DFT Frequency (4A)

- Each Row of the DFT Matrix
-

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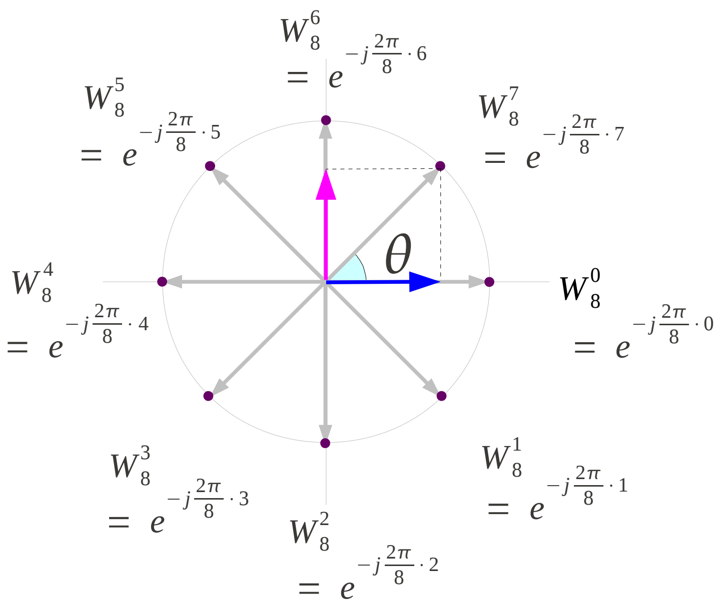
Normalized Frequency



$$\theta = \omega t = 2\pi f t$$

$$\omega = 2\pi f = 2\pi \frac{1}{T}$$

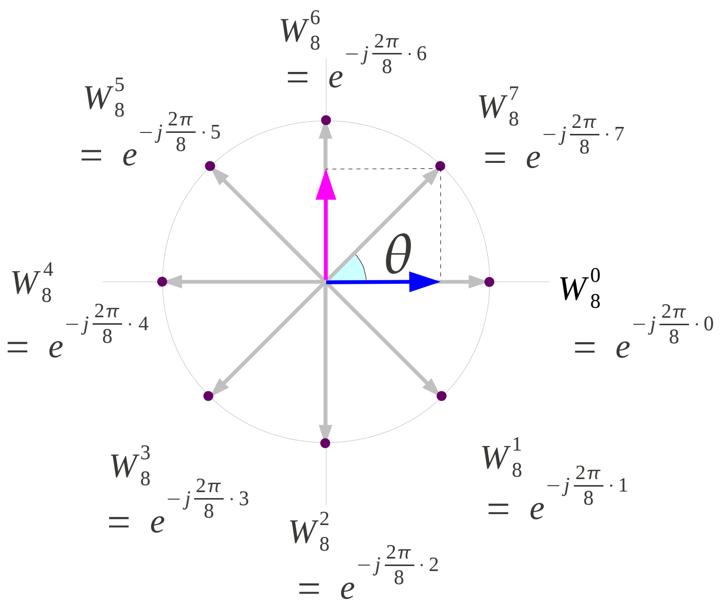
$$W_N^{nk} \triangleq e^{-j(2\pi/N)nk}$$

	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7	
k=1	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	1 cycle

N=8 samples

Period: T   Fundamental Frequency : f = 1 /T

Normalized Frequency

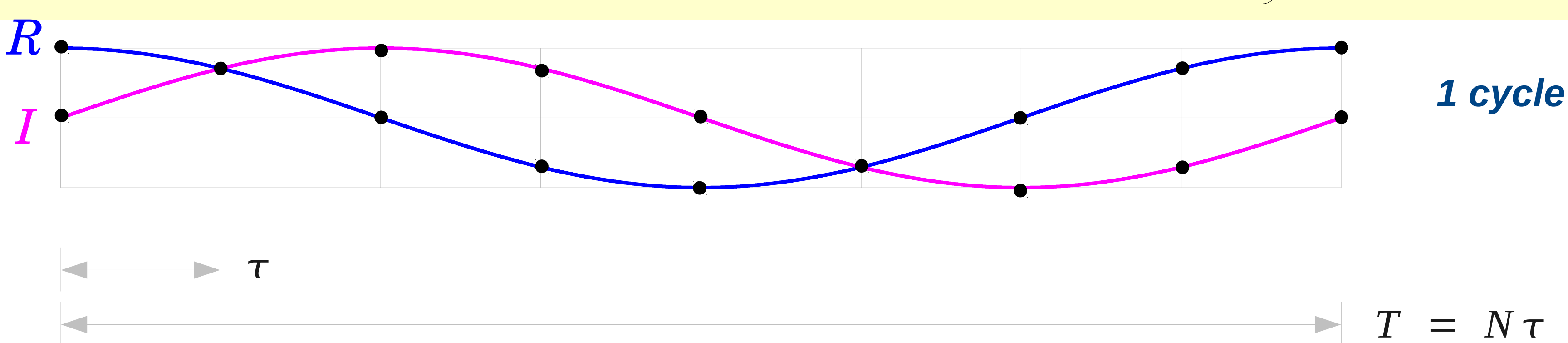


$$\theta = \omega t = 2\pi f t$$

$$\omega = 2\pi f = 2\pi \frac{1}{T}$$

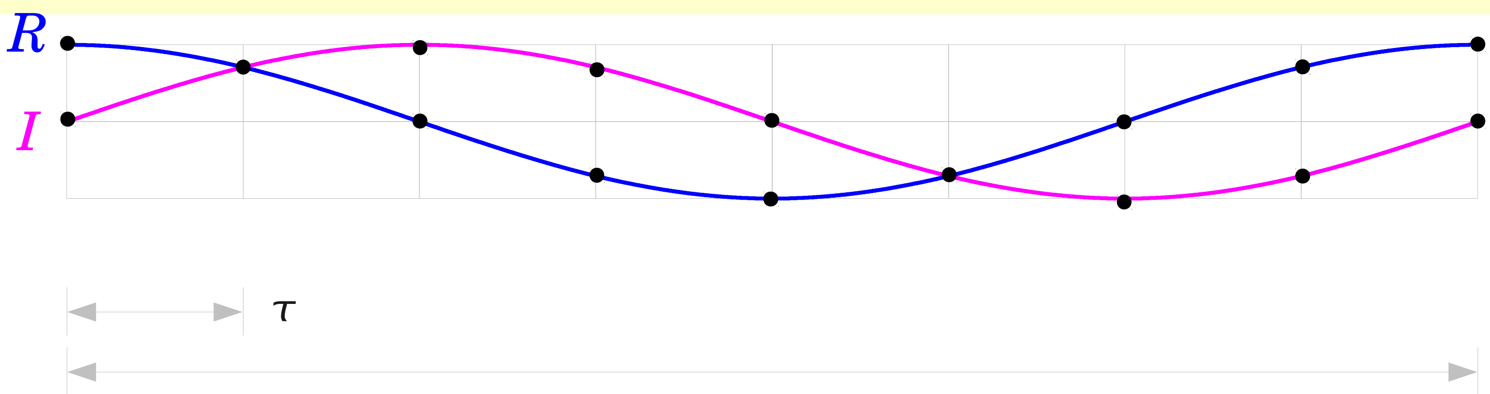
$$W_N^{nk} \triangleq e^{-j(2\pi/N)nk}$$

$$\left(e^{+j\cdot\frac{\pi}{4}\cdot 0} \quad e^{+j\cdot\frac{\pi}{4}\cdot 1} \quad e^{+j\cdot\frac{\pi}{4}\cdot 2} \quad e^{+j\cdot\frac{\pi}{4}\cdot 3} \quad e^{+j\cdot\frac{\pi}{4}\cdot 4} \quad e^{+j\cdot\frac{\pi}{4}\cdot 5} \quad e^{+j\cdot\frac{\pi}{4}\cdot 6} \quad e^{+j\cdot\frac{\pi}{4}\cdot 7} \right)$$



Normalized Frequency

$$\left(e^{+j\cdot\frac{\pi}{4}\cdot 0} \quad e^{+j\cdot\frac{\pi}{4}\cdot 1} \quad e^{+j\cdot\frac{\pi}{4}\cdot 2} \quad e^{+j\cdot\frac{\pi}{4}\cdot 3} \quad e^{+j\cdot\frac{\pi}{4}\cdot 4} \quad e^{+j\cdot\frac{\pi}{4}\cdot 5} \quad e^{+j\cdot\frac{\pi}{4}\cdot 6} \quad e^{+j\cdot\frac{\pi}{4}\cdot 7} \right)$$



1 cycle

$T = N\tau$

Sampling Time τ

Sequence Time Length $T = N\tau$

Sampling Frequency $f_s = \dfrac{1}{\tau}$ *(samples per second)*

1st Harmonic Freq $f_1 = \dfrac{1}{T} = \dfrac{1}{N\tau} = \dfrac{f_s}{N}$

Normalized Frequency

Sampling Time	τ	*(seconds per sample)*
Sequence Time Length	$T = N\tau$	
Sampling Frequency	$f_s = \dfrac{1}{\tau}$	*(samples per second)*

1st Harmonic Freq $\quad f_1 = \dfrac{1}{T} = \dfrac{1}{N\tau} = \dfrac{1}{N} f_s$

nth Harmonic Freq $\quad f_n = \dfrac{n}{T} = \dfrac{n}{N\tau} = \dfrac{n}{N} f_s \qquad n = 0,\ 1,\ 2,\ \ldots,\ N-1$

Normalized Frequency

$$\frac{f_n}{f_s} = \frac{n}{N}$$

(cycles per sample) $= \dfrac{\text{(cycles per second)}}{\text{(samples per second)}}$

Normalized Frequency

Sampling Time τ

Sequence Time Length $T = N\tau$

Sampling Frequency $f_s = \dfrac{1}{\tau}$ *(samples per second)*

Normalized Frequency

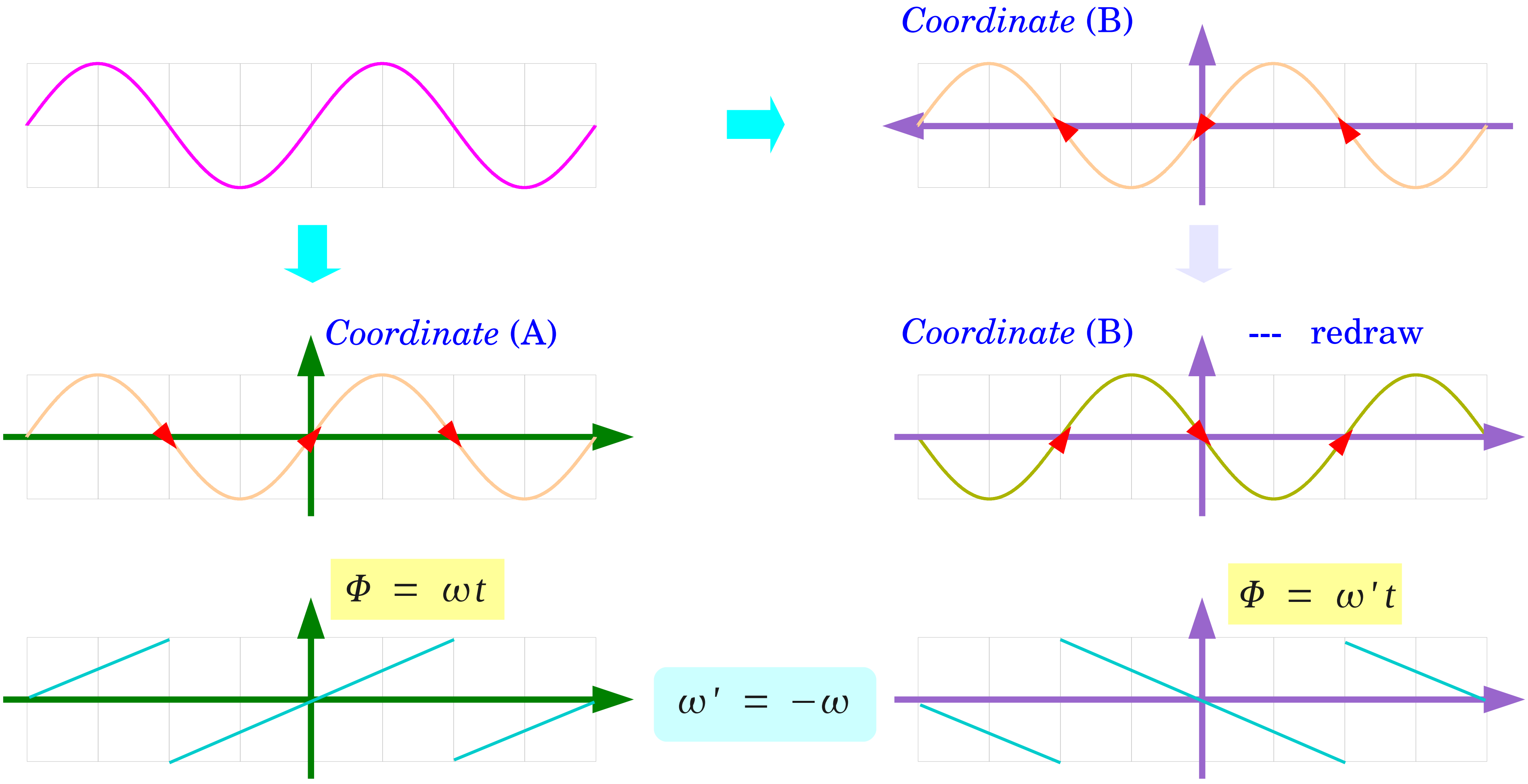
$$\frac{f_n}{f_s} = \frac{n}{N}$$

(cycles per sample)

$$
\begin{aligned}
f_1 &= 1 \cdot f_1 \\
f_2 &= 2 \cdot f_1 \\
f_3 &= 3 \cdot f_1 \\
&\cdots \\
f_{N-1} &= (N-1) \cdot f_1
\end{aligned}
$$

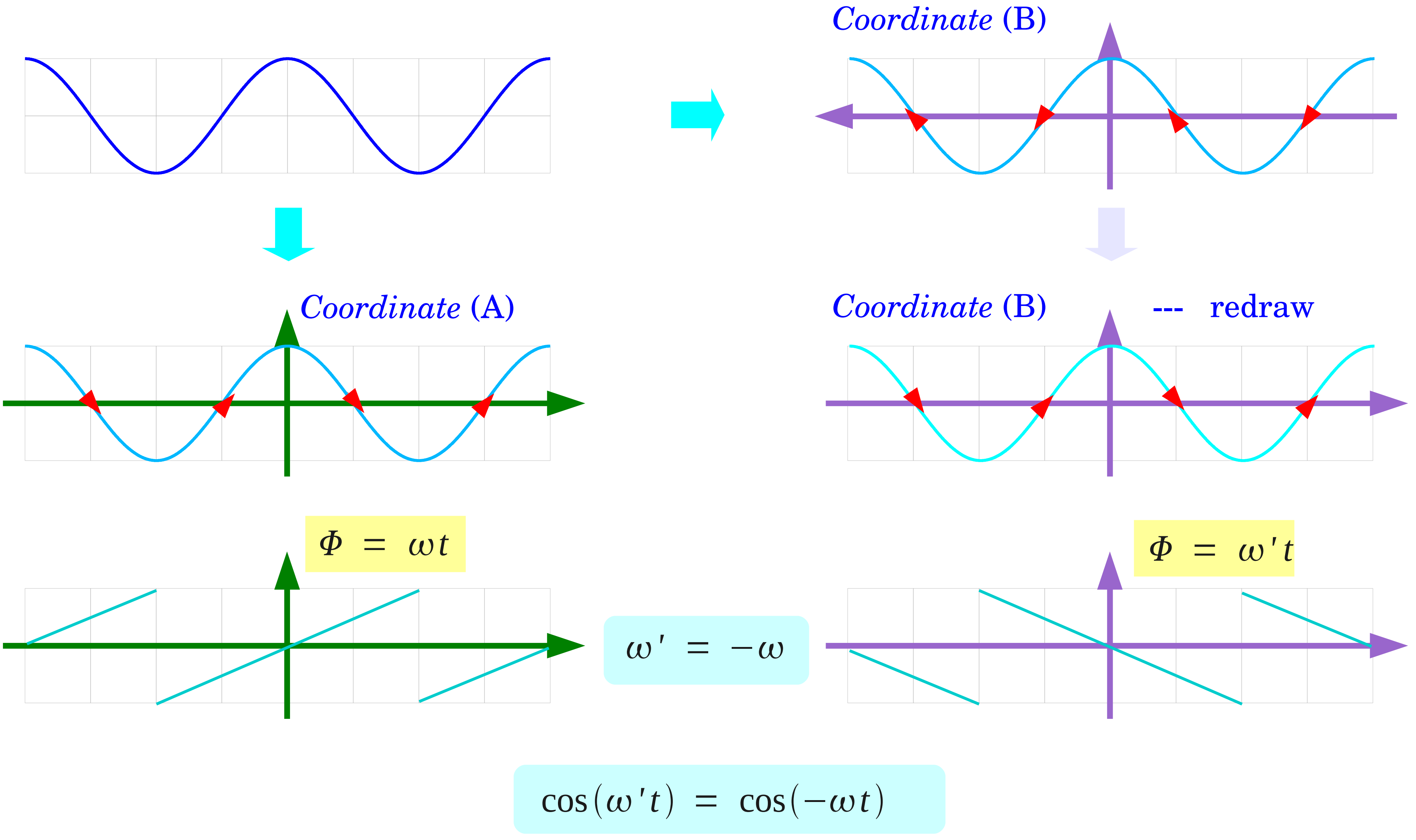
$$
\begin{aligned}
&1/N \\
&2/N \\
&3/N \\
&(N-1)/N
\end{aligned}
$$

Negative Frequency (1)



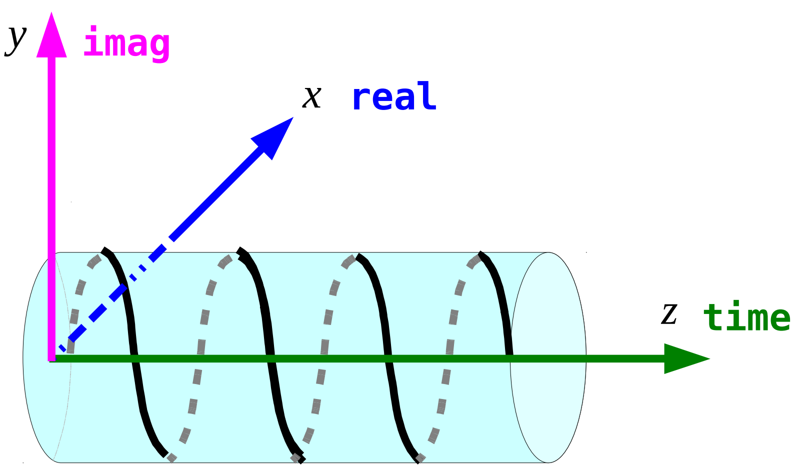
$$\sin(\omega' t) = -\sin(-\omega t)$$

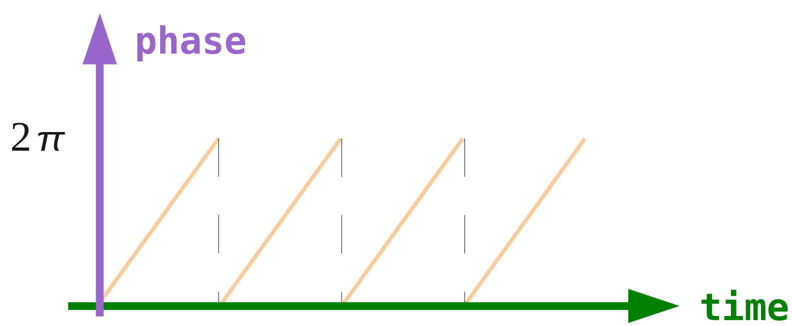
Negative Frequency (2)

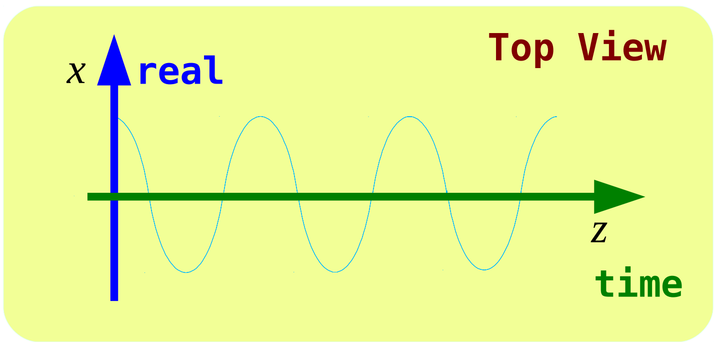


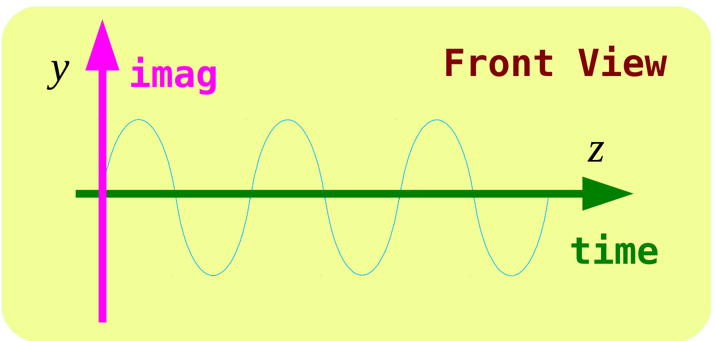
Euler Equation (1)

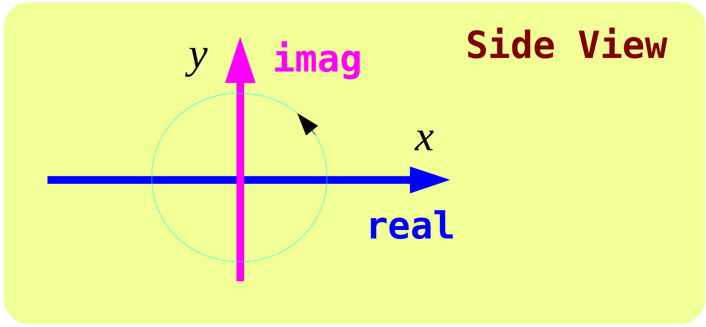
$$e^{+j\omega t} = \cos\omega t + j\sin\omega t$$





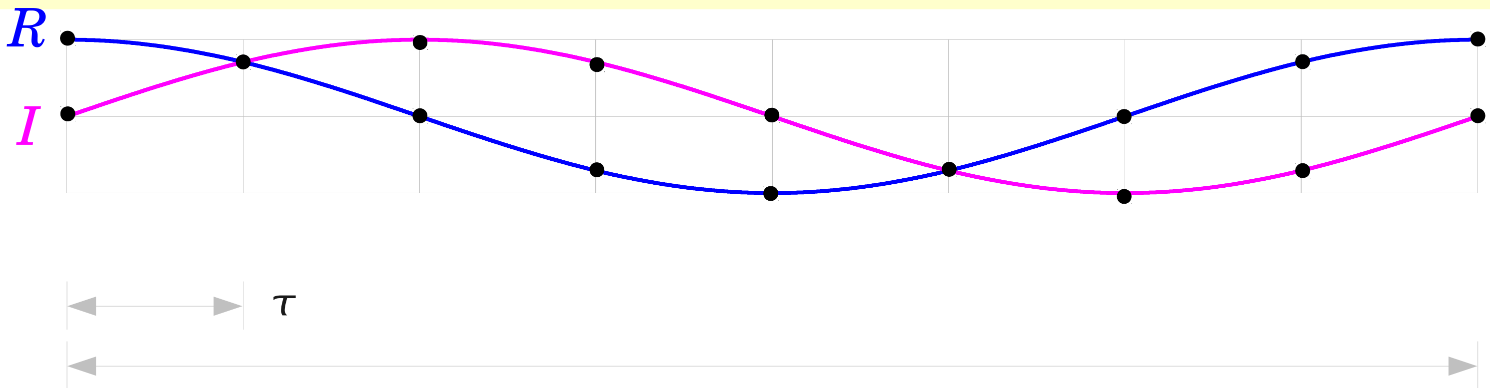






Fundamental Frequency

$$\left(e^{+j\cdot\frac{\pi}{4}\cdot 0} \quad e^{+j\cdot\frac{\pi}{4}\cdot 1} \quad e^{+j\cdot\frac{\pi}{4}\cdot 2} \quad e^{+j\cdot\frac{\pi}{4}\cdot 3} \quad e^{+j\cdot\frac{\pi}{4}\cdot 4} \quad e^{+j\cdot\frac{\pi}{4}\cdot 5} \quad e^{+j\cdot\frac{\pi}{4}\cdot 6} \quad e^{+j\cdot\frac{\pi}{4}\cdot 7} \right)$$



1 cycle

$T = N\tau$

Sampling Time τ

Sequence Time Length $T = N\tau$

Sampling Frequency $f_s = \frac{1}{\tau}$

1st Harmonic Freq $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

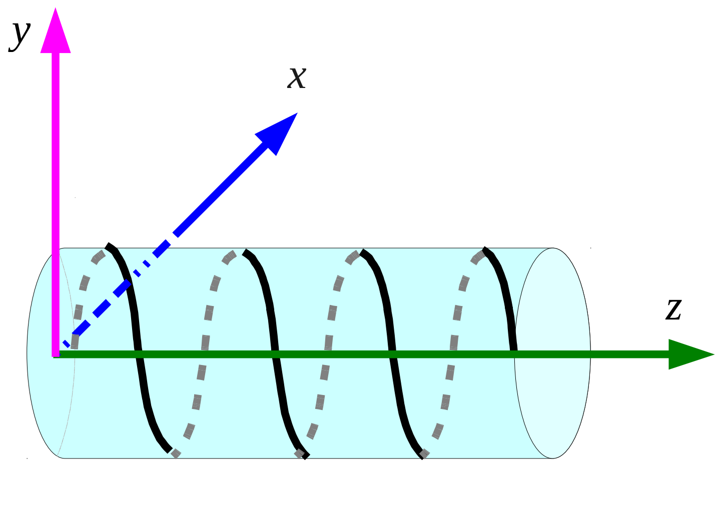
$$f_2 = 2\cdot f_1$$
$$f_3 = 3\cdot f_1$$
$$\cdots$$
$$f_{N-1} = (N-1)\cdot f_1$$

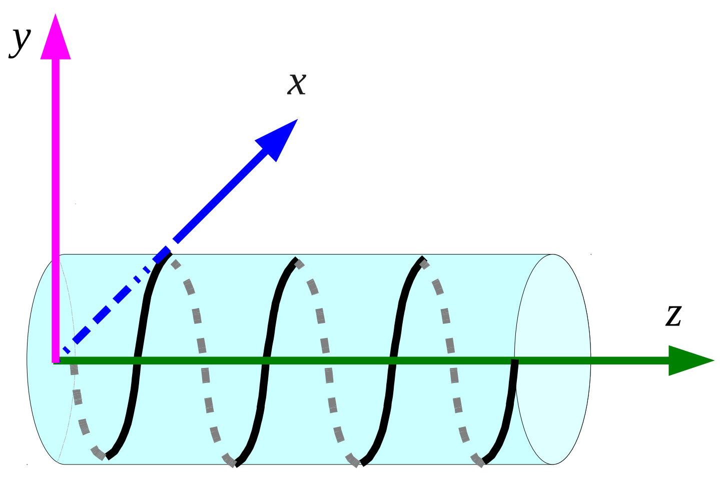
Fundamental Frequency f_o

The Lowest Frequency in a harmonic series.

$$f_0 = f_1 = \frac{f_s}{N}$$

Negative Frequency (3)





Frequency View of a DFT Matrix

row 0	$f = 0$	(0 cycle)	0
row 1	$f = -1 * f_o$	(1 cycle)	$\frac{-1}{N}$
row 2	$f = -2 * f_o$	(2 cycles)	$\frac{-2}{N}$
row $(\frac{N}{2}-1)$	$f = -(\frac{N}{2}-1) * f_o$	($\frac{N}{2}-1$ cycles)	$\frac{-1}{2} + \frac{1}{N}$
row $(\frac{N}{2})$	$f = -(\frac{N}{2}) * f_o$	($\frac{N}{2}$ cycles)	$\frac{-1}{2}$
row $(\frac{N}{2}+1)$	$f = +(\frac{N}{2}-1) * f_o$	($\frac{N}{2}-1$ cycles)	$\frac{1}{2} - \frac{1}{N}$
row $N-2$	$f = +2 * f_o$	(2 cycles)	$\frac{2}{N}$
row $N-1$	$f = +1 * f_o$	(1 cycle)	$\frac{1}{N}$

Rows 1 to $(\frac{N}{2}-1)$: $\frac{N}{2} - 1$

Rows $(\frac{N}{2}+1)$ to $N-1$: $\frac{N}{2} - 1$

Normalized Frequency

$$f_o = \frac{f_s}{N}$$

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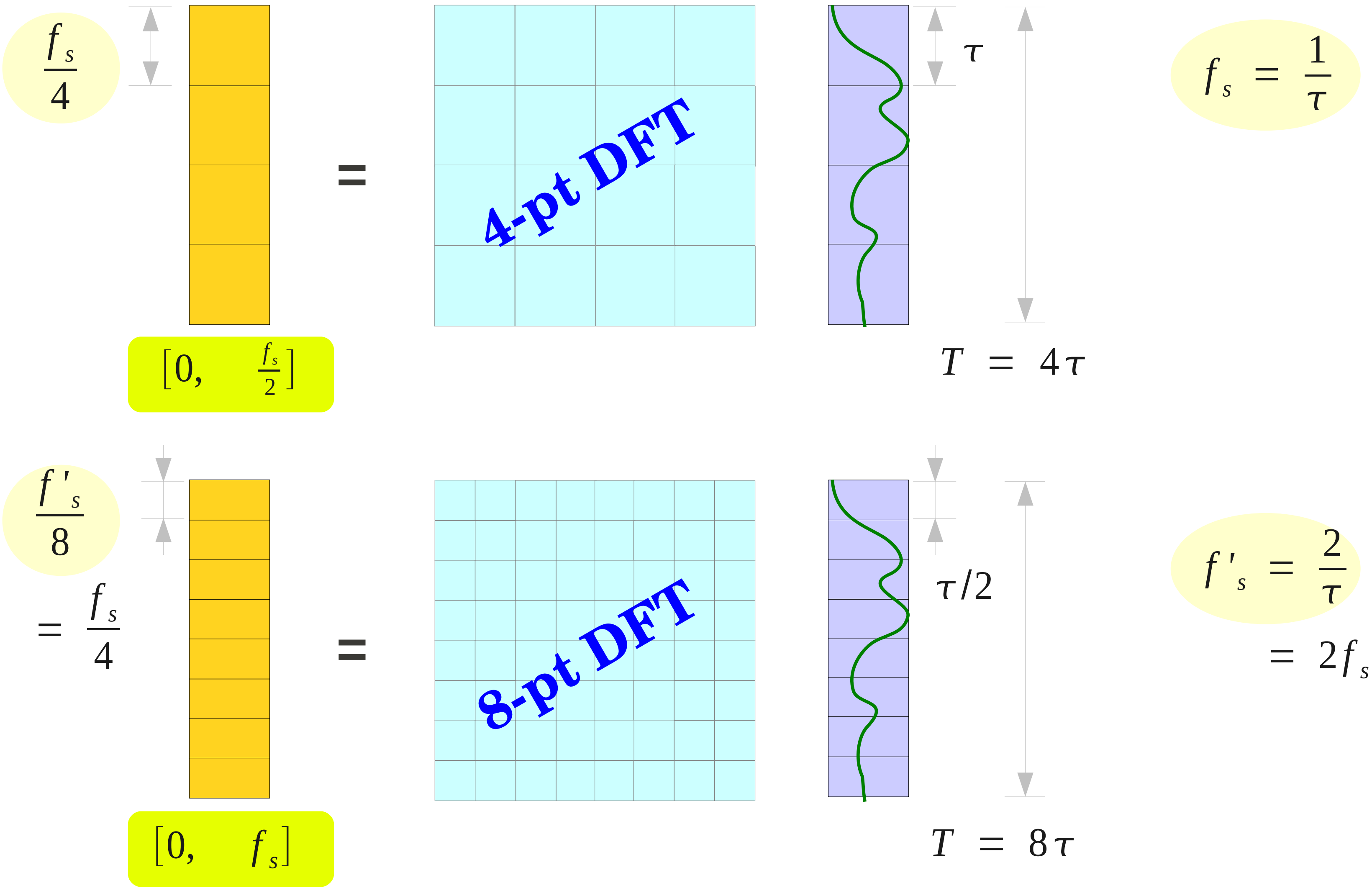
Frequency View of a X[i] Vector

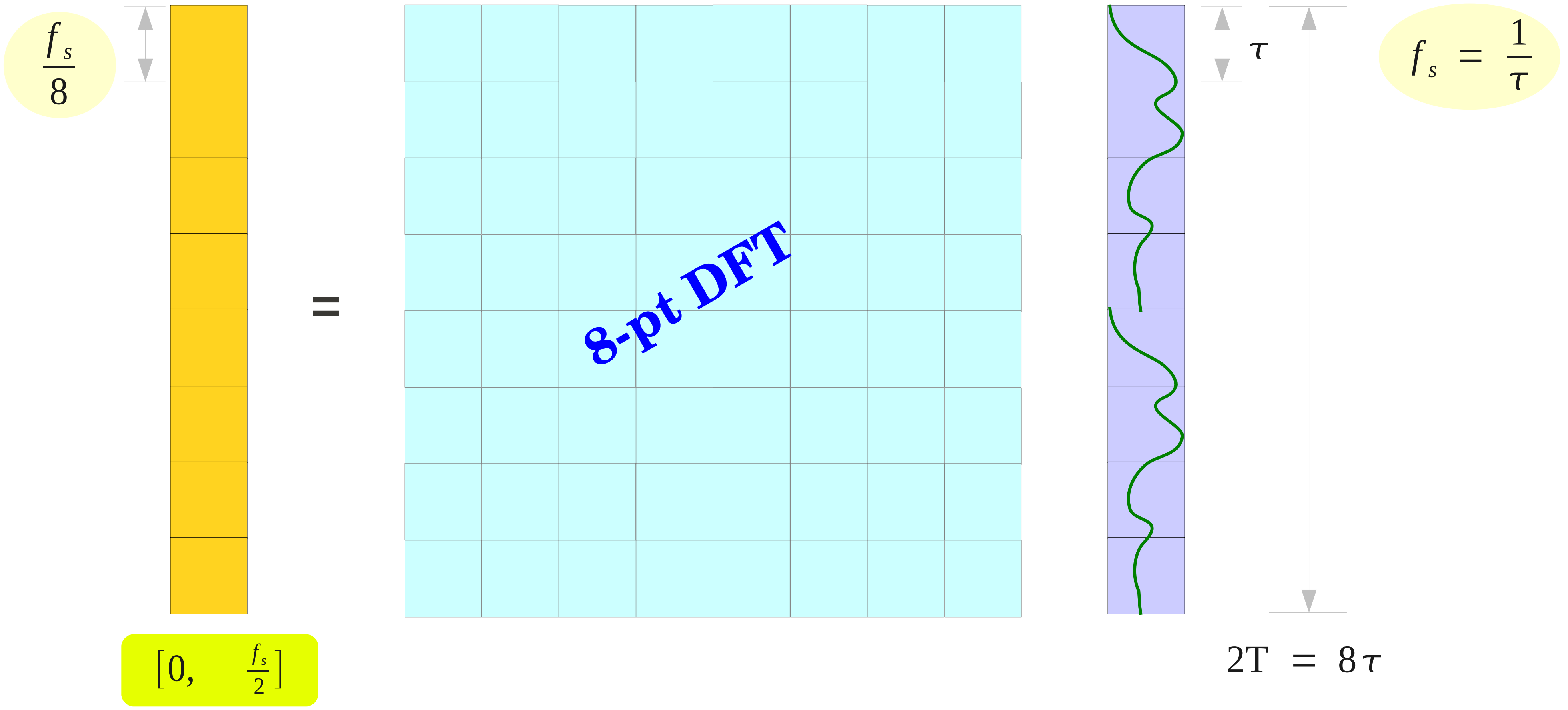
			Normalized Frequency	
$X[0]$	$f = 0$	0	0	$(0\,cycle)$
$X[1]$	$f = +1 * f_o$	$\frac{1}{N} f_s$	$\frac{1}{N}$	$(1\,cycle)$
$X[2]$	$f = +2 * f_o$	$\frac{2}{N} f_s$	$\frac{2}{N}$	$(2\,cycles)$
$X[\frac{N}{2}-1]$	$f = +(\frac{N}{2} - 1) * f_o$	$\frac{(\frac{N}{2} - 1)}{N} f_s$	$\frac{1}{2} - \frac{1}{N}$	$(\frac{N}{2}-1\,cycles)$
$X[\frac{N}{2}]$	$f = +(\frac{N}{2}) * f_o$	$\frac{1}{2} f_s$	$\frac{1}{2}$	$(\frac{N}{2}\,cycles)$
$X[\frac{N}{2}+1]$	$f = -(\frac{N}{2} - 1) * f_o$	$-\frac{(\frac{N}{2} - 1)}{N} f_s$	$\frac{-1}{2} + \frac{1}{N}$	$(\frac{N}{2}-1\,cycles)$
$X[N-2]$	$f = -2 * f_o$	$-\frac{2}{N} f_s$	$\frac{-2}{N}$	$(2\,cycles)$
$X[N-1]$	$f = -1 * f_o$	$-\frac{1}{N} f_s$	$\frac{-1}{N}$	$(1\,cycle)$

$\frac{N}{2} - 1$ (entries $X[1]$ to $X[\frac{N}{2}-1]$)

$\frac{N}{2} - 1$ (entries $X[\frac{N}{2}+1]$ to $X[N-1]$)

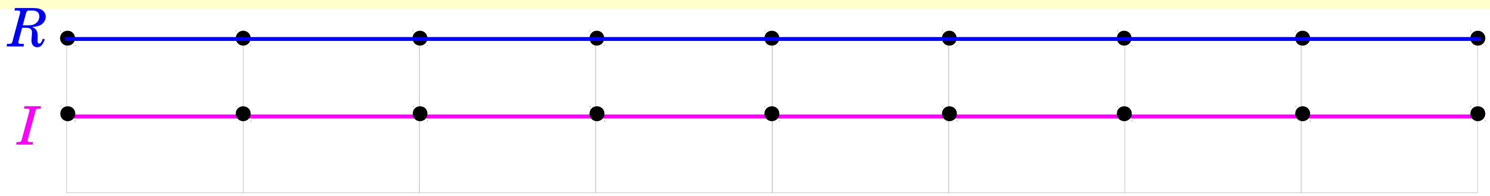


$\frac{f_s}{4}$
=
4-pt DFT
τ
$f_s = \frac{1}{\tau}$
$[0, \frac{f_s}{2}]$
$T = 4\tau$
$\frac{f'_s}{8}$
$= \frac{f_s}{4}$
=
8-pt DFT
$\tau/2$
$f'_s = \frac{2}{\tau}$
$= 2f_s$
$[0, f_s]$
$T = 8\tau$


$\frac{f_s}{8}$
=
8-pt DFT
τ
$f_s = \frac{1}{\tau}$
$[0, \frac{f_s}{2}]$
$2T = 8\tau$

N=8 DFT : The 1st Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \end{pmatrix}$$



0 cycle

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 0, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot(\tfrac{0}{8})\cdot f_s\cdot t$$

$\boldsymbol{X}[0]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

Sampling Time τ

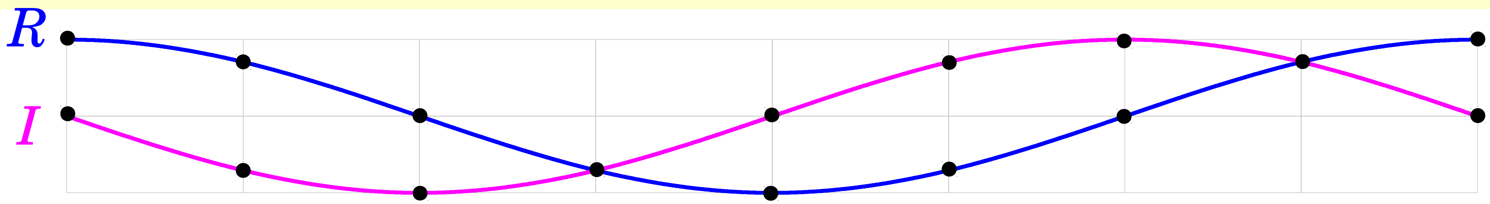
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

Zero Frequency

N=8 DFT : The 2nd Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix}$$



1 cycle

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 1, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot\left(\tfrac{1}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[1]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

Sampling Time τ

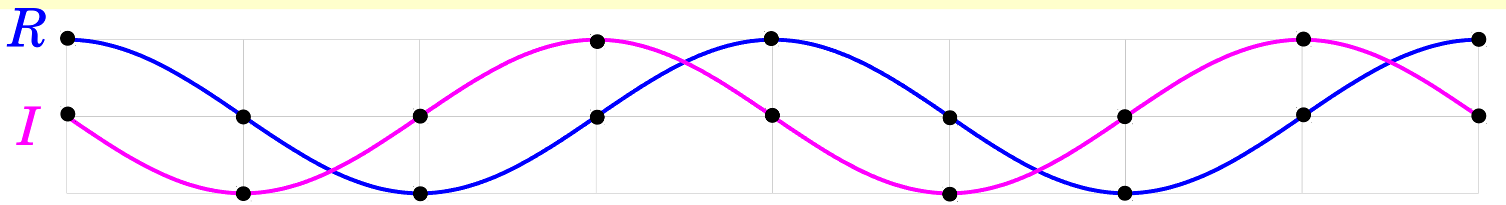
Sampling Frequency $f_s = \dfrac{1}{\tau}$

Sequence Time Length $T = N\tau$

1st Harmonic Freq $f_1 = \dfrac{1}{T} = \dfrac{1}{N\tau} = \dfrac{f_s}{N}$

N=8 DFT : The 3rd Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \end{pmatrix}$$



2 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 2, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot\left(\tfrac{2}{8}\right)\cdot f_s\cdot t$$

$\mathbf{X}[2]$ *measures how much of the above signal component is present in* $\mathbf{x}$.

τ

$T = N\tau$

Sampling Time	τ	Sampling Frequency	$f_s = \frac{1}{\tau}$
Sequence Time Length	$T = N\tau$	2nd Harmonic Freq	$f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2 f_s}{N}$

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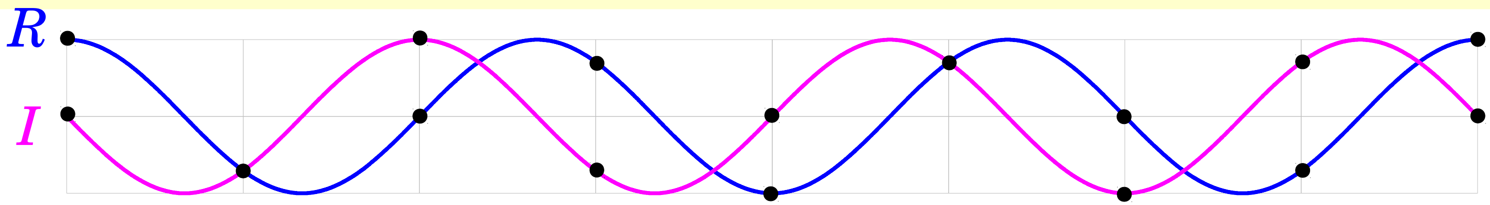
N=8 DFT : The 4th Row of the DFT Matrix

$$\left(e^{-j\cdot\frac{\pi}{4}\cdot 0} \quad e^{-j\cdot\frac{\pi}{4}\cdot 3} \quad e^{-j\cdot\frac{\pi}{4}\cdot 6} \quad e^{-j\cdot\frac{\pi}{4}\cdot 1} \quad e^{-j\cdot\frac{\pi}{4}\cdot 4} \quad e^{-j\cdot\frac{\pi}{4}\cdot 7} \quad e^{-j\cdot\frac{\pi}{4}\cdot 2} \quad e^{-j\cdot\frac{\pi}{4}\cdot 5} \right)$$



3 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 3, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot\left(\tfrac{3}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[3]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

Sampling Time τ

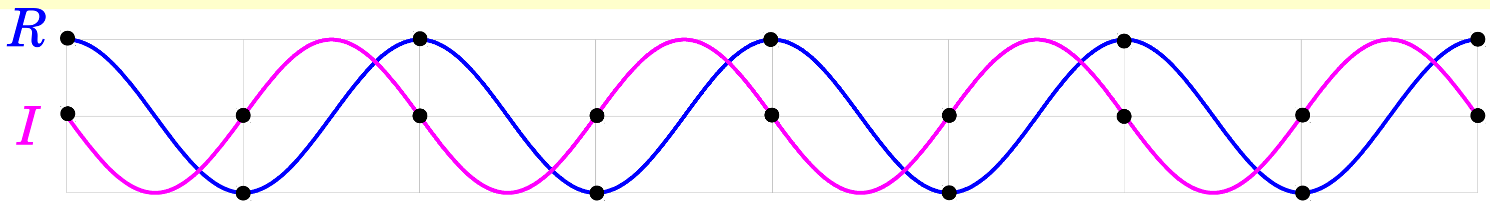
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

3rd Harmonic Freq $f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3 f_s}{N}$

N=8 DFT : The 5th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \end{pmatrix}$$



4 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 4, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot\left(\tfrac{4}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[4]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

τ

$T = N\tau$

Sampling Time τ

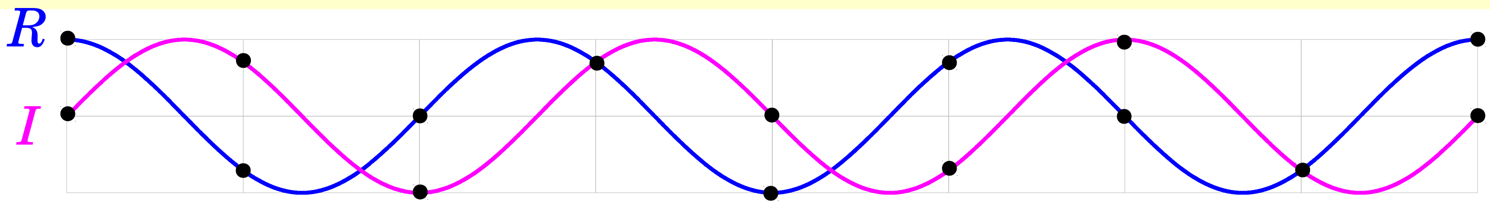
Sampling Frequency $f_s = \dfrac{1}{\tau}$

Sequence Time Length $T = N\tau$

4th Harmonic Freq $f_4 = \dfrac{4}{T} = \dfrac{4}{N\tau} = \dfrac{4 f_s}{N}$

N=8 DFT : The 6th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \end{pmatrix}$$



3 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 5, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(\omega' t) = \cos(-(-\omega)t)$

I ➡ *sampled values of* $\sin(\omega' t) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$

$$2\pi\cdot\left(\frac{-3}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[5]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

$T = N\tau$

Sampling Time τ

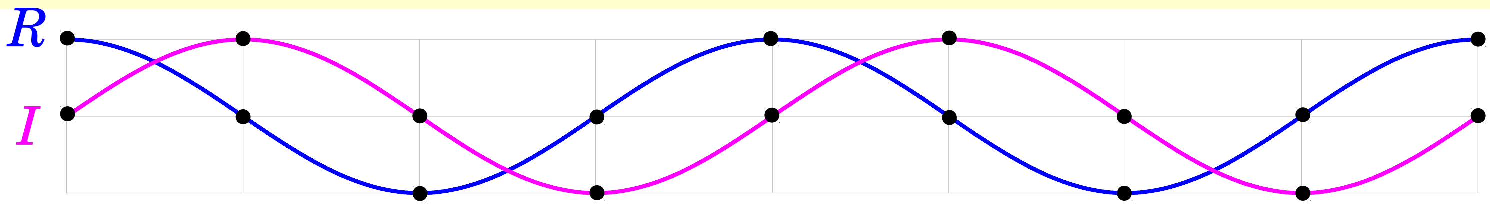
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

3rd Harmonic Freq $f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3 f_s}{N}$

N=8 DFT : The 7th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \end{pmatrix}$$



2 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 2, \quad n = 0, 1, \ldots, 7$$

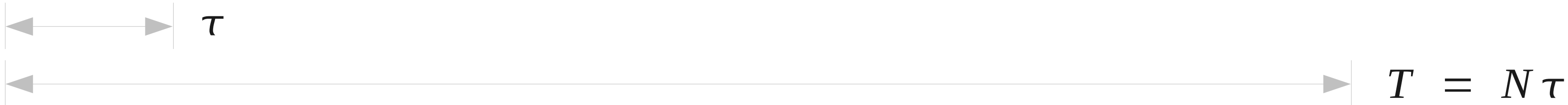
R ➡ *sampled values of* $\cos(\omega' t) = \cos(-(-\omega)t)$

I ➡ *sampled values of* $\sin(\omega' t) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$

$$2\pi\cdot\left(\frac{-2}{8}\right)\cdot f_s\cdot t$$

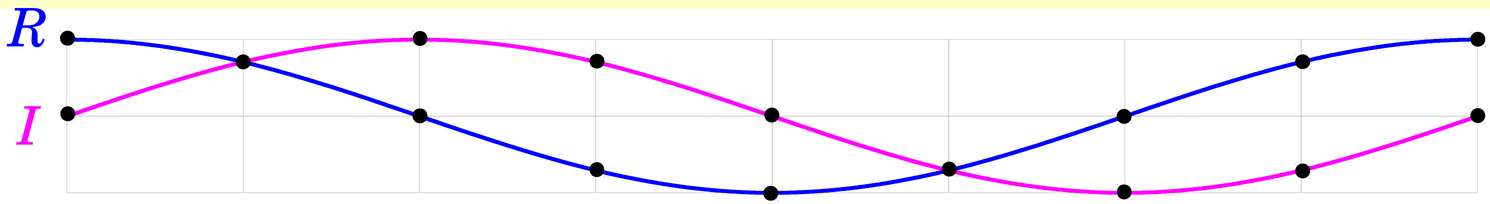
$\boldsymbol{X}[6]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.



Sampling Time	τ	*Sampling Frequency*	$f_s = \frac{1}{\tau}$
Sequence Time Length	$T = N\tau$	*2nd Harmonic Freq*	$f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2 f_s}{N}$

N=8 DFT : The 8th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1} \end{pmatrix}$$



1 cycle

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 7, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(\omega' t) = \cos(-(-\omega)t)$

I ➡ *sampled values of* $\sin(\omega' t) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$

$$2\pi\cdot\left(\tfrac{-1}{8}\right)\cdot f_s\cdot t$$

$\boldsymbol{X}[7]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

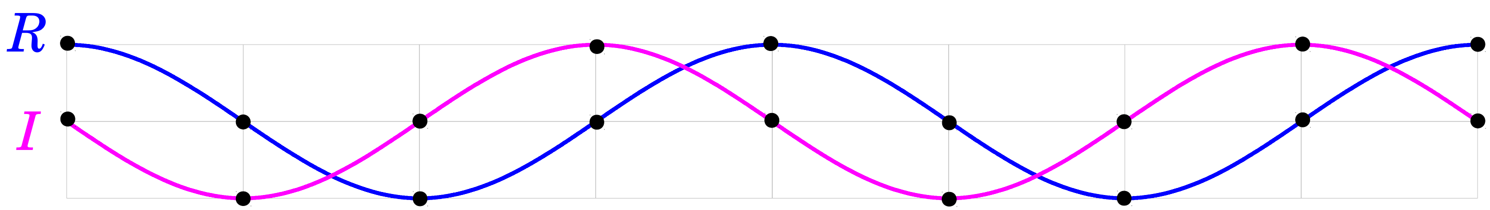
Sampling Time τ

Sampling Frequency $f_s = \dfrac{1}{\tau}$

Sequence Time Length $T = N\tau$

1st Harmonic Freq $f_1 = \dfrac{1}{T} = \dfrac{1}{N\tau} = \dfrac{f_s}{N}$

Negative Frequency



2 cycles

$$W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn} \qquad k = 2, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi \cdot \left(\tfrac{2}{8}\right) \cdot f_s \cdot t$$

$\boldsymbol{X}[2]$ *measures how much of the above signal component is present in* $\boldsymbol{x}$.

Sampling Time τ

Sampling Frequency $f_s = \dfrac{1}{\tau}$

Sequence Time Length $T = N\tau$

2nd Harmonic Freq $f_2 = \dfrac{2}{T} = \dfrac{2}{N\tau} = \dfrac{2 f_s}{N}$

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1/19/11

N=8 DFT : DFT Matrix in + or – Frequencies

$$\omega_0 = 2\pi \cdot \frac{f_s}{N}$$

0th row:	*samples of*	$\cos(\ 0\,\omega_0)t + j \cdot \sin(\ 0\,\omega_0)t$	(0 cycle)
1th row:	*samples of*	$\cos(-1\,\omega_0)t + j \cdot \sin(-1\,\omega_0)t$	(1 cycle)
2th row:	*samples of*	$\cos(-2\,\omega_0)t + j \cdot \sin(-2\,\omega_0)t$	(2 cycles)
3th row:	*samples of*	$\cos(-3\,\omega_0)t + j \cdot \sin(-3\,\omega_0)t$	(3 cycles)
4th row:	*samples of*	$\cos(-4\,\omega_0)t + j \cdot \sin(-4\,\omega_0)t$	(4 cycles)
5th row:	*samples of*	$\cos(-5\,\omega_0)t + j \cdot \sin(-5\,\omega_0)t$	(5 cycles)
6th row:	*samples of*	$\cos(-6\,\omega_0)t + j \cdot \sin(-6\,\omega_0)t$	(6 cycles)
7th row:	*samples of*	$\cos(-7\,\omega_0)t + j \cdot \sin(-7\,\omega_0)t$	(7 cycles)

=

0th row:	*samples of*	$\cos(\ 0\,\omega_0)t + j \cdot \sin(\ 0\,\omega_0)t$	(0 cycle)
1th row:	*samples of*	$\cos(+7\,\omega_0)t + j \cdot \sin(+7\,\omega_0)t$	(7 cycles)
2th row:	*samples of*	$\cos(+6\,\omega_0)t + j \cdot \sin(+6\,\omega_0)t$	(6 cycles)
3th row:	*samples of*	$\cos(+5\,\omega_0)t + j \cdot \sin(+5\,\omega_0)t$	(5 cycles)
4th row:	*samples of*	$\cos(+4\,\omega_0)t + j \cdot \sin(+4\,\omega_0)t$	(4 cycles)
5th row:	*samples of*	$\cos(+3\,\omega_0)t + j \cdot \sin(+3\,\omega_0)t$	(3 cycles)
6th row:	*samples of*	$\cos(+2\,\omega_0)t + j \cdot \sin(+2\,\omega_0)t$	(2 cycles)
7th row:	*samples of*	$\cos(+1\,\omega_0)t + j \cdot \sin(+1\,\omega_0)t$	(1 cycles)

N=8 DFT : DFT Matrix in Both Frequencies

$\omega_0 = 2\pi \cdot \frac{f_s}{N}$

0th row:	*samples of*	$\cos(\ \ 0\,\omega_0)t + j \cdot \sin(\ \ 0\,\omega_0)t$	(0 cycle)
1th row:	*samples of*	$\cos(-1\,\omega_0)t + j \cdot \sin(-1\,\omega_0)t$	(1 cycle)
2th row:	*samples of*	$\cos(-2\,\omega_0)t + j \cdot \sin(-2\,\omega_0)t$	(2 cycles)
3th row:	*samples of*	$\cos(-3\,\omega_0)t + j \cdot \sin(-3\,\omega_0)t$	(3 cycles)
4th row:	*samples of*	$\cos(-4\,\omega_0)t + j \cdot \sin(-4\,\omega_0)t$	(4 cycles)
5th row:	*samples of*	$\cos(-5\,\omega_0)t + j \cdot \sin(-5\,\omega_0)t$	(5 cycles)
6th row:	*samples of*	$\cos(-6\,\omega_0)t + j \cdot \sin(-6\,\omega_0)t$	(6 cycles)
7th row:	*samples of*	$\cos(-7\,\omega_0)t + j \cdot \sin(-7\,\omega_0)t$	(7 cycles)

$=$

0th row:	*samples of*	$\cos(\ \ 0\,\omega_0)t + j \cdot \sin(\ \ 0\,\omega_0)t$	(0 cycle)
1th row:	*samples of*	$\cos(-1\,\omega_0)t + j \cdot \sin(-1\,\omega_0)t$	(1 cycle)
2th row:	*samples of*	$\cos(-2\,\omega_0)t + j \cdot \sin(-2\,\omega_0)t$	(2 cycles)
3th row:	*samples of*	$\cos(-3\,\omega_0)t + j \cdot \sin(-3\,\omega_0)t$	(3 cycles)
4th row:	*samples of*	$\cos(-4\,\omega_0)t + j \cdot \sin(-4\,\omega_0)t$	(4 cycles)
5th row:	*samples of*	$\cos(+3\,\omega_0)t + j \cdot \sin(+3\,\omega_0)t$	(3 cycles)
6th row:	*samples of*	$\cos(+2\,\omega_0)t + j \cdot \sin(+2\,\omega_0)t$	(2 cycles)
7th row:	*samples of*	$\cos(+1\,\omega_0)t + j \cdot \sin(+1\,\omega_0)t$	(1 cycles)

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann