# DFT Analysis (8A)

- X[1]
- X[2]
- X[3]
- X[4]
- X[5]
- X[6]
- X[7]

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# N=8 DFT

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

# N=8 IDFT

$$x[n] = \frac{1}{N}\sum_{k=0}^{7} W_8^{-kn} X[k] \qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} \\ W_8^{0} & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7} \\ W_8^{0} & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^{-8} & W_8^{-10} & W_8^{-12} & W_8^{-14} \\ W_8^{0} & W_8^{-3} & W_8^{-6} & W_8^{-9} & W_8^{-12} & W_8^{-15} & W_8^{-18} & W_8^{-21} \\ W_8^{0} & W_8^{-4} & W_8^{-8} & W_8^{-12} & W_8^{-16} & W_8^{-20} & W_8^{-24} & W_8^{-28} \\ W_8^{0} & W_8^{-5} & W_8^{-10} & W_8^{-15} & W_8^{-20} & W_8^{-25} & W_8^{-30} & W_8^{-35} \\ W_8^{0} & W_8^{-6} & W_8^{-12} & W_8^{-18} & W_8^{-24} & W_8^{-30} & W_8^{-36} & W_8^{-42} \\ W_8^{0} & W_8^{-7} & W_8^{-14} & W_8^{-21} & W_8^{-28} & W_8^{-35} & W_8^{-42} & W_8^{-49} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix}$$

# N=8 DFT Matrix (1)

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad\qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix}
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

$A$

# N=8 IDFT Matrix (1)

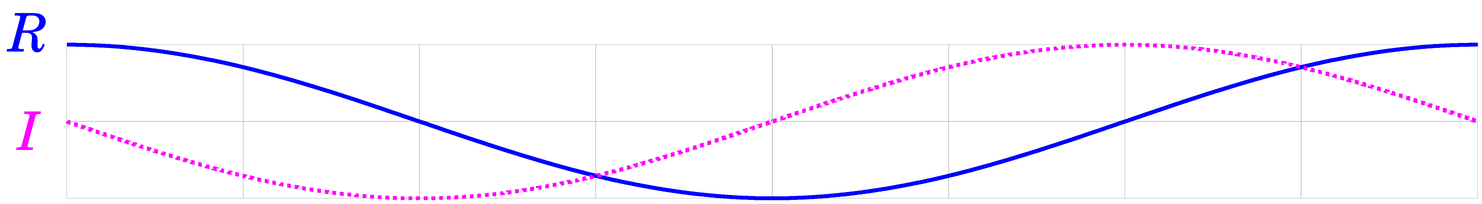
$$x[n] = \frac{1}{N} \sum_{k=0}^{7} W_8^{-kn} \, X[k] \qquad W_8^{-kn} = e^{+j\left(\frac{2\pi}{8}\right)kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix}
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 7} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 5} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 3} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

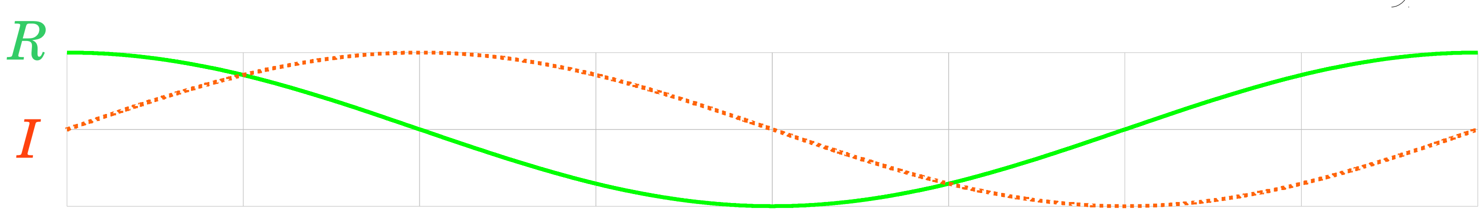
B

# N=8 DFT : X[1]

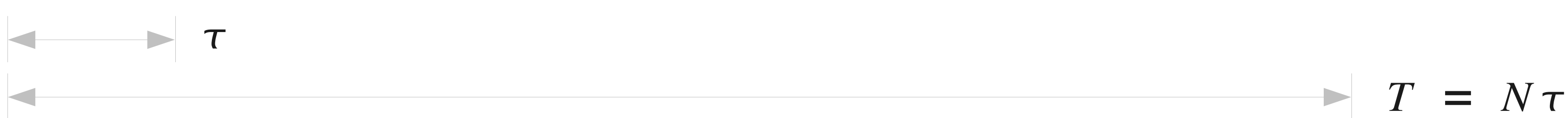
$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7} \end{pmatrix} \bullet$$




$$\begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$




***X*[1] *measures how much of the above signal component is present in* $\boldsymbol{x}$.**



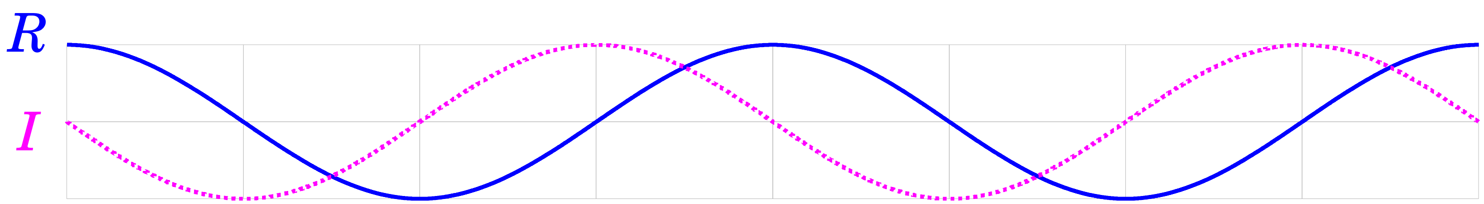

*Sampling Time* $\tau$

*Sampling Frequency* $f_s = \frac{1}{\tau}$

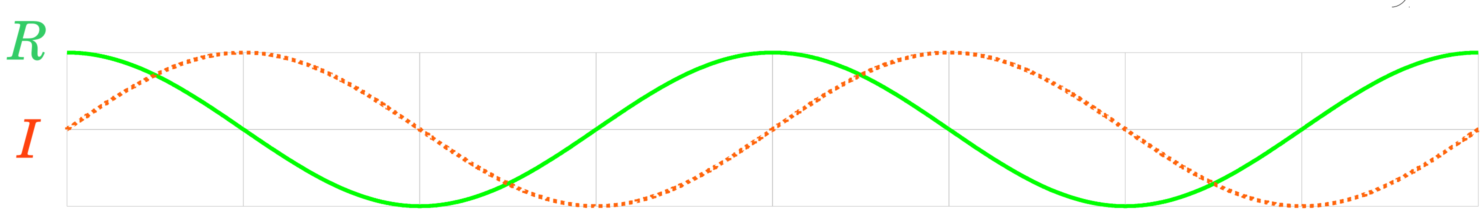
*Sequence Time Length* $T = N\tau$

*1st Harmonic Freq* $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \end{pmatrix} \bullet$$




$$\begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$



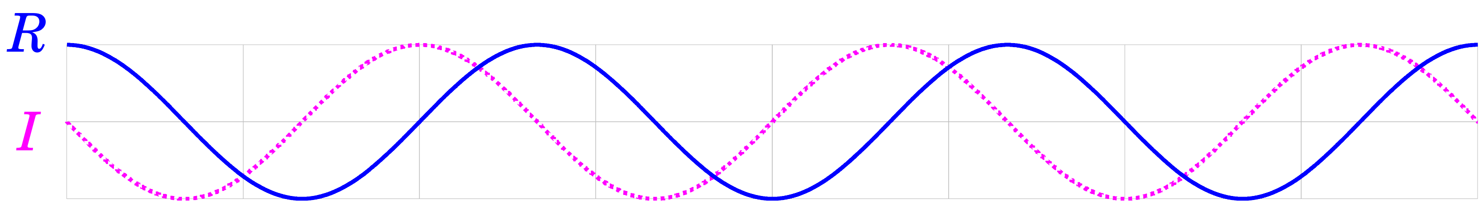

***X*[2] *measures how much of the above signal component is present in* x.**


Sampling Time $\tau$ — Sampling Frequency $f_s = \frac{1}{\tau}$

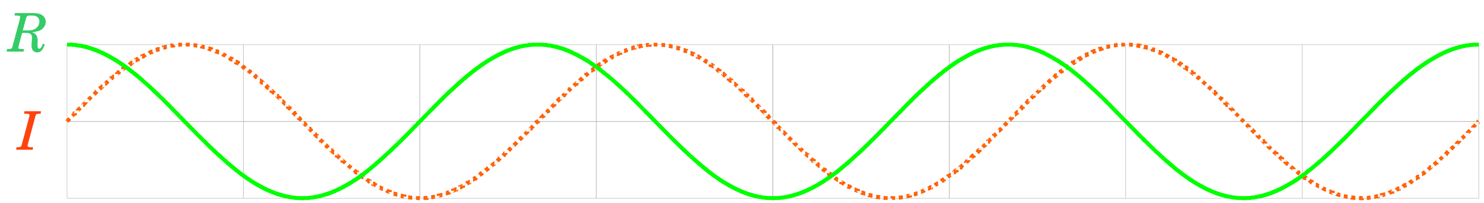
Sequence Time Length $T = N\tau$ — 2[rd] Harmonic Freq $f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2f_s}{N}$

# N=8 DFT : X[3]

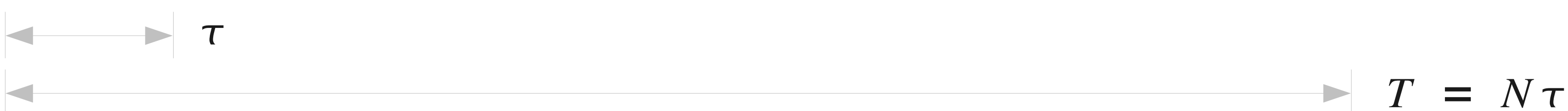
$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \end{pmatrix} \bullet$$




$$\begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$



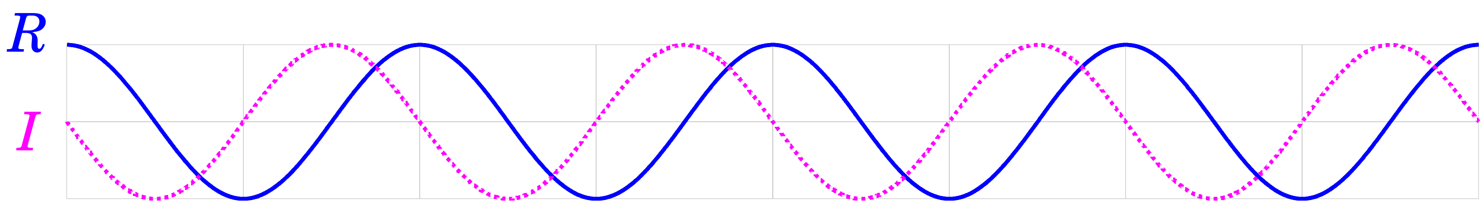

*$\boldsymbol{X}[3]$ measures how much of the above signal component is present in $\boldsymbol{x}$.*



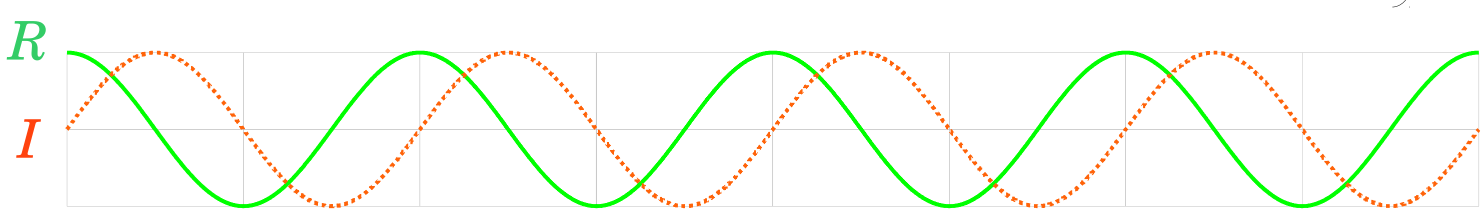

| | | | |
|---|---|---|---|
| *Sampling Time* | $\tau$ | *Sampling Frequency* | $f_s = \frac{1}{\tau}$ |
| *Sequence Time Length* | $T = N\tau$ | *3rd Harmonic Freq* | $f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3 f_s}{N}$ |

# N=8 DFT : X[4]

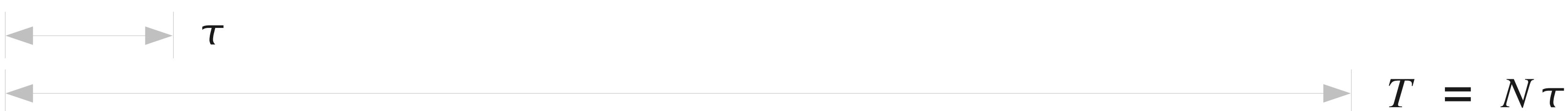
$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \end{pmatrix} \bullet$$




$$\begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$



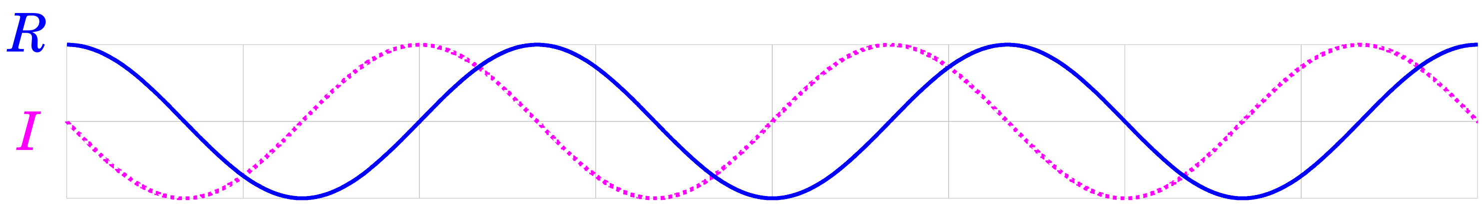

***X*[4] *measures how much of the above signal component is present in* x.**



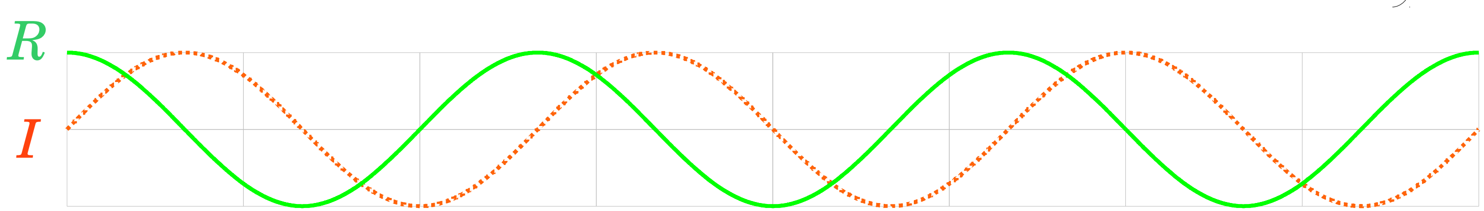

| | | | |
|---|---|---|---|
| *Sampling Time* | $\tau$ | *Sampling Frequency* | $f_s = \frac{1}{\tau}$ |
| *Sequence Time Length* | $T = N\tau$ | *4th Harmonic Freq* | $f_4 = \frac{4}{T} = \frac{4}{N\tau} = \frac{4 f_s}{N}$ |

# N=8 DFT : X[5]

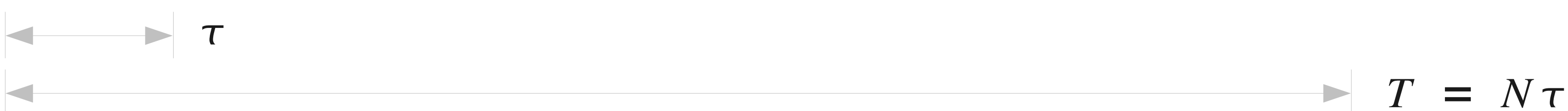
$$\left( e^{-j\cdot\frac{\pi}{4}\cdot 0} \quad e^{-j\cdot\frac{\pi}{4}\cdot 5} \quad e^{-j\cdot\frac{\pi}{4}\cdot 2} \quad e^{-j\cdot\frac{\pi}{4}\cdot 7} \quad e^{-j\cdot\frac{\pi}{4}\cdot 4} \quad e^{-j\cdot\frac{\pi}{4}\cdot 1} \quad e^{-j\cdot\frac{\pi}{4}\cdot 6} \quad e^{-j\cdot\frac{\pi}{4}\cdot 3} \right) \bullet$$




$$\left( x[0] \quad x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \quad x[6] \quad x[7] \right)^T$$



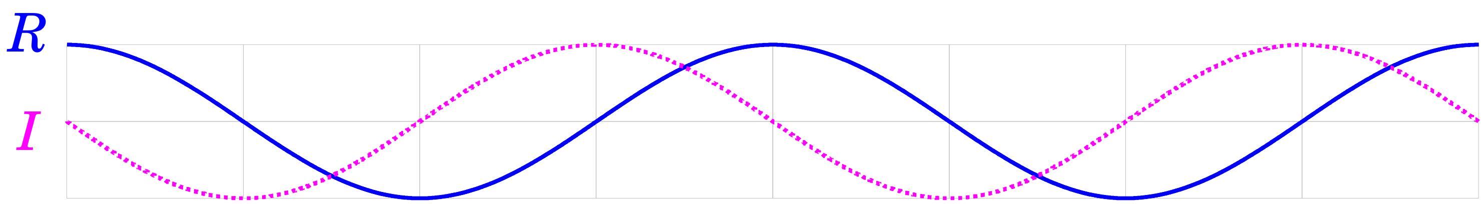

***X*[5] *measures how much of the above signal component is present in* x.**



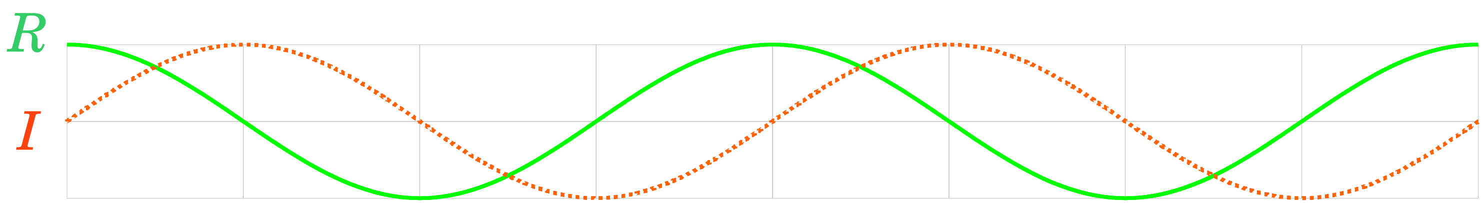

| | | | |
|---|---|---|---|
| *Sampling Time* | $\tau$ | *Sampling Frequency* | $f_s = \frac{1}{\tau}$ |
| *Sequence Time Length* | $T = N\tau$ | *3rd Harmonic Freq* | $f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3 f_s}{N}$ |

# N=8 DFT : X[6]

$$\begin{bmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \end{bmatrix} \bullet$$




$$\begin{bmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{bmatrix}^T$$




***X*[6] *measures how much of the above signal component is present in* $\boldsymbol{x}$.**

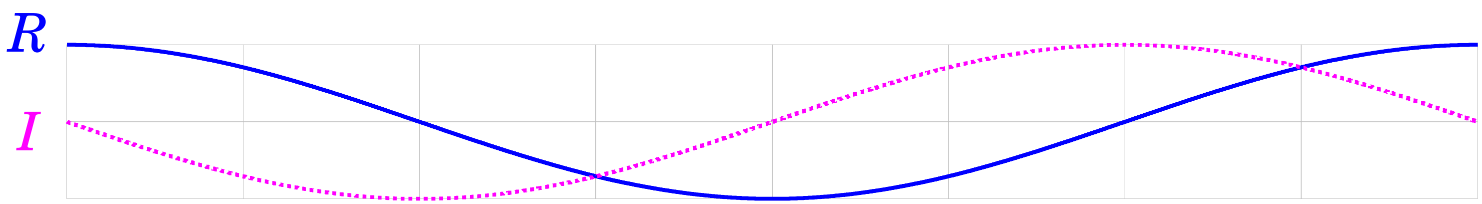
$\tau$

$T = N\tau$

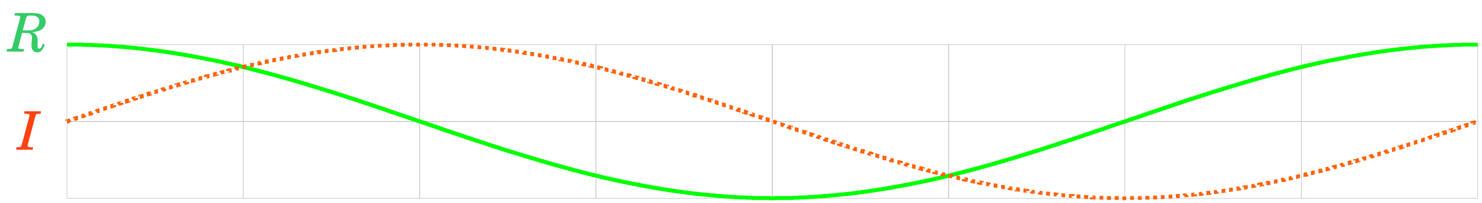
| | | | |
|---|---|---|---|
| *Sampling Time* | $\tau$ | *Sampling Frequency* | $f_s = \frac{1}{\tau}$ |
| *Sequence Time Length* | $T = N\tau$ | *2rd Harmonic Freq* | $f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2f_s}{N}$ |

# N=8 DFT : X[7]

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1} \end{pmatrix} \bullet$$




$$\begin{pmatrix} x[0] & x[1] & x[2] & x[3] & x[4] & x[5] & x[6] & x[7] \end{pmatrix}^T$$




***X*[7] *measures how much of the above signal component is present in* x.**


$T = N\tau$

*Sampling Time* $\tau$

*Sampling Frequency* $f_s = \frac{1}{\tau}$

*Sequence Time Length* $T = N\tau$

*1st Harmonic Freq* $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

## References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann