DFT Matrix Properties (3B)

- X[1]
- X[2]
- X[3]
- X[4]
- X[5]
- X[6]
- X[7]

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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N=8 DFT

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

N=8 IDFT

$$x[n] = \frac{1}{N} \sum_{k=0}^{7} W_8^{-kn} X[k] \qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} \\ W_8^{0} & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7} \\ W_8^{0} & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^{-8} & W_8^{-10} & W_8^{-12} & W_8^{-14} \\ W_8^{0} & W_8^{-3} & W_8^{-6} & W_8^{-9} & W_8^{-12} & W_8^{-15} & W_8^{-18} & W_8^{-21} \\ W_8^{0} & W_8^{-4} & W_8^{-8} & W_8^{-12} & W_8^{-16} & W_8^{-20} & W_8^{-24} & W_8^{-28} \\ W_8^{0} & W_8^{-5} & W_8^{-10} & W_8^{-15} & W_8^{-20} & W_8^{-25} & W_8^{-30} & W_8^{-35} \\ W_8^{0} & W_8^{-6} & W_8^{-12} & W_8^{-18} & W_8^{-24} & W_8^{-30} & W_8^{-36} & W_8^{-42} \\ W_8^{0} & W_8^{-7} & W_8^{-14} & W_8^{-21} & W_8^{-28} & W_8^{-35} & W_8^{-42} & W_8^{-49} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix}$$

N=8 DFT Matrix (1)

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad\qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix}
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

A

N=8 IDFT Matrix (1)

$$x[n] = \frac{1}{N} \sum_{k=0}^{7} W_8^{-kn} \, X[k] \qquad\qquad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix}
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 7} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 5} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 3} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

B

N=8 DFT & IDFT Matrix (1)

		0	1	2	3	4	5	6	7	
Row 0		$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	DFT
		$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	IDFT
Row 1		$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	DFT
		$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	IDFT
Row 2		$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	DFT
		$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	IDFT
Row 3		$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	DFT
		$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	IDFT

N=8 DFT & IDFT Matrix (2)

		0	1	2	3	4	5	6	7	
Row 4		$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	DFT
		$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	IDFT
Row 5		$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	DFT
		$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	IDFT
Row 6		$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	DFT
		$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	IDFT
Row 7		$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	DFT
		$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	IDFT

Product AB – Diagonal Elements

$C = A \cdot B$

$$[C]_{(i,j)} = [A]_{(row\ i)} \cdot [B]_{(col\ j)}$$

$$C_{(i,i)} = N$$

$$C_{(1,1)} \quad \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix} \bullet \begin{pmatrix} e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix}^T$$

+1 +1 +1 +1 +1 +1 +1 +1 = N

Product AB – Off-Diagonal Elements

$C = A \cdot B$

$$[\boldsymbol{C}]_{(i,j)} = [\boldsymbol{A}]_{(row\ \ i)} \cdot [\boldsymbol{B}]_{(col\ \ j)}$$

$$C_{(i,j)} = 0$$

$C_{(1,2)}$

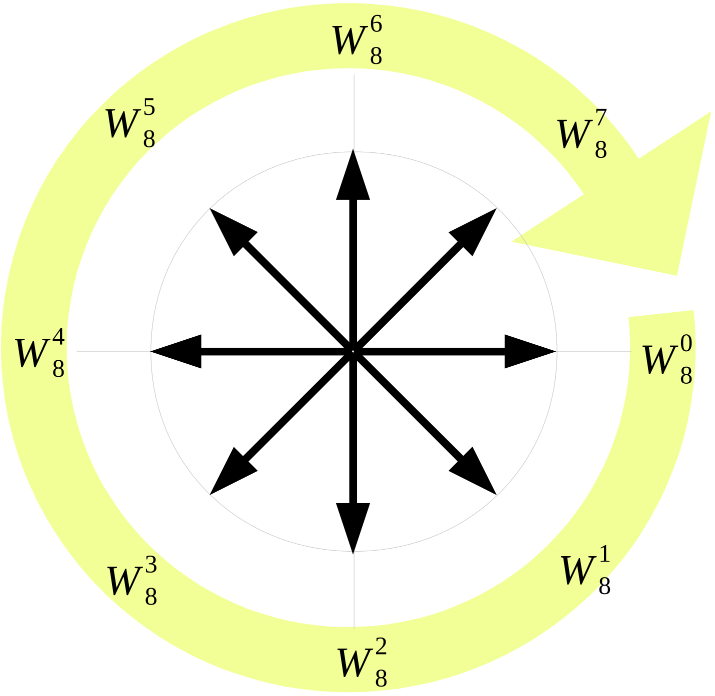
$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix} \bullet$$

$$\begin{pmatrix} e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} \end{pmatrix}^T$$

$$e^{+j\cdot\frac{\pi}{4}\cdot 0} + e^{+j\cdot\frac{\pi}{4}\cdot 1} + e^{+j\cdot\frac{\pi}{4}\cdot 2} + e^{+j\cdot\frac{\pi}{4}\cdot 3} + e^{+j\cdot\frac{\pi}{4}\cdot 4} + e^{+j\cdot\frac{\pi}{4}\cdot 5} + e^{+j\cdot\frac{\pi}{4}\cdot 6} + e^{+j\cdot\frac{\pi}{4}\cdot 7} = 0$$

Root of Unity

$$\sum_{k=0}^{N-1} W_N^k = \sum_{k=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)k} = 0$$



$$z \equiv e^{-j\left(\frac{2\pi}{N}\right)}$$

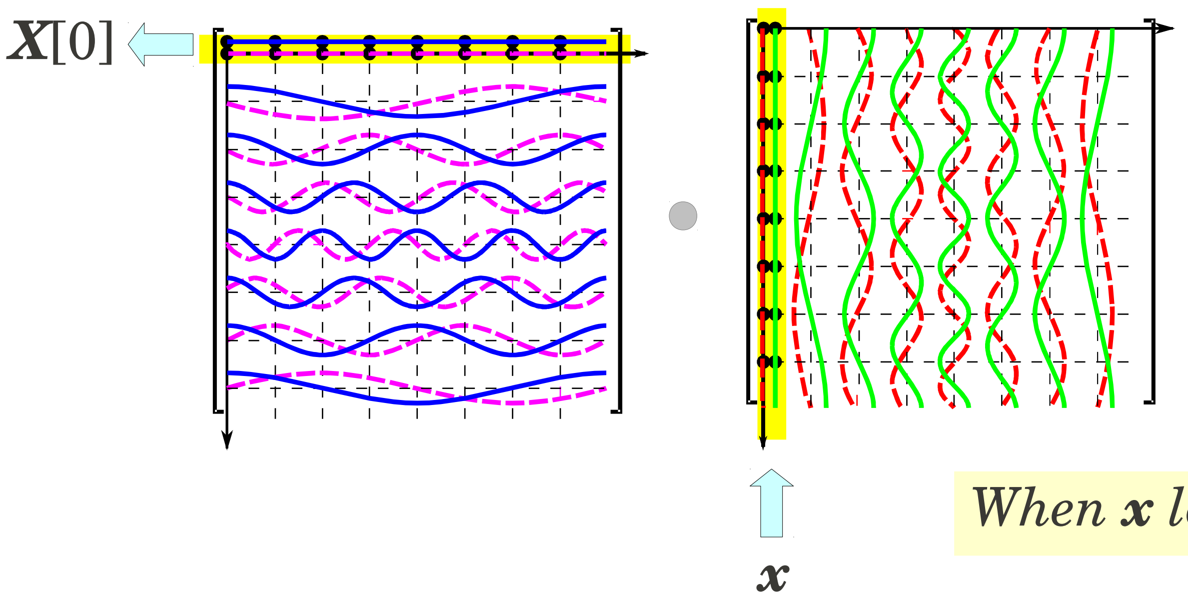
$$z^N = e^{-j\left(\frac{2\pi}{N}\right)N} = 1$$

$$\sum_{k=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)k} = \frac{z^N - 1}{z - 1} = 0$$

$$W_8^0+W_8^1+W_8^2+W_8^3+W_8^4+W_8^5+W_8^6+W_8^7 = 0$$

N=8 DFT : Inner Product X[0]

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$



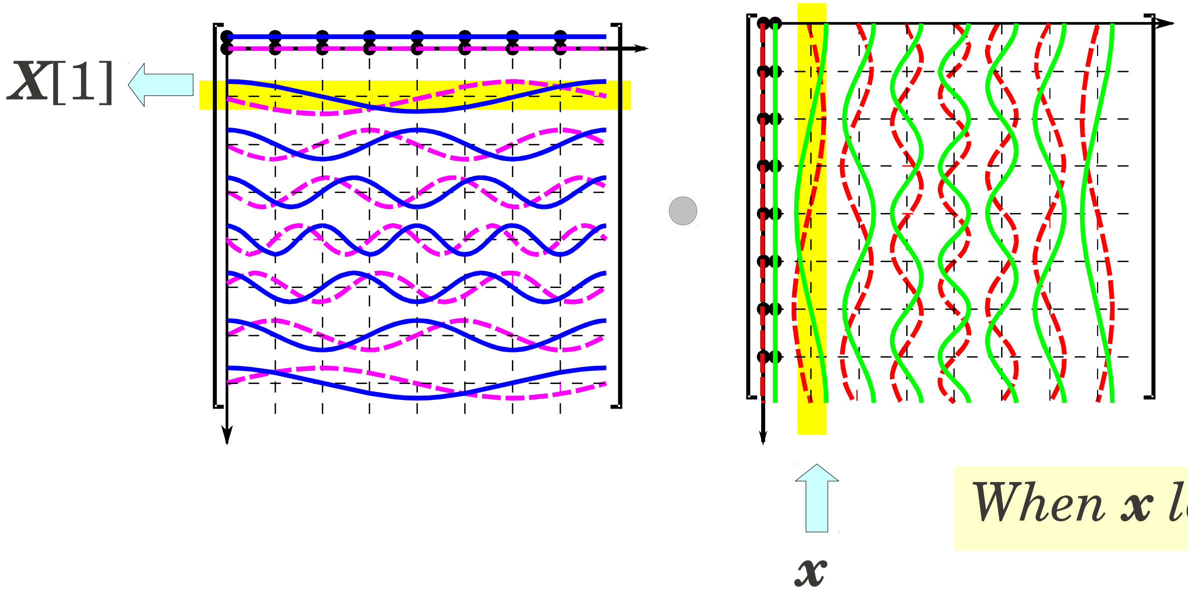
$$\boldsymbol{x} = \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

*When **x** looks like this, **X**[0] is max.*

$$\boldsymbol{X}[0] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

N=8 DFT : Inner Product X[1]

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$



$$\boldsymbol{x} = \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

*When **x** looks like this, **X[1]** is max.*

$$\boldsymbol{X}[1] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

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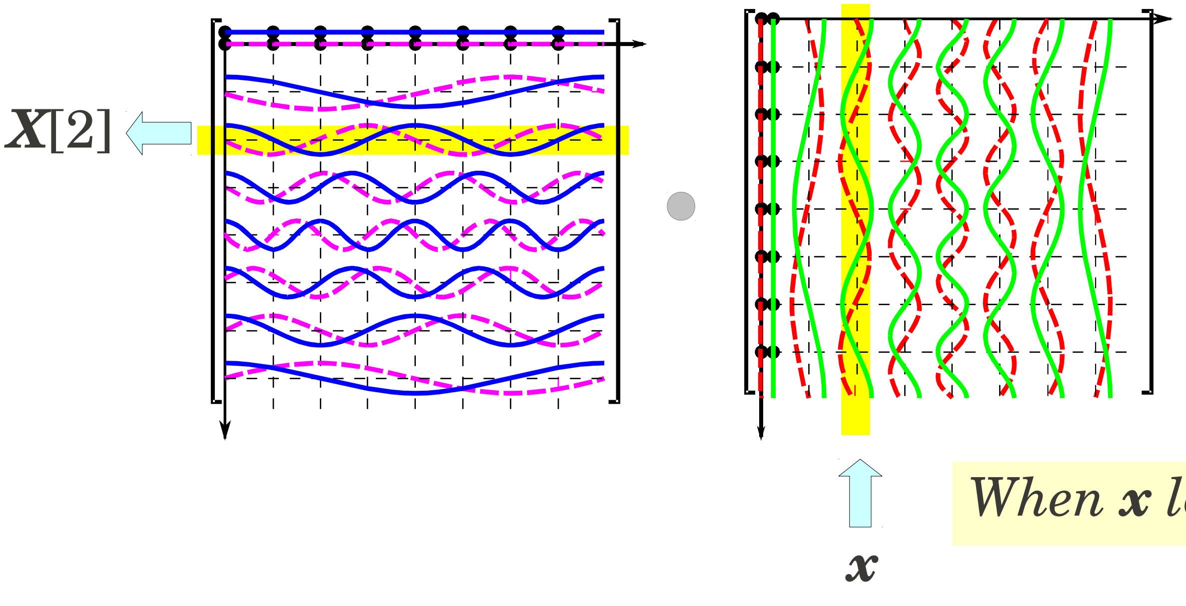
N=8 DFT : Inner Product X[2]

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$



$$\boldsymbol{x} = \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

When $\boldsymbol{x}$ *looks like this,* ***X*****[2]** *is max.*

$$\boldsymbol{X}[2] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

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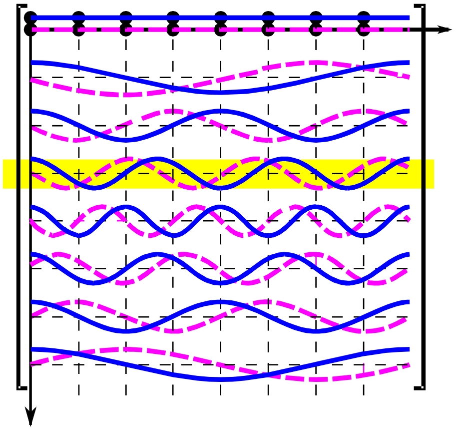
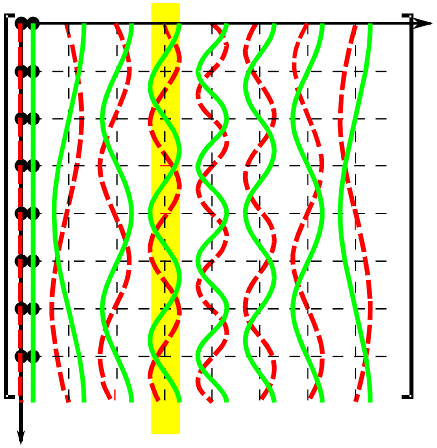
$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

X[3]





$$\boldsymbol{x} = \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

x

*When **x** looks like this, **X**[3] is max.*

$$\boldsymbol{X}[3] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

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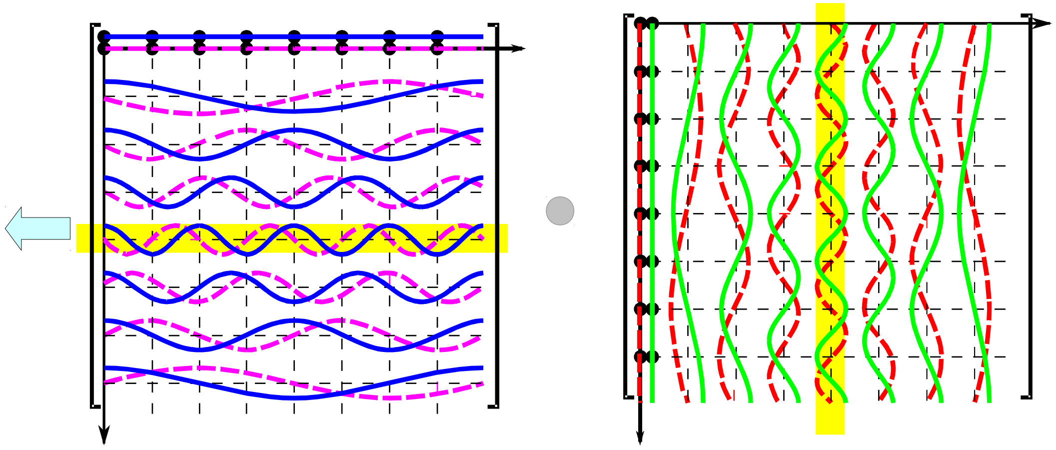
N=8 DFT : Inner Product X[4]

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$\boldsymbol{X}[4]$



$$\boldsymbol{x} = \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

*When **x** looks like this, **X**[4] is max.*

$\boldsymbol{x}$

$$\boldsymbol{X}[4] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

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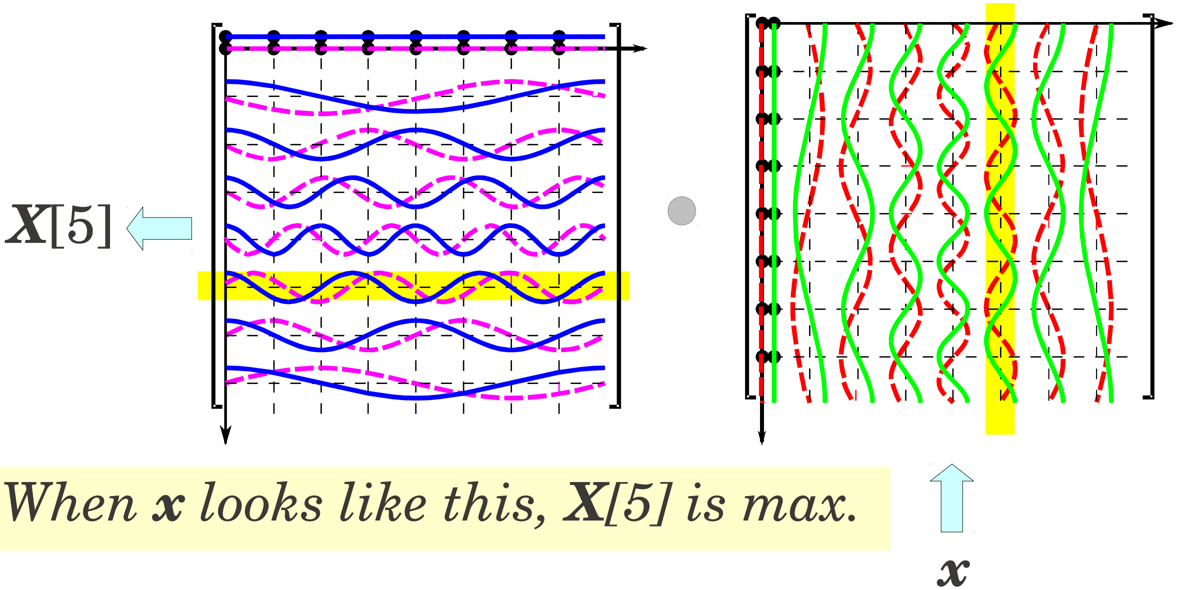
N=8 DFT : Inner Product X[5]

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$



$$\boldsymbol{x} = \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

*When **x** looks like this, **X**[5] is max.*

$$\boldsymbol{X}[5] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

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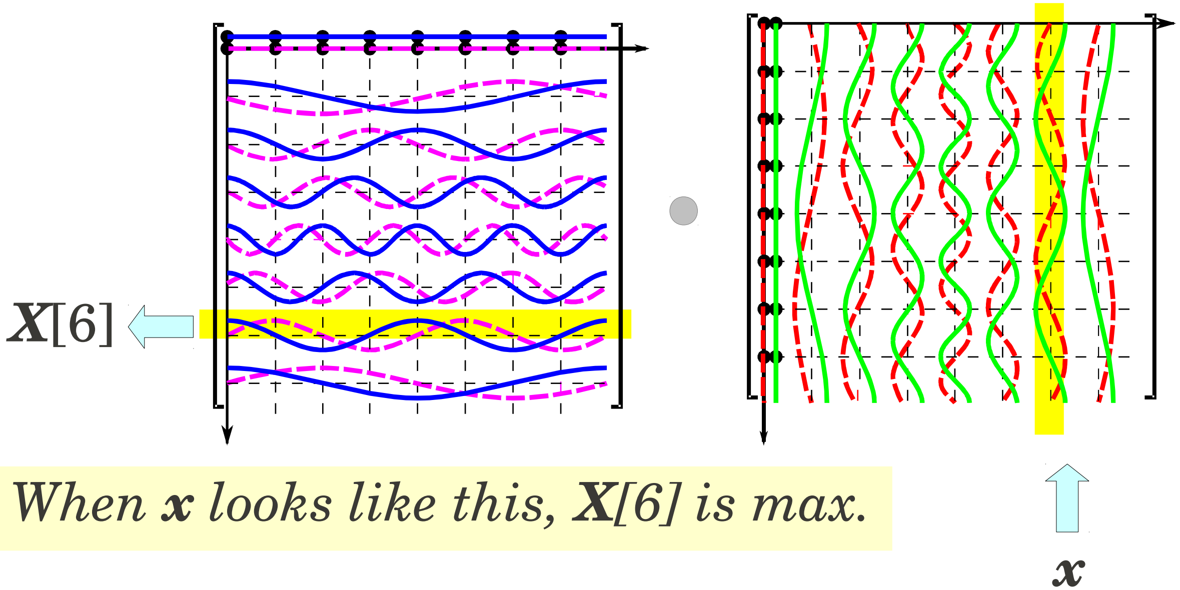
N=8 DFT : Inner Product X[6]

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$



$$\boldsymbol{x} = \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

*When **x** looks like this, **X**[6] is max.*

$$\boldsymbol{X}[6] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

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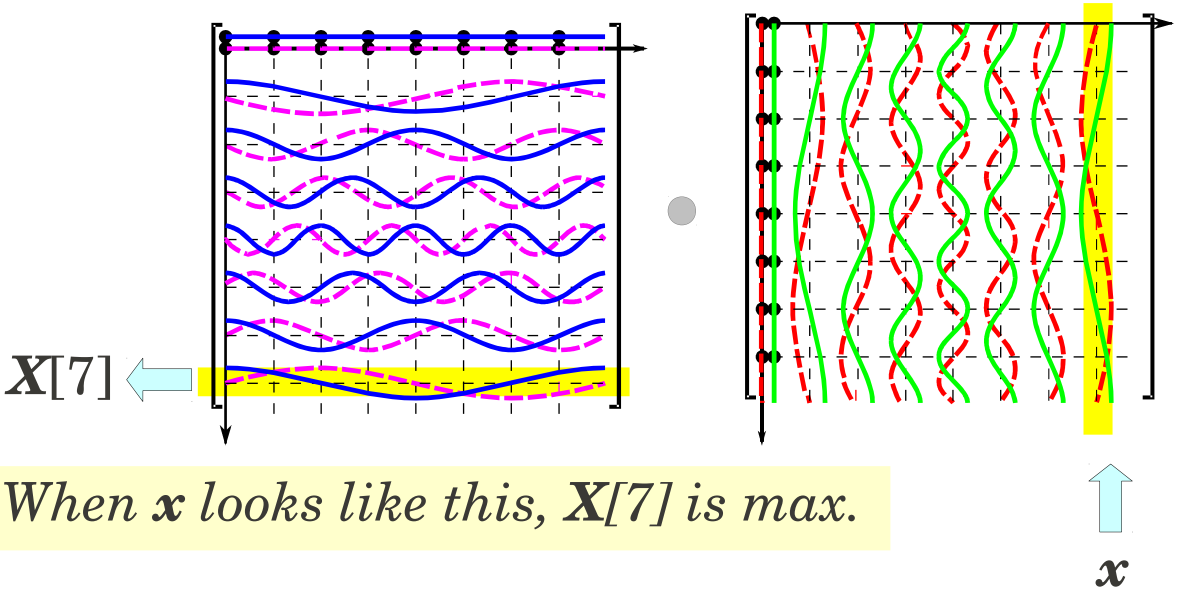
N=8 DFT : Inner Product X[7]

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad\qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$



$$\boldsymbol{x} = \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix}$$

*When **x** looks like this, **X**[7] is max.*

$$\boldsymbol{X}[7] = \begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1} \end{pmatrix} \bullet \begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann