

DFT Frequency (9A)

- Each Row of the DFT Matrix
-

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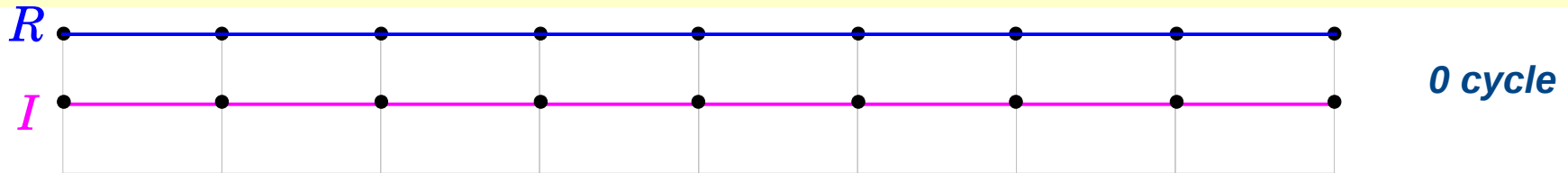
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N=8 DFT : The 1st Row of the DFT Matrix

$$\left(e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \right)$$



$$W_8^{kn} = e^{-j \left(\frac{2\pi}{8} \right) kn} \quad k = 0, \quad n = 0, 1, \dots, 7$$

R \rightarrow sampled values of $\cos(-\omega t) = \cos(\omega t)$

I \rightarrow sampled values of $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$

$$2\pi \cdot \left(\frac{0}{8} \right) \cdot f_s \cdot t$$

X[0] measures how much of the above signal component is present in **x**.



$$T = N\tau$$

Sampling Time

$$\tau$$

Sampling Frequency $f_s = \frac{1}{\tau}$

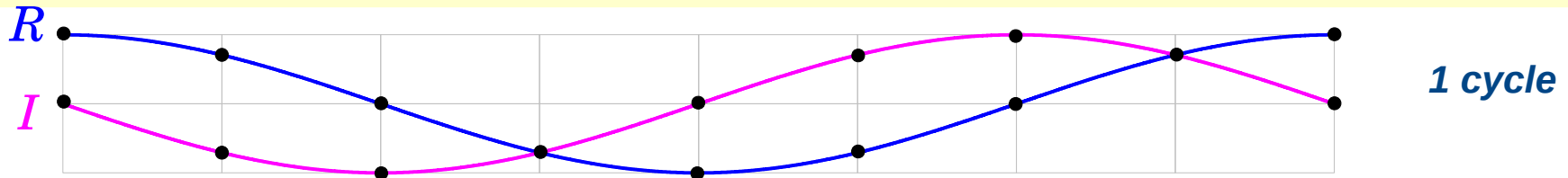
Sequence Time Length

$$T = N\tau$$

Zero Frequency

N=8 DFT : The 2nd Row of the DFT Matrix

$$\left(e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 1} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 3} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 5} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 7} \right)$$



$$W_8^{kn} = e^{-j\frac{2\pi}{8}kn} \quad k = 1, \quad n = 0, 1, \dots, 7$$

R \rightarrow sampled values of $\cos(-\omega t) = \cos(\omega t)$

I \rightarrow sampled values of $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$

$$2\pi \cdot \left(\frac{1}{8}\right) \cdot f_s \cdot t$$

X[1] measures how much of the above signal component is present in **x**.



Sampling Time τ

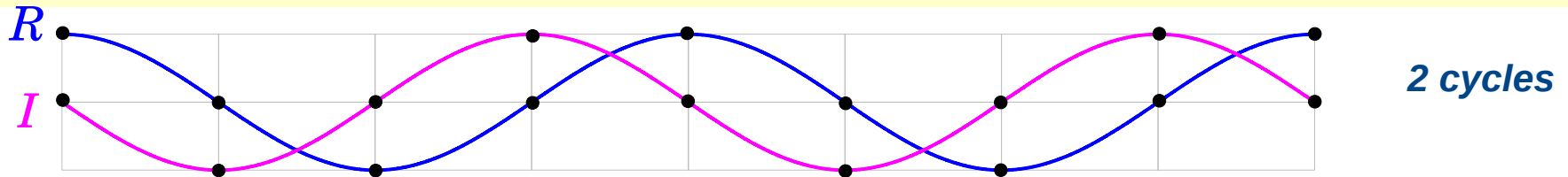
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

1st Harmonic Freq $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

N=8 DFT : The 3rd Row of the DFT Matrix

$$\left(e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 6} \right)$$



$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \quad k = 2, \quad n = 0, 1, \dots, 7$$

R \rightarrow sampled values of $\cos(-\omega t) = \cos(\omega t)$

I \rightarrow sampled values of $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$

$$2\pi \cdot \left(\frac{2}{8}\right) \cdot f_s \cdot t$$

X[2] measures how much of the above signal component is present in **x**.



$$T = N\tau$$

Sampling Time τ

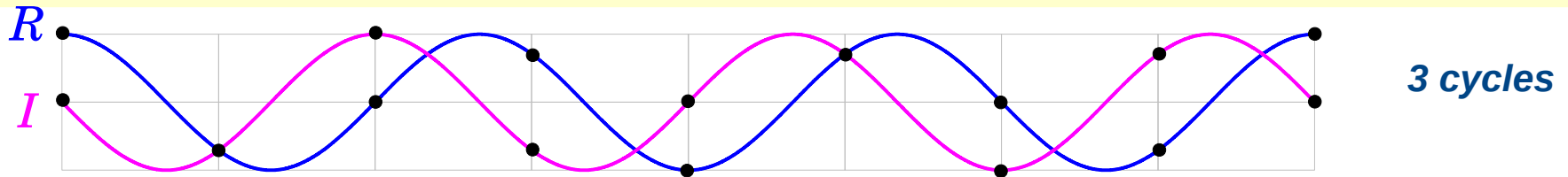
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

2nd Harmonic Freq $f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2f_s}{N}$

N=8 DFT : The 4th Row of the DFT Matrix

$$\left(e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 3} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 1} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 7} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 5} \right)$$



$$W_8^{kn} = e^{-j\frac{2\pi}{8}kn} \quad k = 3, \quad n = 0, 1, \dots, 7$$

R \rightarrow sampled values of $\cos(-\omega t) = \cos(\omega t)$

I \rightarrow sampled values of $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$

$$2\pi \cdot \left(\frac{3}{8}\right) \cdot f_s \cdot t$$

X[3] measures how much of the above signal component is present in **x**.



$$T = N\tau$$

Sampling Time τ

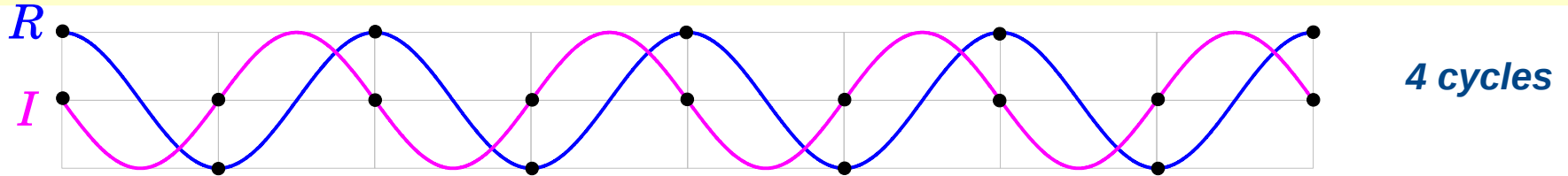
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

3rd Harmonic Freq $f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3f_s}{N}$

N=8 DFT : The 5th Row of the DFT Matrix

$$\left(e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 4} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 4} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 4} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 4} \right)$$



$$W_8^{kn} = e^{-j \left(\frac{2\pi}{8} \right) kn} \quad k = 4, \quad n = 0, 1, \dots, 7$$

R \rightarrow sampled values of $\cos(-\omega t) = \cos(\omega t)$

I \rightarrow sampled values of $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$

$$2\pi \cdot \left(\frac{4}{8} \right) \cdot f_s \cdot t$$

X[4] measures how much of the above signal component is present in **x**.



$$T = N\tau$$

Sampling Time τ

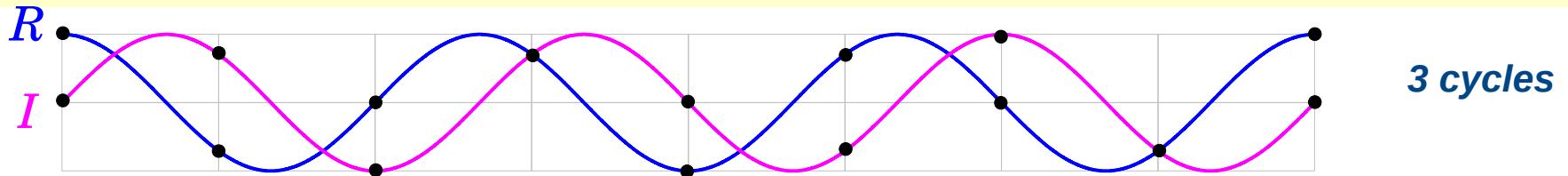
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

4th Harmonic Freq $f_4 = \frac{4}{T} = \frac{4}{N\tau} = \frac{4f_s}{N}$

N=8 DFT : The 6th Row of the DFT Matrix

$$\left(e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 5} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 7} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 1} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 3} \right)$$



$$W_8^{kn} = e^{-j\frac{2\pi}{8}kn} \quad k = 5, \quad n = 0, 1, \dots, 7$$

R \rightarrow sampled values of $\cos(\omega't) = \cos(-(-\omega)t)$

I \rightarrow sampled values of $\sin(\omega't) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$

$$2\pi \cdot \left(\frac{-3}{8}\right) \cdot f_s \cdot t$$

X[5] measures how much of the above signal component is present in **x**.



$$T = N\tau$$

Sampling Time τ

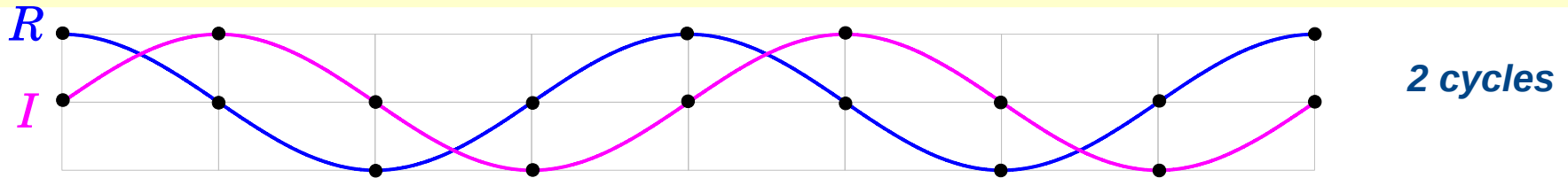
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

3rd Harmonic Freq $f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3f_s}{N}$

N=8 DFT : The 7th Row of the DFT Matrix

$$\left(e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 6} \quad e^{-j \cdot \frac{\pi}{4} \cdot 4} \quad e^{-j \cdot \frac{\pi}{4} \cdot 2} \quad e^{-j \cdot \frac{\pi}{4} \cdot 0} \quad e^{-j \cdot \frac{\pi}{4} \cdot 6} \quad e^{-j \cdot \frac{\pi}{4} \cdot 4} \quad e^{-j \cdot \frac{\pi}{4} \cdot 2} \right)$$



$$W_8^{kn} = e^{-j \left(\frac{2\pi}{8} \right) kn} \quad k = 2, \quad n = 0, 1, \dots, 7$$

R \rightarrow sampled values of $\cos(\omega' t) = \cos(-(-\omega)t)$

I \rightarrow sampled values of $\sin(\omega' t) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$

$$2\pi \cdot \left(\frac{-2}{8} \right) \cdot f_s \cdot t$$

X[6] measures how much of the above signal component is present in **x**.



Sampling Time τ

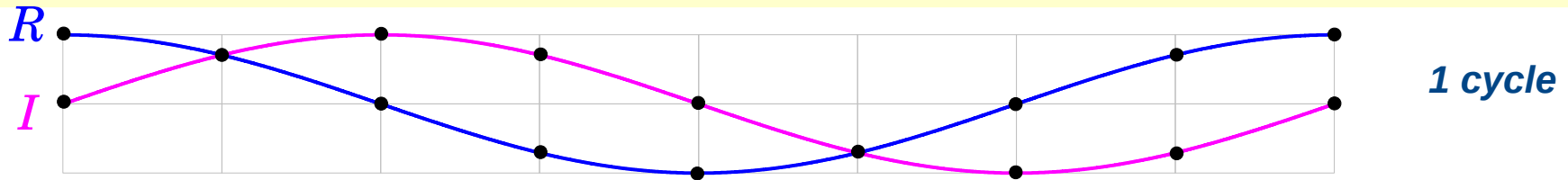
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

2nd Harmonic Freq $f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2f_s}{N}$

N=8 DFT : The 8th Row of the DFT Matrix

$$\left(e^{-j\frac{\pi}{4}\cdot 0} \quad e^{-j\frac{\pi}{4}\cdot 7} \quad e^{-j\frac{\pi}{4}\cdot 6} \quad e^{-j\frac{\pi}{4}\cdot 5} \quad e^{-j\frac{\pi}{4}\cdot 4} \quad e^{-j\frac{\pi}{4}\cdot 3} \quad e^{-j\frac{\pi}{4}\cdot 2} \quad e^{-j\frac{\pi}{4}\cdot 1} \right)$$



$$W_8^{kn} = e^{-j\frac{2\pi}{8}kn} \quad k = 7, \quad n = 0, 1, \dots, 7$$

R \rightarrow sampled values of $\cos(\omega't) = \cos(-(-\omega)t)$

$$-\omega t = -2\pi f t$$

I \rightarrow sampled values of $\sin(\omega't) = \sin(-(-\omega)t)$

$$2\pi \cdot \left(\frac{-1}{8}\right) \cdot f_s \cdot t$$

X[7] measures how much of the above signal component is present in **x**.



$$T = N\tau$$

Sampling Time

$$\tau$$

Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length

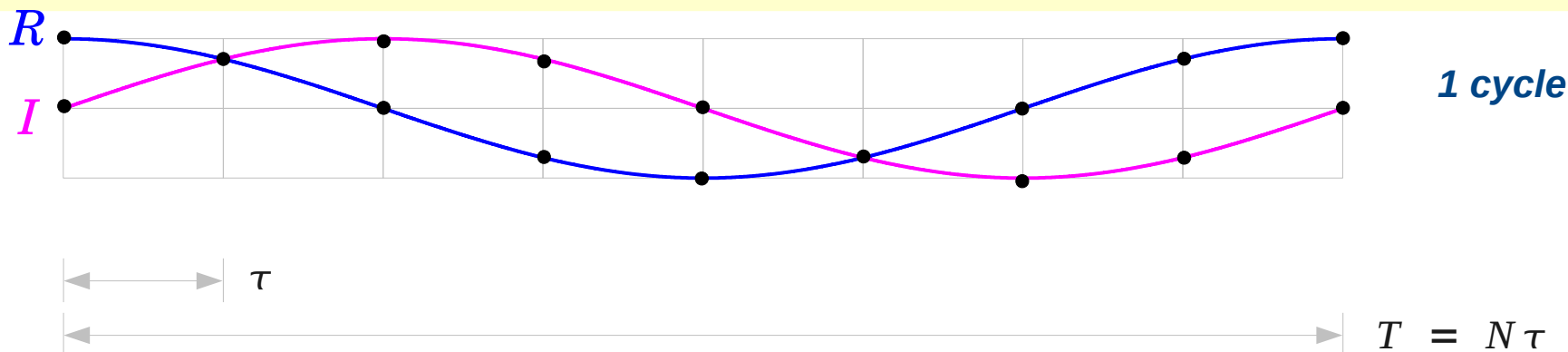
$$T = N\tau$$

1st Harmonic Freq

$$f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$$

Fundamental Frequency

$$\left(e^{+j\frac{\pi}{4}\cdot 0} \quad e^{+j\frac{\pi}{4}\cdot 1} \quad e^{+j\frac{\pi}{4}\cdot 2} \quad e^{+j\frac{\pi}{4}\cdot 3} \quad e^{+j\frac{\pi}{4}\cdot 4} \quad e^{+j\frac{\pi}{4}\cdot 5} \quad e^{+j\frac{\pi}{4}\cdot 6} \quad e^{+j\frac{\pi}{4}\cdot 7} \right)$$



Sampling Time

τ

Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length

$T = N\tau$

1st Harmonic Freq $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

$f_2 = 2 \cdot f_1$

$f_3 = 3 \cdot f_1$

...

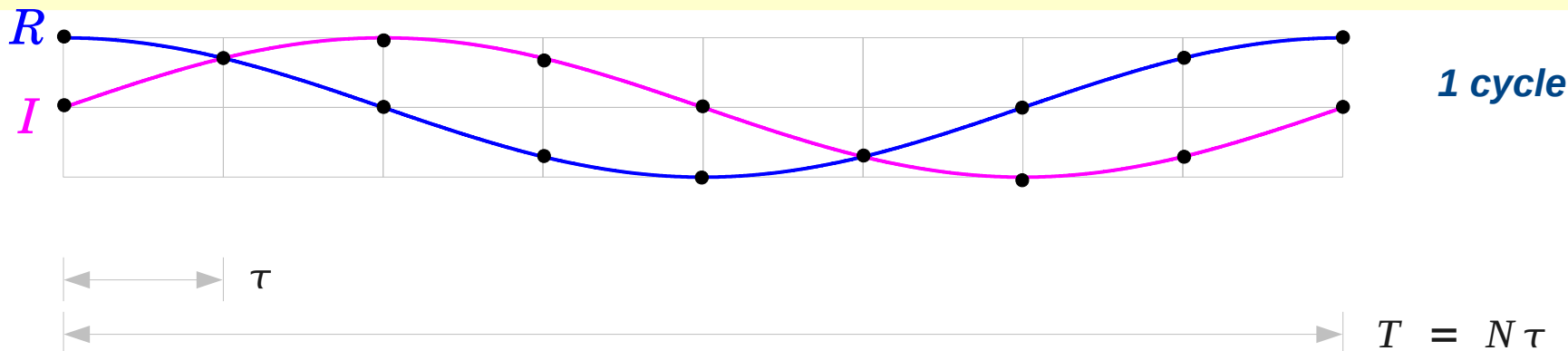
$f_{N-1} = (N-1) \cdot f_1$

Fundamental Frequency f_0

The Lowest Frequency in a harmonic series. $f_0 = f_1 = \frac{f_s}{N}$

Normalized Frequency

$$\left(e^{+j\frac{\pi}{4}\cdot 0} \quad e^{+j\frac{\pi}{4}\cdot 1} \quad e^{+j\frac{\pi}{4}\cdot 2} \quad e^{+j\frac{\pi}{4}\cdot 3} \quad e^{+j\frac{\pi}{4}\cdot 4} \quad e^{+j\frac{\pi}{4}\cdot 5} \quad e^{+j\frac{\pi}{4}\cdot 6} \quad e^{+j\frac{\pi}{4}\cdot 7} \right)$$



Sampling Time τ

Sampling Frequency $f_s = \frac{1}{\tau}$ (samples per second)

Sequence Time Length $T = N\tau$

1st Harmonic Freq $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

$$f_1 = 1 \cdot f_1$$

$$1/N$$

$$f_2 = 2 \cdot f_1$$

$$2/N$$

$$f_3 = 3 \cdot f_1$$

$$3/N$$

...

$$f_{N-1} = (N-1) \cdot f_1$$

$$(N-1)/N$$

$$n = 0, 1, 2, \dots, N-1$$

Normalized Frequency (cycles per sample)

$$f_n = \frac{n \cdot f_s}{N}$$

$$\frac{f_n}{f_s} = \frac{n}{N}$$

N=8 DFT : DFT Matrix in + or - Frequencies

$$\omega_0 = 2\pi \cdot \frac{f_s}{N}$$

0th row:	<i>samples of</i>	$\cos(0\omega_0)t + j \cdot \sin(0\omega_0)t$	(0 cycle)
1th row:	<i>samples of</i>	$\cos(-1\omega_0)t + j \cdot \sin(-1\omega_0)t$	(1 cycle)
2th row:	<i>samples of</i>	$\cos(-2\omega_0)t + j \cdot \sin(-2\omega_0)t$	(2 cycles)
3th row:	<i>samples of</i>	$\cos(-3\omega_0)t + j \cdot \sin(-3\omega_0)t$	(3 cycles)
4th row:	<i>samples of</i>	$\cos(-4\omega_0)t + j \cdot \sin(-4\omega_0)t$	(4 cycles)
5th row:	<i>samples of</i>	$\cos(-5\omega_0)t + j \cdot \sin(-5\omega_0)t$	(5 cycles)
6th row:	<i>samples of</i>	$\cos(-6\omega_0)t + j \cdot \sin(-6\omega_0)t$	(6 cycles)
7th row:	<i>samples of</i>	$\cos(-7\omega_0)t + j \cdot \sin(-7\omega_0)t$	(7 cycles)

==

0th row:	<i>samples of</i>	$\cos(0\omega_0)t + j \cdot \sin(0\omega_0)t$	(0 cycle)
1th row:	<i>samples of</i>	$\cos(+7\omega_0)t + j \cdot \sin(+7\omega_0)t$	(7 cycles)
2th row:	<i>samples of</i>	$\cos(+6\omega_0)t + j \cdot \sin(+6\omega_0)t$	(6 cycles)
3th row:	<i>samples of</i>	$\cos(+5\omega_0)t + j \cdot \sin(+5\omega_0)t$	(5 cycles)
4th row:	<i>samples of</i>	$\cos(+4\omega_0)t + j \cdot \sin(+4\omega_0)t$	(4 cycles)
5th row:	<i>samples of</i>	$\cos(+3\omega_0)t + j \cdot \sin(+3\omega_0)t$	(3 cycles)
6th row:	<i>samples of</i>	$\cos(+2\omega_0)t + j \cdot \sin(+2\omega_0)t$	(2 cycles)
7th row:	<i>samples of</i>	$\cos(+1\omega_0)t + j \cdot \sin(+1\omega_0)t$	(1 cycles)

N=8 DFT : DFT Matrix in Both Frequencies

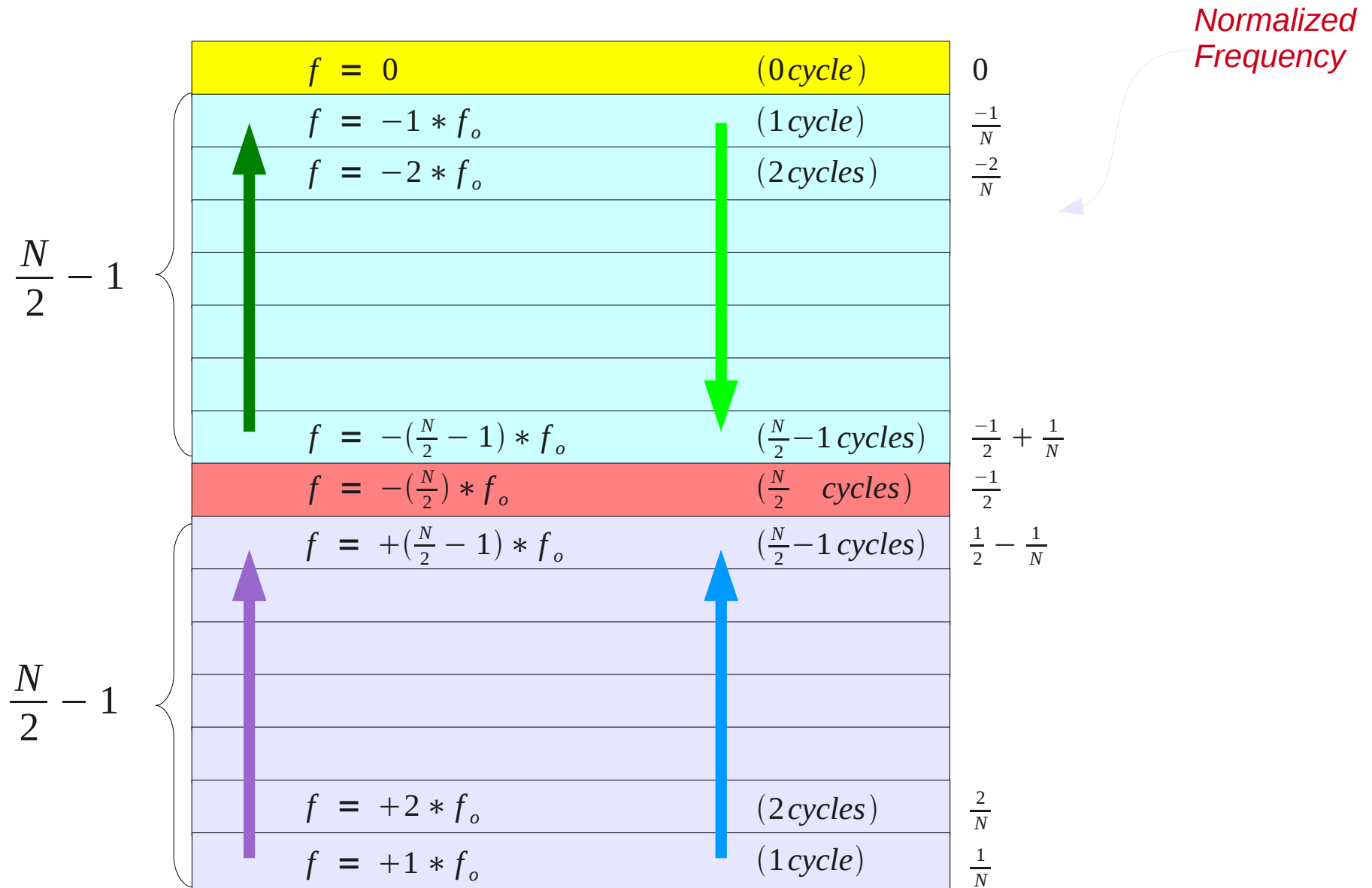
$$\omega_0 = 2\pi \cdot \frac{f_s}{N}$$

0th row:	<i>samples of</i>	$\cos(0\omega_0)t + j \cdot \sin(0\omega_0)t$	(0 cycle)
1th row:	<i>samples of</i>	$\cos(-1\omega_0)t + j \cdot \sin(-1\omega_0)t$	(1 cycle)
2th row:	<i>samples of</i>	$\cos(-2\omega_0)t + j \cdot \sin(-2\omega_0)t$	(2 cycles)
3th row:	<i>samples of</i>	$\cos(-3\omega_0)t + j \cdot \sin(-3\omega_0)t$	(3 cycles)
4th row:	<i>samples of</i>	$\cos(-4\omega_0)t + j \cdot \sin(-4\omega_0)t$	(4 cycles)
5th row:	<i>samples of</i>	$\cos(-5\omega_0)t + j \cdot \sin(-5\omega_0)t$	(5 cycles)
6th row:	<i>samples of</i>	$\cos(-6\omega_0)t + j \cdot \sin(-6\omega_0)t$	(6 cycles)
7th row:	<i>samples of</i>	$\cos(-7\omega_0)t + j \cdot \sin(-7\omega_0)t$	(7 cycles)

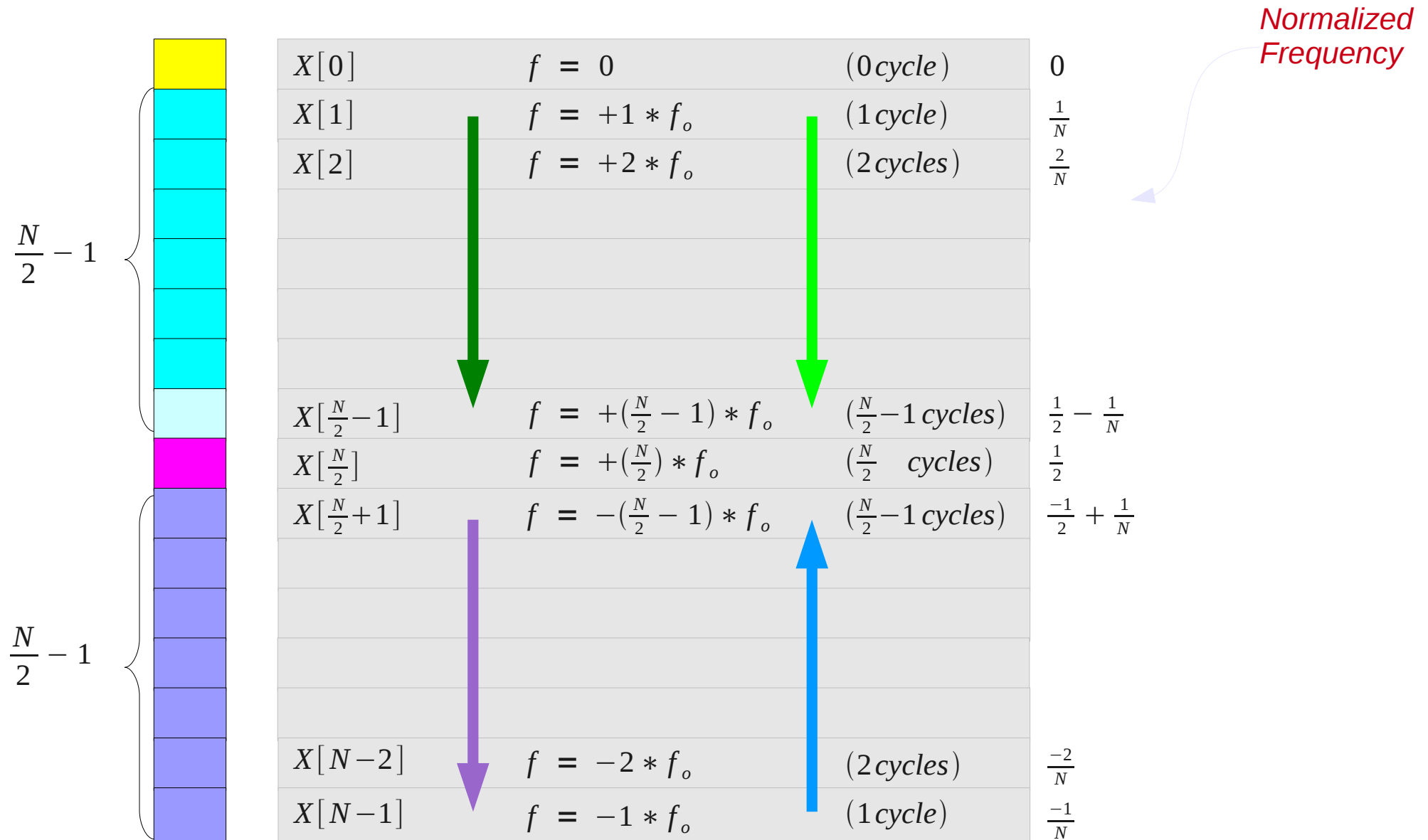
==

0th row:	<i>samples of</i>	$\cos(0\omega_0)t + j \cdot \sin(0\omega_0)t$	(0 cycle)
1th row:	<i>samples of</i>	$\cos(-1\omega_0)t + j \cdot \sin(-1\omega_0)t$	(1 cycle)
2th row:	<i>samples of</i>	$\cos(-2\omega_0)t + j \cdot \sin(-2\omega_0)t$	(2 cycles)
3th row:	<i>samples of</i>	$\cos(-3\omega_0)t + j \cdot \sin(-3\omega_0)t$	(3 cycles)
4th row:	<i>samples of</i>	$\cos(-4\omega_0)t + j \cdot \sin(-4\omega_0)t$	(4 cycles)
5th row:	<i>samples of</i>	$\cos(+3\omega_0)t + j \cdot \sin(+3\omega_0)t$	(3 cycles)
6th row:	<i>samples of</i>	$\cos(+2\omega_0)t + j \cdot \sin(+2\omega_0)t$	(2 cycles)
7th row:	<i>samples of</i>	$\cos(+1\omega_0)t + j \cdot \sin(+1\omega_0)t$	(1 cycles)

Frequency View of a DFT Matrix



Frequency View of a $X[i]$ Vector



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A “graphical interpretation” of the DFT and FFT, by Steve Mann