DFT Analysis (9A)

- Each Row of the DFT Matrix
-

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6/7/10

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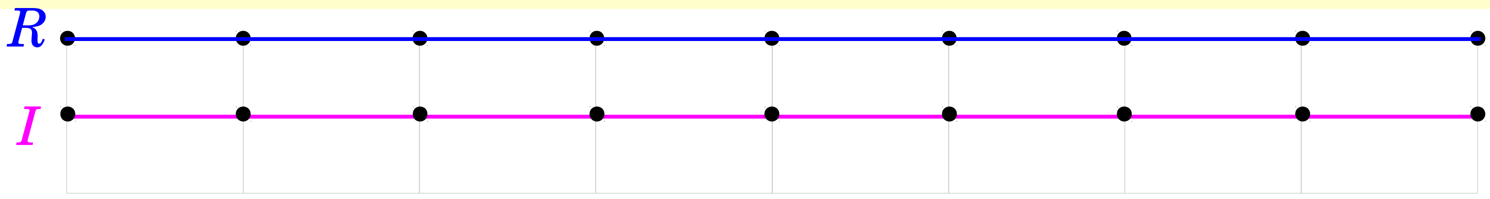
N=8 DFT : The 1st Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \end{pmatrix}$$



0 cycle

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 0, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot(\tfrac{0}{8})\cdot f_s\cdot t$$

$\boldsymbol{X}[0]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

$T = N\tau$

Sampling Time τ

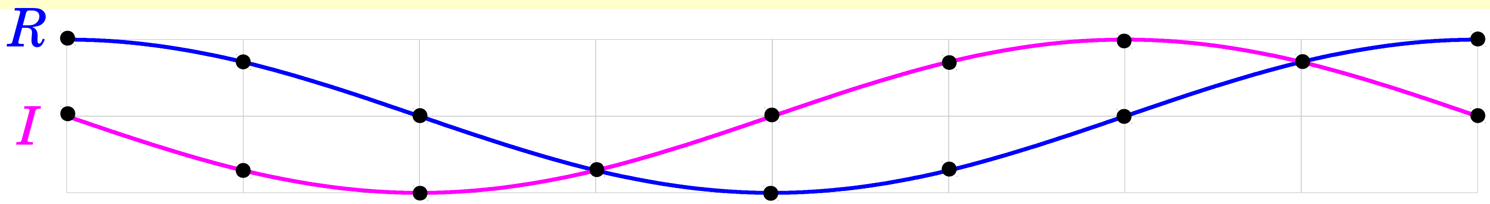
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

Zero Frequency

N=8 DFT : The 2nd Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix}$$



$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 1, \quad n = 0, 1, \ldots, 7$$

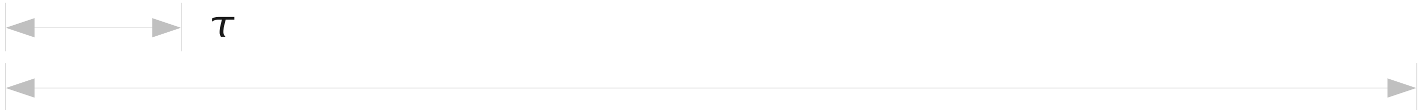
R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$

$$2\pi\cdot\left(\tfrac{1}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[1]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.



$$T = N\tau$$

Sampling Time τ

Sampling Frequency $f_s = \dfrac{1}{\tau}$

Sequence Time Length $T = N\tau$

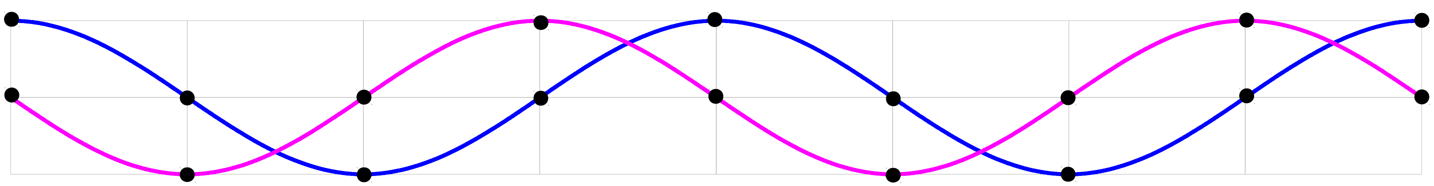
1st Harmonic Freq $f_1 = \dfrac{1}{T} = \dfrac{1}{N\tau} = \dfrac{f_s}{N}$

N=8 DFT : The 3rd Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \end{pmatrix}$$

R

I



2 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 2, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

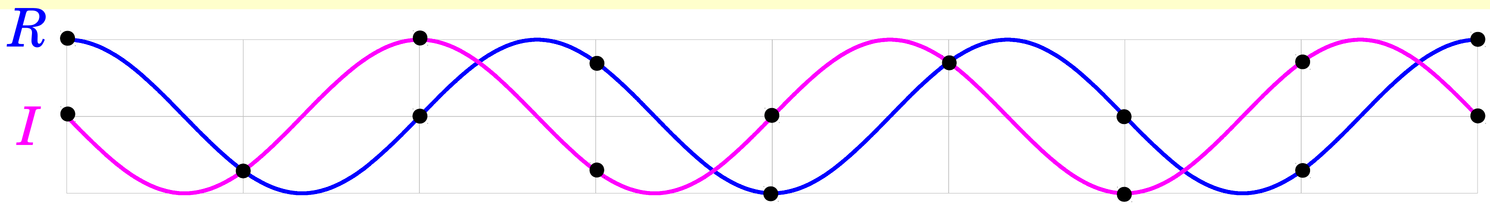
$$\omega t = 2\pi f t$$
$$2\pi\cdot\left(\tfrac{2}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[2]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

Sampling Time	τ	*Sampling Frequency*	$f_s = \frac{1}{\tau}$
Sequence Time Length	$T = N\tau$	*2nd Harmonic Freq*	$f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2 f_s}{N}$

N=8 DFT : The 4th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \end{pmatrix}$$



3 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 3, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot\left(\tfrac{3}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[3]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

τ

$T = N\tau$

Sampling Time $\quad \tau$

Sampling Frequency $\quad f_s = \frac{1}{\tau}$

Sequence Time Length $\quad T = N\tau$

3rd Harmonic Freq $\quad f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3 f_s}{N}$

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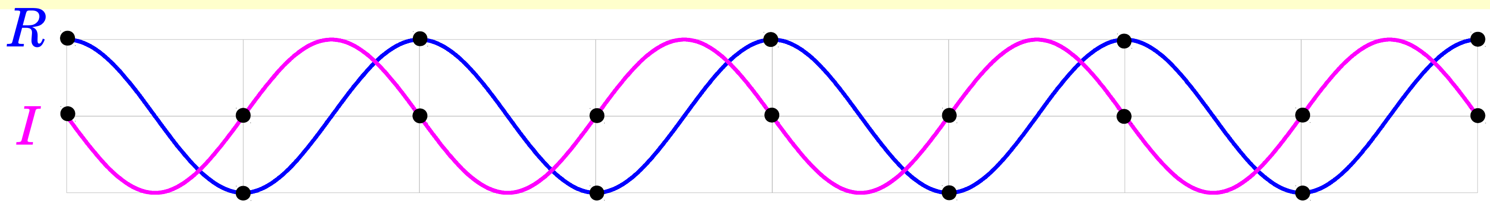
N=8 DFT : The 5th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \end{pmatrix}$$



4 cycles

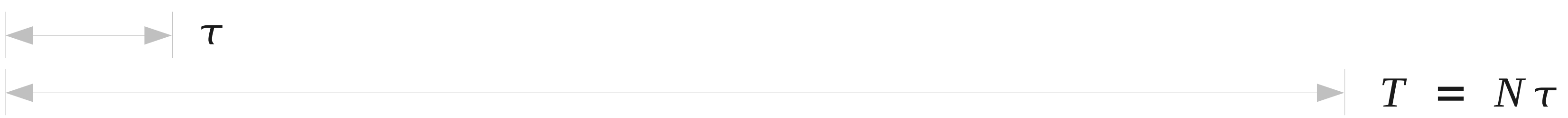
$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 4, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot\left(\tfrac{4}{8}\right)\cdot f_s\cdot t$$

$X[4]$ *measures how much of the above signal component is present in* x.



Sampling Time τ

Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

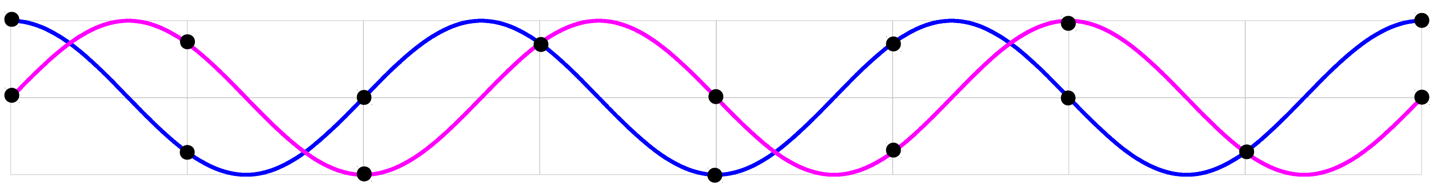
4th Harmonic Freq $f_4 = \frac{4}{T} = \frac{4}{N\tau} = \frac{4 f_s}{N}$

N=8 DFT : The 6th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \end{pmatrix}$$

R

I



3 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 5, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(\omega' t) = \cos(-(-\omega)t)$

I ➡ *sampled values of* $\sin(\omega' t) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$

$$2\pi\cdot\left(\frac{-3}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[5]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

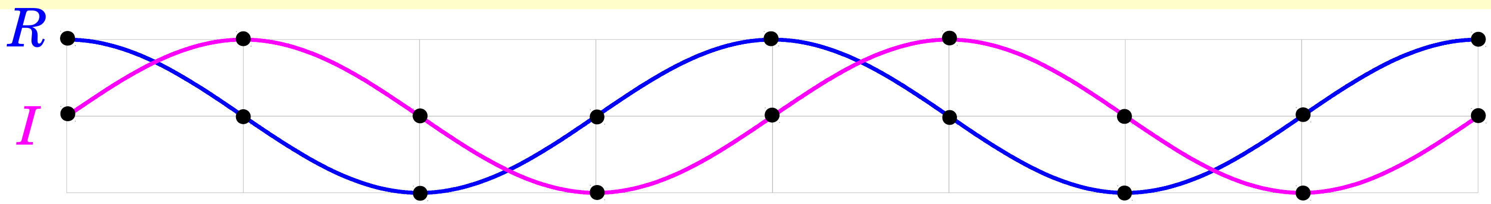
τ

$T = N\tau$

Sampling Time	τ	*Sampling Frequency*	$f_s = \frac{1}{\tau}$
Sequence Time Length	$T = N\tau$	*3rd Harmonic Freq*	$f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3 f_s}{N}$

N=8 DFT : The 7th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \end{pmatrix}$$



2 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 2, \quad n = 0, 1, \dots, 7$$

R ➡ *sampled values of* $\cos(\omega' t) = \cos(-(-\omega)t)$

I ➡ *sampled values of* $\sin(\omega' t) = \sin(-(-\omega)t)$

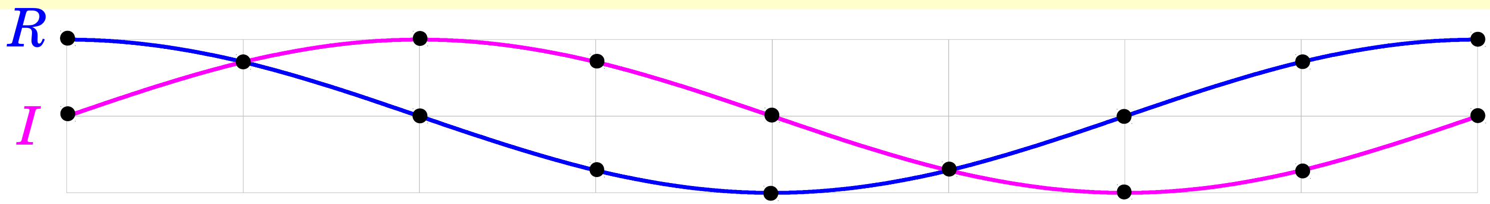
$$-\omega t = -2\pi f t$$
$$2\pi\cdot\left(\tfrac{-2}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[6]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

Sampling Time	τ	Sampling Frequency	$f_s = \frac{1}{\tau}$
Sequence Time Length	$T = N\tau$	2nd Harmonic Freq	$f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2 f_s}{N}$

N=8 DFT : The 8th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1} \end{pmatrix}$$



1 cycle

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 7, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(\omega' t) = \cos(-(-\omega)t)$

I ➡ *sampled values of* $\sin(\omega' t) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$

$$2\pi \cdot (\tfrac{-1}{8}) \cdot f_s \cdot t$$

$\boldsymbol{X}[7]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

τ

$T = N\tau$

Sampling Time	τ	Sampling Frequency	$f_s = \frac{1}{\tau}$
Sequence Time Length	$T = N\tau$	1st Harmonic Freq	$f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann