DLTI Impulse Response (1A)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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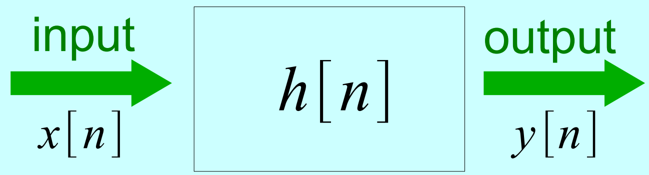
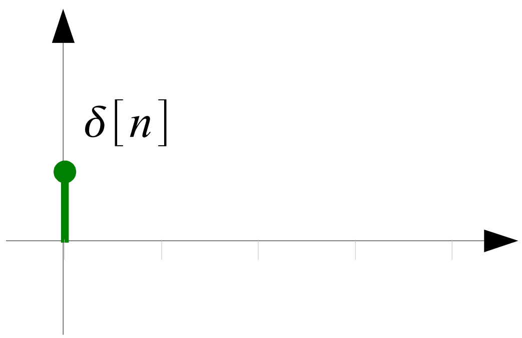
Finite Impulse Response (1)

y[n] as a convolution sum

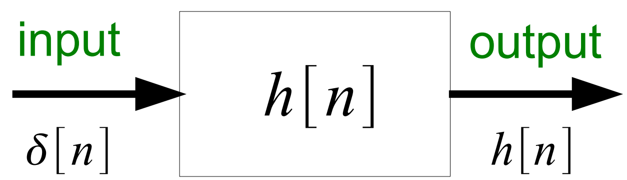
Impulse Response

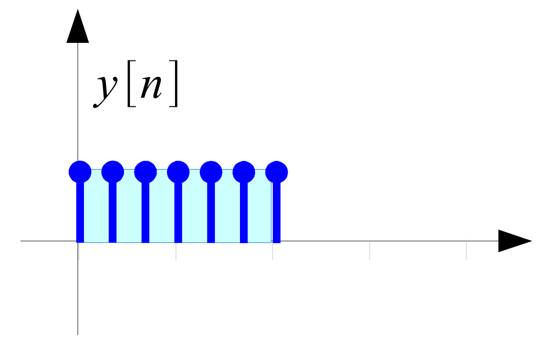


$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$



Impulse Response

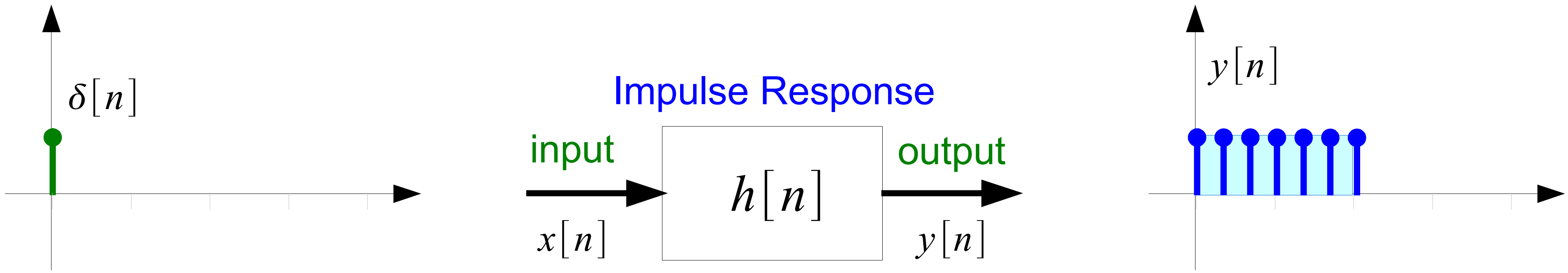




Special Case: ***h*[*n*]** has a finite duration

$$y[n] = \sum_{k=0}^{M} h[k]\, x[n-k]$$

Finite Impulse Response (2)



Special Case: ***h*[*n*]** has a finite duration

y[n] as a convolution sum

$$y[n] = \sum_{k=0}^{M} h[k]\, x[n-k]$$

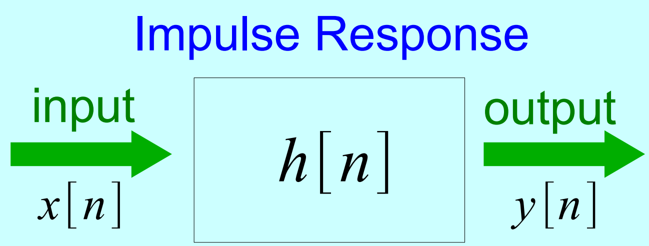
y[n] as a difference equation

$$y[n] = b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$

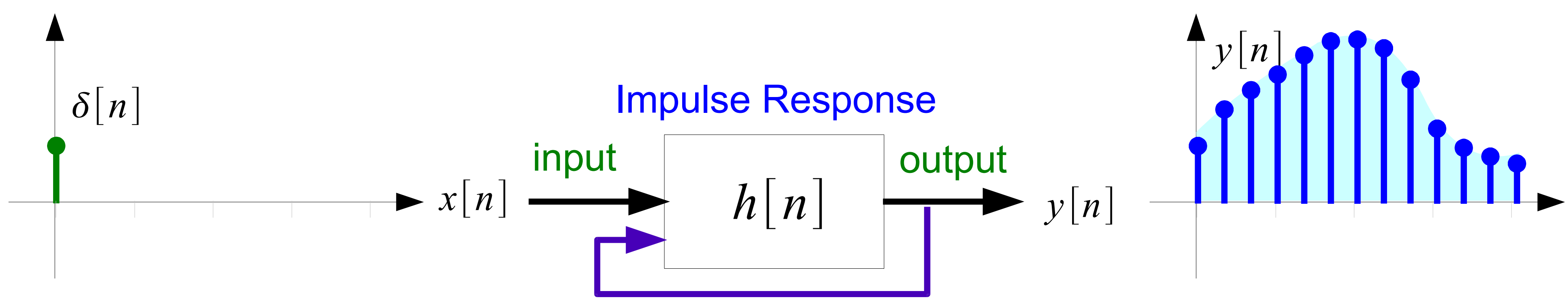
FIR (Finite Impulse Response) Filter

Infinite Impulse Response (1)

y[n] as a convolution sum



$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$



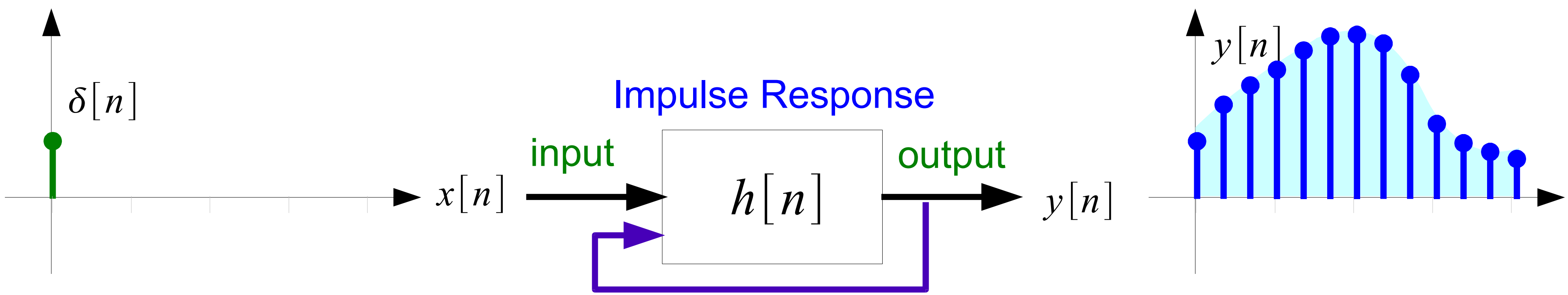
Special Case: ***Feedback***

***h*[*n*]** has a <u>infinite</u> duration

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Young Won Lim
11년 5월 4일

Infinite Impulse Response (2)



Special Case: ***Feedback***

h[n] has a infinite duration

y[n] as a convolution sum

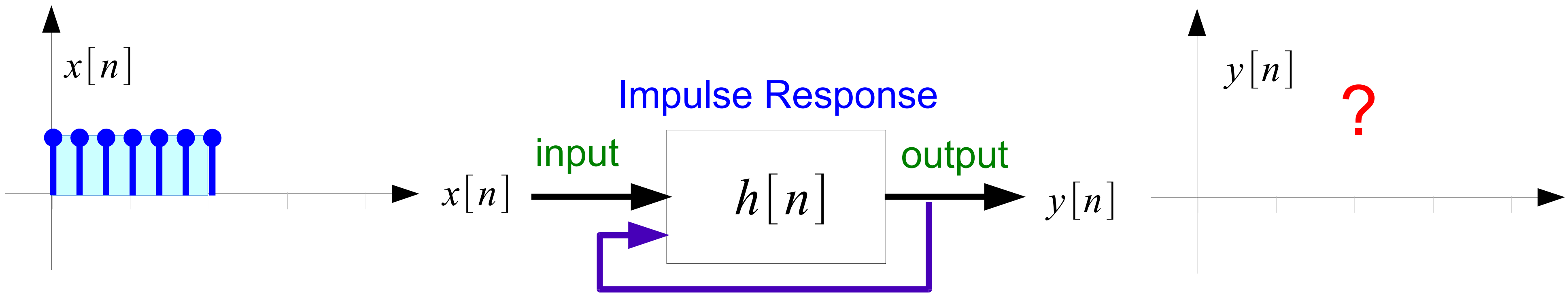
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

y[n] as a difference equation

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \cdots + a_N y[n-N] + b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$

IIR (Infinite Impulse Response) Filter

Infinite Impulse Response (3)



$$y[n] = a_1 \, y[n-1] + b_1 \, x[n]$$

$$y[0] = a_1 \, y[-1] + b_1 \, x[0] = b_1 \qquad = b_1$$

(0)

$$y[1] = a_1 \, y[0] + b_1 \, x[1] = a_1 b_1 + b_1 \qquad = b_1 (a_1 + 1)$$

$$y[2] = a_1 \, y[1] + b_1 \, x[2] = a_1 (a_1 b_1 + b_1) + b_1 \qquad = b_1 (a_1^2 + a_1 + 1)$$

$$y[3] = a_1 \, y[2] + b_1 \, x[3] = a_1 (a_1^2 b_1 + a_1 b_1 + b_1) + b_1 \qquad = b_1 (a_1^3 + a_1^2 + a_1 + 1)$$

$$y[M] = a_1 \, y[M-1] + b_1 \, x[M] \qquad = b_1 (a_1^M + a_1^{M-1} + \cdots + a_1 + 1)$$

$$\begin{cases} S_N = (a_1^M + a_1^{M-1} + \cdots + a_1 + 1) \\ a_1 S_N = (a_1^{M+1} + a_1^M + \cdots + a_1^2 + a_1) \end{cases}$$

$$S_N = (a_1^M + a_1^{M-1} + \cdots + a_1 + 1)$$

Infinite Impulse Response (4)

$$y[n] = a_1\, y[n-1] + b_1\, x[n]$$

$$y[M] = a_1\, y[M-1] + b_1\, x[M] = b_1(a_1^M + a_1^{M-1} + \cdots + a_1 + 1)$$

Geometric Sequence

$$S_N = (a_1^M + a_1^{M-1} + \cdots + a_1 + 1)$$

$$a_1 S_N = (a_1^{M+1} + a_1^M + \cdots + a_1^2 + a_1)$$

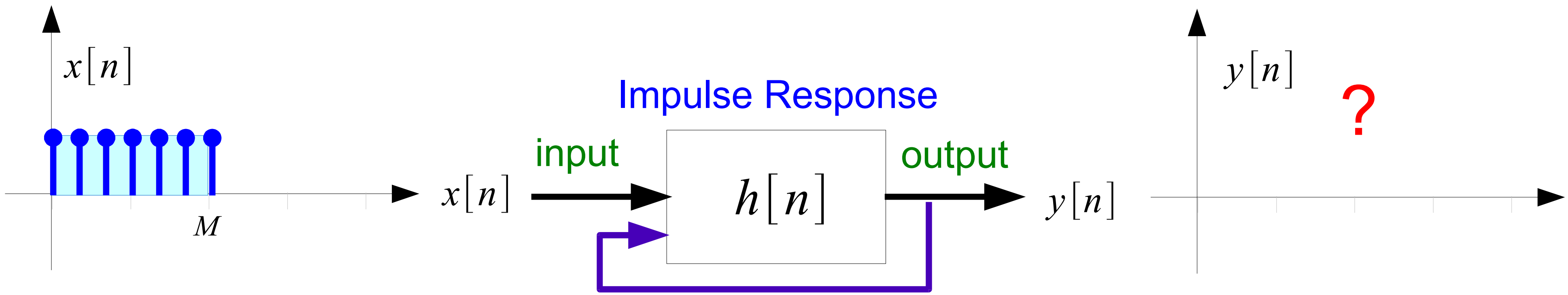
$$(1-a_1) S_N = 1 - a_1^{M+1}$$

$$S_N = \begin{cases} \dfrac{1-a_1^{M+1}}{1-a_1} & (a_1 \neq 1) \\ M & (a_1 = 1) \end{cases}$$

$$\lim_{N\to\infty} S_N = \frac{1}{1-a_1} \qquad (|a_1| < 1)$$

$$y[M] = b_1(a_1^M + a_1^{M-1} + \cdots + a_1 + 1) = b_1 \frac{1-a_1^{M+1}}{1-a_1}$$

Infinite Impulse Response (4)



$$y[n] = a_1 y[n-1] + b_1 x[n]$$

$$y[M] = b_1(a_1^M + a_1^{M-1} + \cdots + a_1 + 1) = b_1 \frac{1 - a_1^{M+1}}{1 - a_1}$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] R.D. Strum, et al., Discrete Systems and Digital Signal Processing