

# FFT (6B)

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- DFT Matrix Property
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# N=8 DFT

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

# N=8 IDFT

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7} \\ W_8^0 & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^{-8} & W_8^{-10} & W_8^{-12} & W_8^{-14} \\ W_8^0 & W_8^{-3} & W_8^{-6} & W_8^{-9} & W_8^{-12} & W_8^{-15} & W_8^{-18} & W_8^{-21} \\ W_8^0 & W_8^{-4} & W_8^{-8} & W_8^{-12} & W_8^{-16} & W_8^{-20} & W_8^{-24} & W_8^{-28} \\ W_8^0 & W_8^{-5} & W_8^{-10} & W_8^{-15} & W_8^{-20} & W_8^{-25} & W_8^{-30} & W_8^{-35} \\ W_8^0 & W_8^{-6} & W_8^{-12} & W_8^{-18} & W_8^{-24} & W_8^{-30} & W_8^{-36} & W_8^{-42} \\ W_8^0 & W_8^{-7} & W_8^{-14} & W_8^{-21} & W_8^{-28} & W_8^{-35} & W_8^{-42} & W_8^{-49} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix}$$

# N=8 DFT Matrix (1)

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 7} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 6} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 5} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 3} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 1} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

# N=8 IDFT Matrix (1)

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix} e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 7} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 6} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 5} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 3} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 1} \end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

# N=8 DFT Matrix (1)

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 7} \\ 0 & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 6} \\ 0 & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 6} & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 5} \\ 0 & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} \\ 0 & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 3} \\ 0 & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} \\ 0 & e^{-j\frac{\pi}{4}\cdot 7} & e^{-j\frac{\pi}{4}\cdot 6} & e^{-j\frac{\pi}{4}\cdot 5} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 1} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

# N=8 IDFT Matrix (1)

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix} e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 7} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 6} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 5} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 3} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 1} \end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$



# Symmetric Matrices

$$A = \left[ \begin{array}{c} \text{blue square with green diagonal} \end{array} \right] \quad A = A^T$$

$$B = A^*$$

$$B = \left[ \begin{array}{c} \text{green square with green diagonal} \end{array} \right] \quad B = B^T$$

# Matrix Multiplication - AB

$$C = A \cdot B \quad [C]_{(i,j)} = [A]_{(\text{row } i)} \cdot [B]_{(\text{col } j)}$$

$C_{(1,1)}$

$$\begin{array}{cccccccc}
 e^{-j \cdot \frac{\pi}{4} \cdot 0} & e^{-j \cdot \frac{\pi}{4} \cdot 1} & e^{-j \cdot \frac{\pi}{4} \cdot 2} & e^{-j \cdot \frac{\pi}{4} \cdot 3} & e^{-j \cdot \frac{\pi}{4} \cdot 4} & e^{-j \cdot \frac{\pi}{4} \cdot 5} & e^{-j \cdot \frac{\pi}{4} \cdot 6} & e^{-j \cdot \frac{\pi}{4} \cdot 7} \\
 e^{+j \cdot \frac{\pi}{4} \cdot 0} & e^{+j \cdot \frac{\pi}{4} \cdot 1} & e^{+j \cdot \frac{\pi}{4} \cdot 2} & e^{+j \cdot \frac{\pi}{4} \cdot 3} & e^{+j \cdot \frac{\pi}{4} \cdot 4} & e^{+j \cdot \frac{\pi}{4} \cdot 5} & e^{+j \cdot \frac{\pi}{4} \cdot 6} & e^{+j \cdot \frac{\pi}{4} \cdot 7} \\
 \mathbf{1} & \mathbf{+1} & \mathbf{+1} & \mathbf{+1} & \mathbf{+1} & \mathbf{+1} & \mathbf{+1} & \mathbf{+1} & \mathbf{= N}
 \end{array}$$

$C_{(1,2)}$

$$\begin{array}{cccccccc}
 e^{-j \cdot \frac{\pi}{4} \cdot 0} & e^{-j \cdot \frac{\pi}{4} \cdot 1} & e^{-j \cdot \frac{\pi}{4} \cdot 2} & e^{-j \cdot \frac{\pi}{4} \cdot 3} & e^{-j \cdot \frac{\pi}{4} \cdot 4} & e^{-j \cdot \frac{\pi}{4} \cdot 5} & e^{-j \cdot \frac{\pi}{4} \cdot 6} & e^{-j \cdot \frac{\pi}{4} \cdot 7} \\
 e^{+j \cdot \frac{\pi}{4} \cdot 0} & e^{+j \cdot \frac{\pi}{4} \cdot 2} & e^{+j \cdot \frac{\pi}{4} \cdot 4} & e^{+j \cdot \frac{\pi}{4} \cdot 6} & e^{+j \cdot \frac{\pi}{4} \cdot 0} & e^{+j \cdot \frac{\pi}{4} \cdot 2} & e^{+j \cdot \frac{\pi}{4} \cdot 4} & e^{+j \cdot \frac{\pi}{4} \cdot 6} \\
 e^{+j \cdot \frac{\pi}{4} \cdot 0} & \mathbf{+} e^{+j \cdot \frac{\pi}{4} \cdot 1} & \mathbf{+} e^{+j \cdot \frac{\pi}{4} \cdot 2} & \mathbf{+} e^{+j \cdot \frac{\pi}{4} \cdot 3} & \mathbf{+} e^{+j \cdot \frac{\pi}{4} \cdot 4} & \mathbf{+} e^{+j \cdot \frac{\pi}{4} \cdot 5} & \mathbf{+} e^{+j \cdot \frac{\pi}{4} \cdot 6} & \mathbf{+} e^{+j \cdot \frac{\pi}{4} \cdot 7} & \mathbf{= 0}
 \end{array}$$

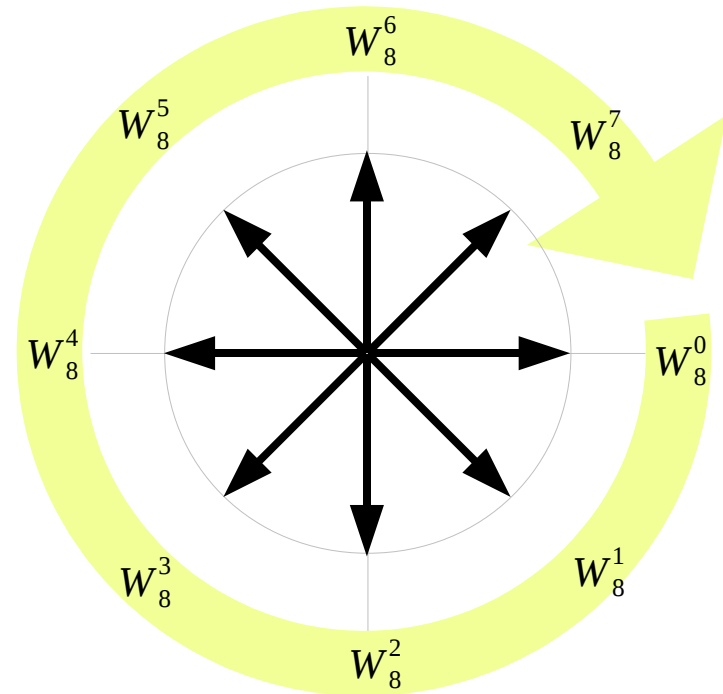
# Root of Unity

$$\sum_{k=0}^{N-1} W_N^k = \sum_{k=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)k} = 0$$

$$z \equiv e^{-j\left(\frac{2\pi}{N}\right)}$$

$$z^N = e^{-j\left(\frac{2\pi}{N}\right)N} = 1$$

$$\sum_{k=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)k} = \frac{z^N - 1}{z - 1} = 0$$



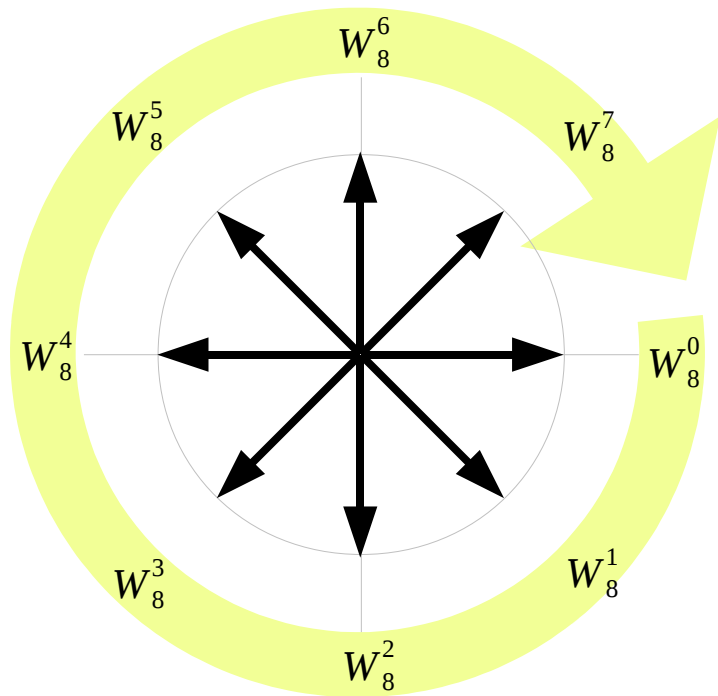
$$W_8^0 + W_8^1 + W_8^2 + W_8^3 + W_8^4 + W_8^5 + W_8^6 + W_8^7 = 0$$



# Complex Phase Periodicity

$$W_8^k = e^{-j\left(\frac{2\pi}{8}\right)k}$$

$$W_8^{-k} = e^{+j\left(\frac{2\pi}{8}\right)k}$$



$$W_8^1 = W_8^{-7}$$

$$W_8^2 = W_8^{-6}$$

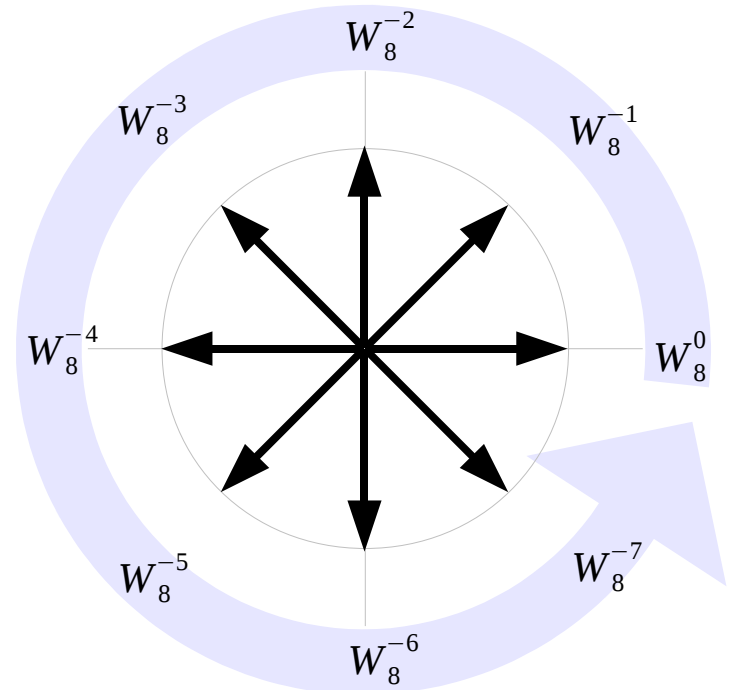
$$W_8^3 = W_8^{-5}$$

$$W_8^4 = W_8^{-4}$$

$$W_8^5 = W_8^{-3}$$

$$W_8^6 = W_8^{-2}$$

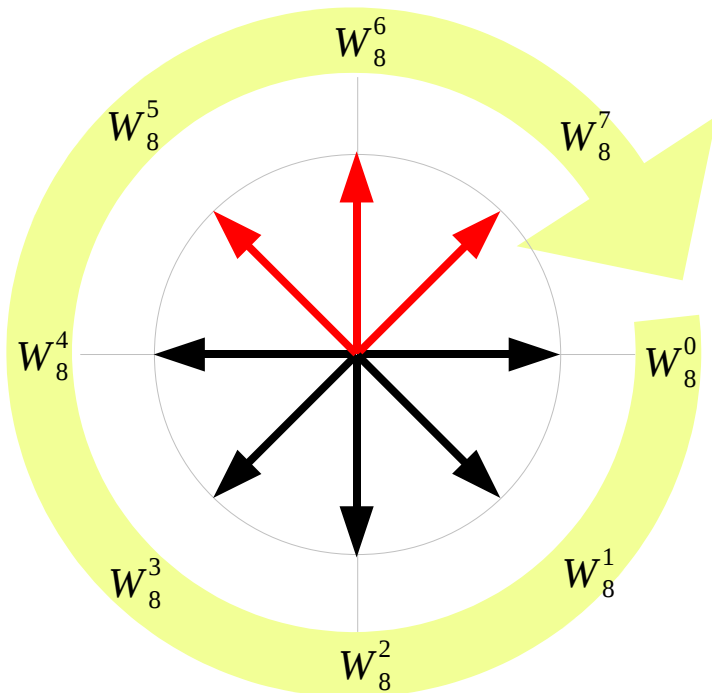
$$W_8^7 = W_8^{-1}$$



$$W_N^{k+N} = W_N^k$$

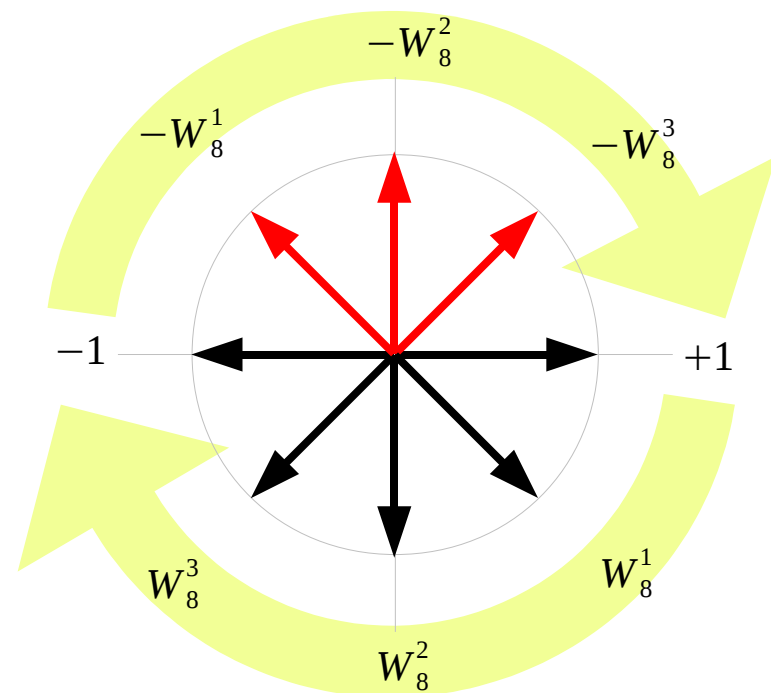
# Complex Phase Symmetry (1)

$$W_8^k = e^{-j\left(\frac{2\pi}{8}\right)k}$$



$$k = 0, 1, 2, 3, 4, 5, 6, 7$$

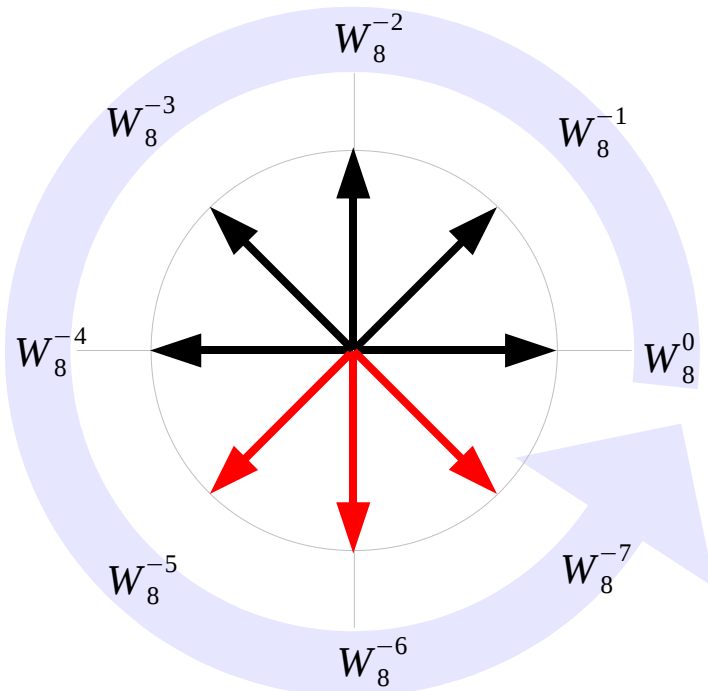
$$W_N^{k+\frac{N}{2}} = -W_N^k$$



$$k = 0, 1, 2, 3$$

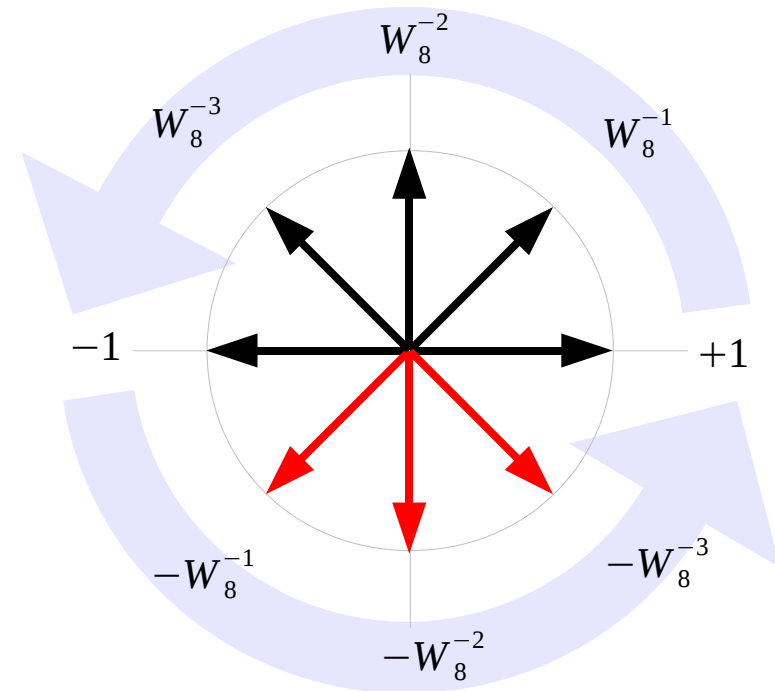
# Complex Phase Symmetry (2)

$$W_8^{-k} = e^{+j\left(\frac{2\pi}{8}\right)k}$$



$$k = 0, 1, 2, 3, 4, 5, 6, 7$$

$$W_N^{-\left(k + \frac{N}{2}\right)} = -W_N^{-k}$$



$$k = 0, 1, 2, 3$$

# Complex Phase - Periodicity

$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{-k} = e^{+j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{k-N} = W_N^k$$

$$W_N^{k+N} = W_N^k$$

$$W_N^{k-N} = e^{-j\left(\frac{2\pi}{N}\right)(k-N)}$$

$$W_N^{k+N} = e^{-j\left(\frac{2\pi}{N}\right)(k+N)}$$

$$\frac{W_N^{k-N}}{W_N^k} = \frac{e^{-j\left(\frac{2\pi}{N}\right)(k-N)}}{e^{-j\left(\frac{2\pi}{N}\right)k}} = e^{j2\pi} = 1$$

$$\frac{W_N^{k+N}}{W_N^k} = \frac{e^{-j\left(\frac{2\pi}{N}\right)(k+N)}}{e^{-j\left(\frac{2\pi}{N}\right)k}} = e^{-j2\pi} = 1$$

$$W_N^{kN} = 1$$

$$W_N^{-kN} = 1$$

$$W_N^{kN} = e^{-j\left(\frac{2\pi}{N}\right)kN} = e^{-j2\pi k} = 1$$

$$W_N^{-kN} = e^{+j\left(\frac{2\pi}{N}\right)kN} = e^{+j2\pi k} = 1$$



# Complex Phase - Symmetry

$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{-k} = e^{+j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{k+\frac{N}{2}} = -W_N^k$$

$$W_N^{-\left(k+\frac{N}{2}\right)} = -W_N^{-k}$$

$$W_N^{k+\frac{N}{2}} = e^{-j\left(\frac{2\pi}{N}\right)\left(k+\frac{N}{2}\right)}$$

$$W_N^{-\left(k+\frac{N}{2}\right)} = e^{+j\left(\frac{2\pi}{N}\right)\left(k+\frac{N}{2}\right)}$$

$$= e^{-j\frac{2\pi}{N}k} \cdot e^{-j\left(\frac{2\pi}{N}\right)\left(\frac{N}{2}\right)}$$

$$= e^{+j\frac{2\pi}{N}k} \cdot e^{+j\left(\frac{2\pi}{N}\right)\left(\frac{N}{2}\right)}$$

$$= e^{-j\frac{2\pi}{N}k} \cdot e^{-j\pi}$$

$$= e^{+j\frac{2\pi}{N}k} \cdot e^{+j\pi}$$

$$= e^{-j\frac{2\pi}{N}k} \cdot (-1)$$

$$= e^{+j\frac{2\pi}{N}k} \cdot (-1)$$

$$W_N^{k+\frac{N}{2}} \cdot W_N^k = -1$$

$$W_N^{-\left(k+\frac{N}{2}\right)} \cdot W_N^{-k} = -1$$

# N=8 DFT Matrix (2)

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$X[0]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$x[0]$
$X[1]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$x[1]$
$X[2]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$x[2]$
$X[3]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$x[3]$
$X[4]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$x[4]$
$X[5]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$x[5]$
$X[6]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$x[6]$
$X[7]$	$e^{-j\frac{\pi}{4}\cdot 0}$	$e^{-j\frac{\pi}{4}\cdot 7}$	$e^{-j\frac{\pi}{4}\cdot 6}$	$e^{-j\frac{\pi}{4}\cdot 5}$	$e^{-j\frac{\pi}{4}\cdot 4}$	$e^{-j\frac{\pi}{4}\cdot 3}$	$e^{-j\frac{\pi}{4}\cdot 2}$	$e^{-j\frac{\pi}{4}\cdot 1}$	$x[7]$

# N=8 DFT Matrix (3)

$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n] \quad W_8^{kn} = e^{-j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 0} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 4} & -e^{-j\frac{\pi}{4}\cdot 1} & -e^{-j\frac{\pi}{4}\cdot 2} & -e^{-j\frac{\pi}{4}\cdot 3} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & -e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & -e^{-j\frac{\pi}{4}\cdot 2} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 3} & -e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 4} & -e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 2} & -e^{-j\frac{\pi}{4}\cdot 1} \\ e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & -e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 0} & e^{-j\frac{\pi}{4}\cdot 4} \\ e^{-j\frac{\pi}{4}\cdot 0} & -e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 2} & -e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 1} & -e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 3} \\ e^{-j\frac{\pi}{4}\cdot 0} & -e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 0} & -e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 2} \\ e^{-j\frac{\pi}{4}\cdot 0} & -e^{-j\frac{\pi}{4}\cdot 3} & -e^{-j\frac{\pi}{4}\cdot 2} & -e^{-j\frac{\pi}{4}\cdot 1} & e^{-j\frac{\pi}{4}\cdot 4} & e^{-j\frac{\pi}{4}\cdot 3} & e^{-j\frac{\pi}{4}\cdot 2} & e^{-j\frac{\pi}{4}\cdot 1} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

# N=8 IDFT Matrix (2)

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix} e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 7} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 6} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 5} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 3} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 7} & e^{+j\frac{\pi}{4}\cdot 6} & e^{+j\frac{\pi}{4}\cdot 5} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 1} \end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

# N=8 IDFT Matrix (3)

$$x[n] = \frac{1}{N} \sum_{k=0}^7 W_8^{-kn} X[k] \quad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix} e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 0} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 4} & -e^{+j\frac{\pi}{4}\cdot 1} & -e^{+j\frac{\pi}{4}\cdot 2} & -e^{+j\frac{\pi}{4}\cdot 3} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & -e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & -e^{+j\frac{\pi}{4}\cdot 2} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 3} & -e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 4} & -e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 2} & -e^{+j\frac{\pi}{4}\cdot 1} \\ e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 0} & e^{+j\frac{\pi}{4}\cdot 4} \\ e^{+j\frac{\pi}{4}\cdot 0} & -e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 2} & -e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 1} & -e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 3} \\ e^{+j\frac{\pi}{4}\cdot 0} & -e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 0} & -e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 2} \\ e^{+j\frac{\pi}{4}\cdot 0} & -e^{+j\frac{\pi}{4}\cdot 3} & -e^{+j\frac{\pi}{4}\cdot 2} & -e^{+j\frac{\pi}{4}\cdot 1} & e^{+j\frac{\pi}{4}\cdot 4} & e^{+j\frac{\pi}{4}\cdot 3} & e^{+j\frac{\pi}{4}\cdot 2} & e^{+j\frac{\pi}{4}\cdot 1} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} \frac{1}{N}$$

## References

[1] <http://en.wikipedia.org/>

[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003