consider a counter example to the statement that we only need a collection of subsets of Ω to form a sigma-field:

$$\Omega = \{1, 2, 3\}$$

\Omega = \{ 1 , 2 , 3 \}

$$\mathcal{F} := \{\emptyset, 1, 2, \Omega\}$$

\mathcal F := \{ \emptyset , 1 , 2 , \Omega \}

$$\{1\} \cup \{2\} = \{1, 2\} \notin \mathcal{F}$$

\{ 1 \} \cup \{ 2 \} = \{ 1, 2 \} \notin \mathcal F

Clearly, $\mathcal{F}$ cannot be a sigma-field.

The point here is that you cannot take any arbitrary collection of subsets of Ω to form a sigma-field, but you need to take a collection of subsets of Ω that satisfies 3 conditions for the set $\mathcal{F}$ to be a sigma-field: For these 3 conditions, see Xiu 2010 p.10, definition of sigma-field.

If you take ALL possible subsets of Ω, then you have a sigma-field, which is the largest sigma-field possible.

The smallest sigma-field is $\mathcal{F} := \{\emptyset, \Omega\}$

\mathcal F := \{ \emptyset , \Omega \}