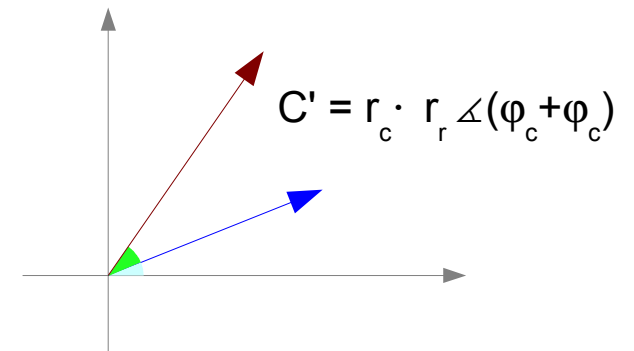
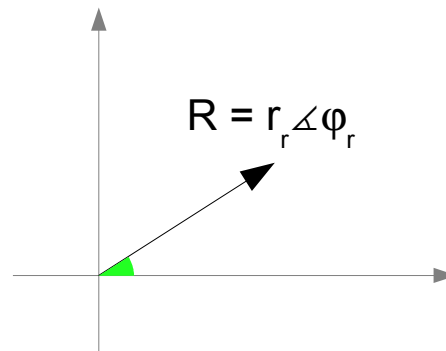
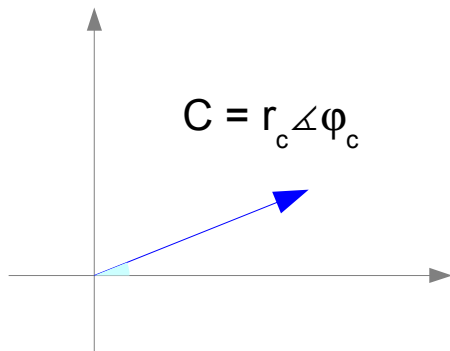
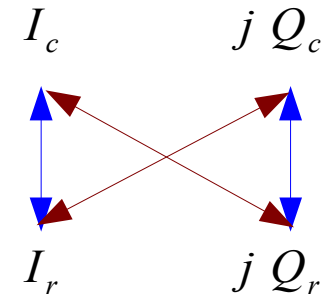
Given Complex Value $C = I_c + j Q_c$

Rotated Complex Value $C' = I_c' + j Q_c'$

Rotation Value $R = I_r + j Q_r$

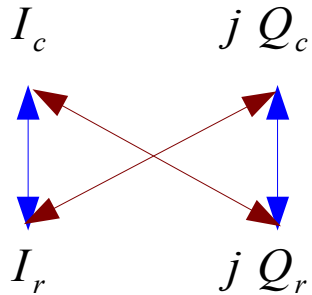
$$C' = C \cdot R$$

$$\begin{aligned} I_c' + j Q_c' &= (I_c + j Q_c) \cdot (I_r + j Q_r) \\ &= (I_c I_r - Q_c Q_r) + j (Q_c I_r + I_c Q_r) \end{aligned}$$



Adding / Subtracting Phase

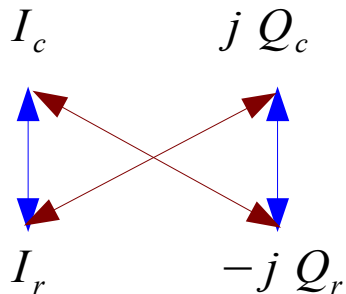
To add R' phase to C



$$C' = C \cdot R$$

$$\begin{aligned} I_c' + j Q_c' &= (I_c + j Q_c) \cdot (I_r + j Q_r) \\ &= (I_c I_r - Q_c Q_r) + j (Q_c I_r + I_c Q_r) \end{aligned}$$

To sub R' phase to C

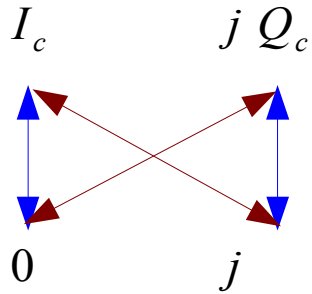


$$C' = C \cdot R^*$$

$$\begin{aligned} I_c' + j Q_c' &= (I_c + j Q_c) \cdot (I_r - j Q_r) \\ &= (I_c I_r + Q_c Q_r) + j (Q_c I_r - I_c Q_r) \end{aligned}$$

Adding / Subtracting 90 Degrees

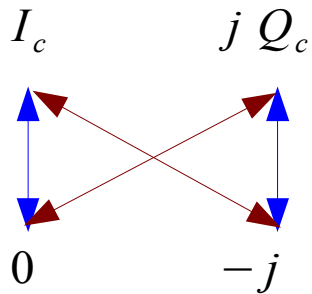
To add R' phase to C



$$C' = C \cdot R$$

$$\begin{aligned} I_c' + j Q_c' &= (I_c + j Q_c) \cdot (0 + j) \\ &= (-Q_c) + j (I_c) \end{aligned}$$

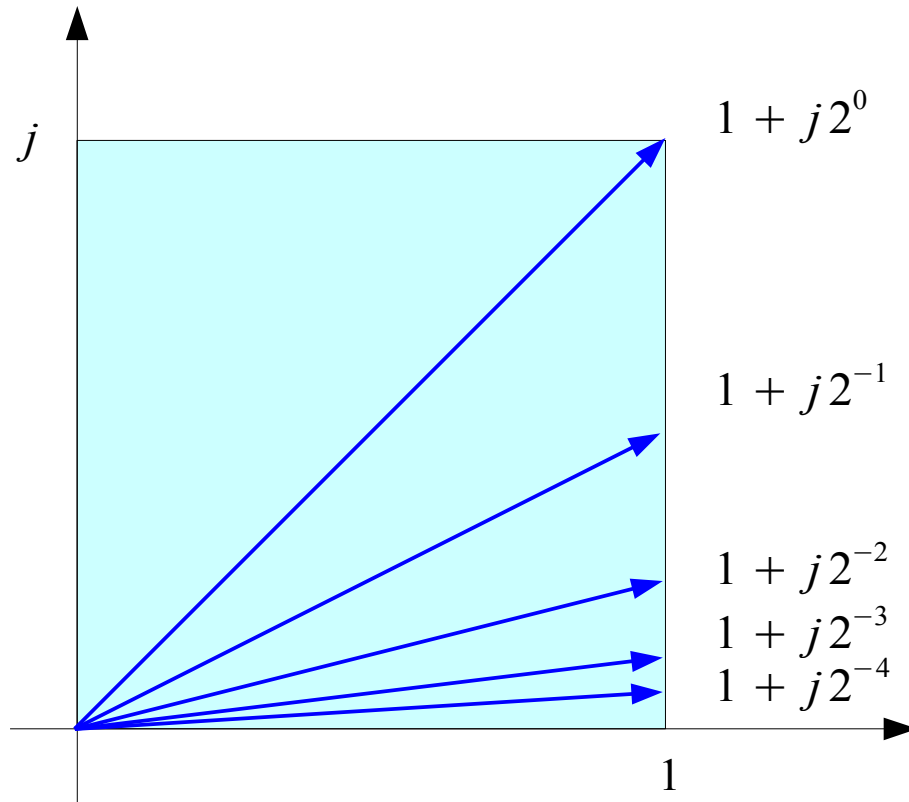
To sub R' phase to C



$$C' = C \cdot R^*$$

$$\begin{aligned} I_c' + j Q_c' &= (I_c + j Q_c) \cdot (0 - j) \\ &= (Q_c) + j (-I_c) \end{aligned}$$

atan(K)



$$\theta_0 = \tan^{-1}(2^0) = 45.00000$$

$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505$$

$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624$$

$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502$$

$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633$$

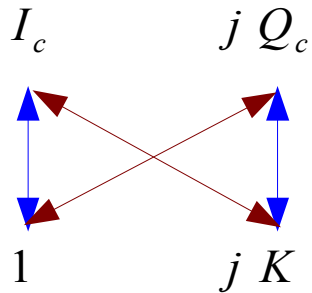
$$\theta_L = \tan^{-1}(2^{-L}) = \tan^{-1}(K)$$

Represent arbitrary angle θ

in terms of $\pm\theta_0, \pm\theta_1, \pm\theta_2, \pm\theta_3, \dots, \pm\theta_L, \dots$ $\left(K = \frac{1}{2^L}, L = 0, 1, 2, \dots\right)$

Adding / Subtracting atan(K)

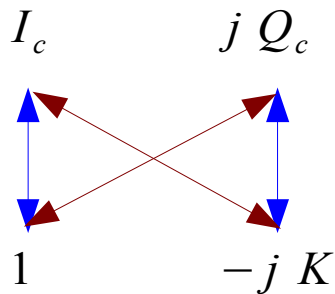
To add R' phase to C



$$C' = C \cdot \boxed{R}$$

$$\begin{aligned} I_c' + j Q_c' &= (I_c + j Q_c) \cdot \boxed{(1 + j K)} \\ &= (I_c - K Q_c) + j (Q_c + K I_c) \\ &= (I_c - 2^{-L} Q_c) + j (Q_c + 2^{-L} I_c) \end{aligned}$$

To sub R' phase to C



$$C' = C \cdot \boxed{R^*}$$


$$\begin{aligned} I_c' + j Q_c' &= (I_c + j Q_c) \cdot \boxed{(1 - j K)} \\ &= (I_c + K Q_c) + j (Q_c - K I_c) \\ &= (I_c + 2^{-L} Q_c) + j (Q_c - 2^{-L} I_c) \end{aligned}$$

$$K = \frac{1}{2^L}, \quad L = 0, 1, 2, \dots$$

Phase and Magnitude of $1 + jK$ (1)

Cumulative Magnitude

L	$K = \frac{1}{2^L}$	$R = 1 + jK$	Phase of R	Magnitude of R	CORDIC Gain
0	1.0	$1 + j1.0$	45°	1.41421356	1.414213562
1	0.5	$1 + j0.5$	26.56505°	1.11803399	1.581138830
2	0.25	$1 + j0.25$	14.03624°	1.03077641	1.629800601
3	0.125	$1 + j0.125$	7.12502°	1.00778222	1.642484066
4	0.0625	$1 + j0.0625$	3.57633°	1.00195122	1.645688916
5	0.03125	$1 + j0.03125$	1.78991°	1.00048816	1.646492279
6	0.015625	$1 + j0.015625$	0.89517°	1.00012206	1.646693254
7	0.007813	$1 + j0.007813$	0.44761°	1.00003052	1.646743507
...

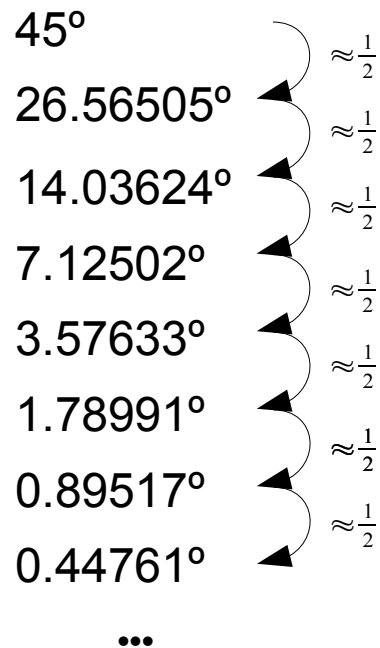

 $\sqrt{1^2 + K^2} > 1.0$


1.647

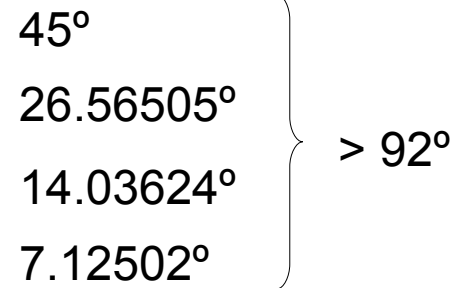
Phase and Magnitude of $1 + jK$ (2)

$K = \frac{1}{2^L}$ \Rightarrow Shift and Add \Rightarrow No multiplier

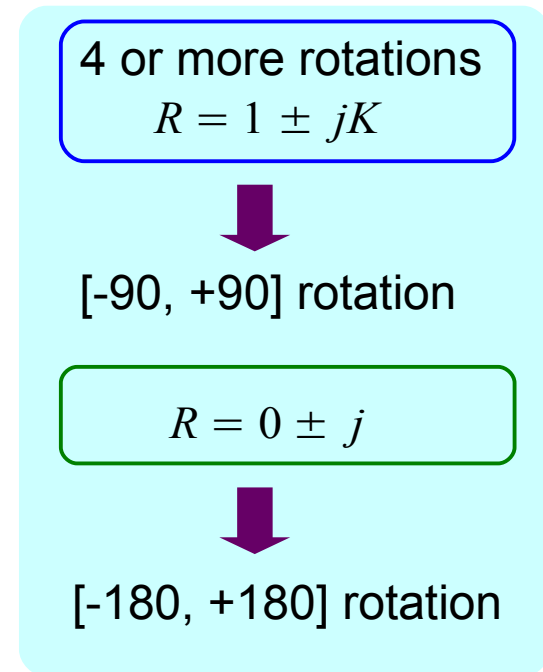
Phase of R



Phase of R



Binary Search



Phase and Magnitude of $1 + jK$ (3)

$$\sqrt{1^2 + K^2} > 1.0$$

Cumulative Magnitude

Magnitude of R

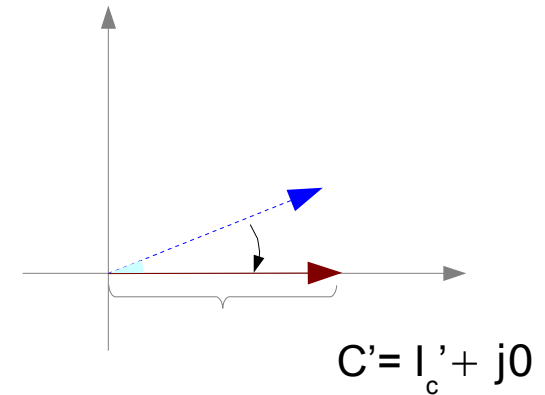
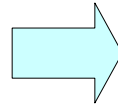
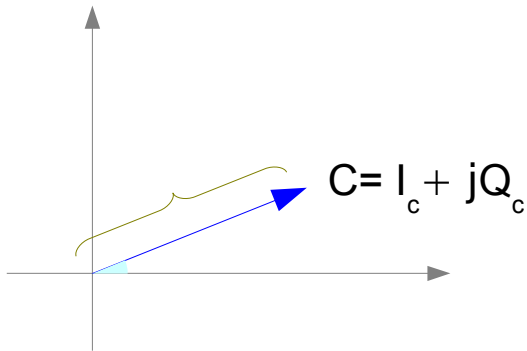
CORDIC Gain

$\sqrt{1^2 + 1^2}$	1.41421356	→	1.414213562
$\sqrt{1^2 + (1/2)^2}$	1.11803399	←	1.581138830
$\sqrt{1^2 + (1/2^2)^2}$	1.03077641	←	1.629800601
$\sqrt{1^2 + (1/2^3)^2}$	1.00778222	←	1.642484066
$\sqrt{1^2 + (1/2^4)^2}$	1.00195122	←	1.645688916
$\sqrt{1^2 + (1/2^5)^2}$	1.00048816	←	1.646492279
$\sqrt{1^2 + (1/2^6)^2}$	1.00012206	←	1.646693254
$\sqrt{1^2 + (1/2^7)^2}$	1.00003052	←	1.646743507
...			...

↓
1.647

The actual CORDIC Gain depends on the number of iterations

Calculating Magnitude



$I'_c \rightarrow$ magnitude

Each iteration, the magnitude is increased by $\sqrt{1^2 + K^2}$

CORDIC Gain (cumulative gain)

$$\simeq 1.647 = 0.607^{-1}$$

*Can't perform gain adjustment
multiplication ← simple shift and add*

Calculating Magnitude (1)

Positive Phase ($Q_c > 0$) → Rotate by -90 degrees



$$\begin{array}{cc}
 I_c & j Q_c \\
 \begin{array}{c} \uparrow \\ \downarrow \\ 0 \end{array} & \begin{array}{c} \uparrow \\ \downarrow \\ -j \end{array} \\
 (Q_c) & + j (-I_c)
 \end{array}$$

Negative Phase ($Q_c < 0$) → Rotate by $+90$ degrees



$$\begin{array}{cc}
 I_c & j Q_c \\
 \begin{array}{c} \uparrow \\ \downarrow \\ 0 \end{array} & \begin{array}{c} \uparrow \\ \downarrow \\ j \end{array} \\
 (-Q_c) & + j (I_c)
 \end{array}$$

Resulting Phase → $[-90, +90]$

Calculating Magnitude (2)

Positive Phase ($Q_c > 0$) \Rightarrow Rotate by $1 - jK$



$$\begin{array}{cc}
 I_c & j Q_c \\
 \uparrow & \uparrow \\
 \downarrow & \downarrow \\
 1 & -j K
 \end{array}$$

$$(I_c + 2^{-L} Q_c) + j (Q_c - 2^{-L} I_c)$$

Negative Phase ($Q_c < 0$) \Rightarrow Rotate by $1 + jK$

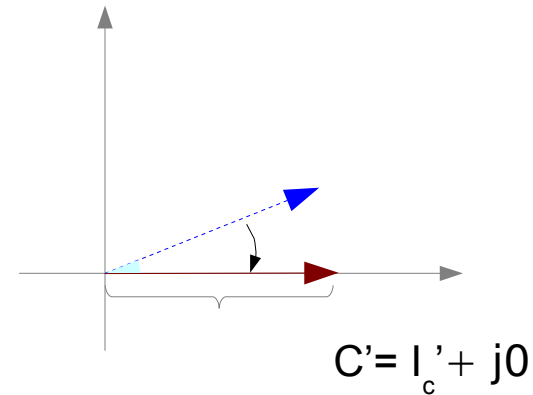
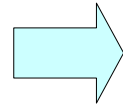
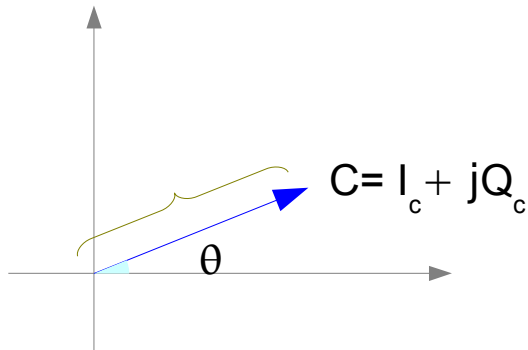


$$\begin{array}{cc}
 I_c & j Q_c \\
 \uparrow & \uparrow \\
 \downarrow & \downarrow \\
 1 & j K
 \end{array}$$

$$(I_c - 2^{-L} Q_c) + j (Q_c + 2^{-L} I_c)$$

After iterations, the result $\Rightarrow I_c + j 0$

Calculating Phase



Phase of R

$$\theta_0 = \tan^{-1}(2^0) = 45^\circ$$

$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505^\circ$$

$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624^\circ$$

$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502^\circ$$

$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633^\circ$$

$$\theta_5 = \tan^{-1}(2^{-5}) = 1.78991^\circ$$

$$\theta_6 = \tan^{-1}(2^{-6}) = 0.89517^\circ$$

$$\theta_7 = \tan^{-1}(2^{-7}) = 0.44761^\circ$$

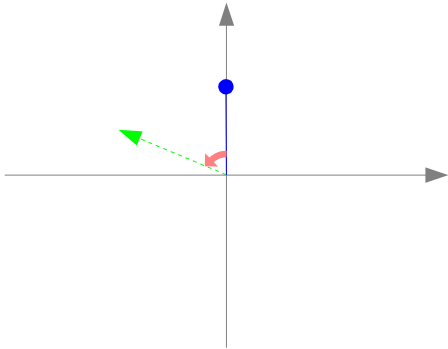
...

$$\theta \pm \theta_0 \pm \theta_1 \pm \theta_2 \pm \theta_3 \dots = 0$$

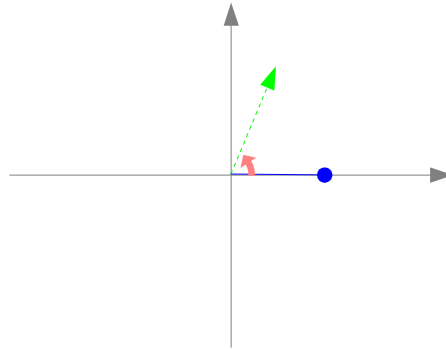
*Accumulate each rotating angles
Then negate the result*

Calculating Sine and Cosine (1)

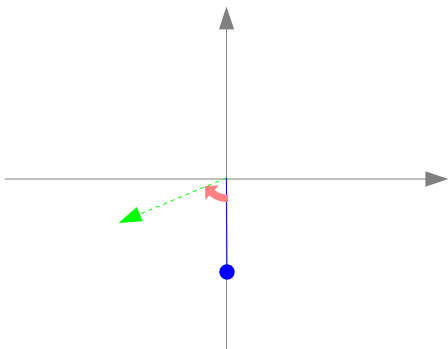
$\Theta > +90$ \Rightarrow starting from $0 + j1$



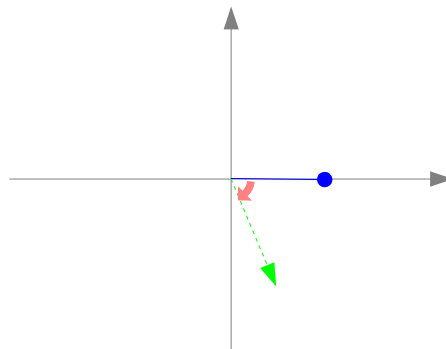
$\Theta < +90$ \Rightarrow starting from $1 + j0$



$\Theta < -90$ \Rightarrow starting from $0 - j1$



$\Theta > -90$ \Rightarrow starting from $1 - j0$



Initialize the accumulate rotation \Rightarrow -90, +90, 0

Calculating Sine and Cosine (2)

In each iteration

\ominus – the accumulated rotation < 0

➔ then *add* the next angle

\ominus – the accumulated rotation > 0

➔ then *subtract* the next angle

The final

I_c ➔ *cosine*

Q_c ➔ *sine*

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
- [4] CORDIC FAQ, www.dspguru.com