FFT (6A)

- Complex Phase Factors
- DFT Symmetry
- DFT Matrix

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12/3/09

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Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} \quad \Longleftrightarrow \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{+j(2\pi/N)kn}$$

$$W_N \triangleq e^{-j(2\pi/N)}$$

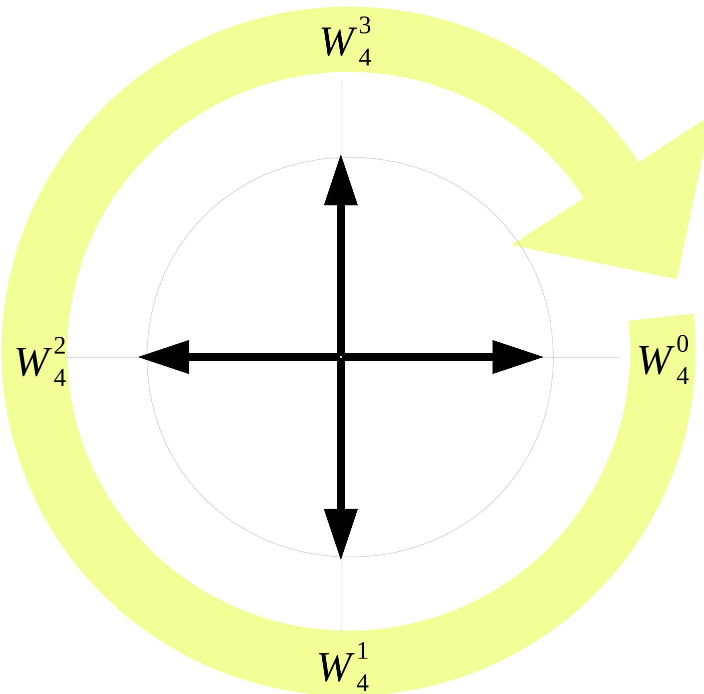
$$W_N^{nk} \triangleq e^{-j(2\pi/N)nk}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn} \quad \Longleftrightarrow \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{-kn}$$

Complex Phase Factor (1)

$$W_4^{k} = e^{-j\left(\frac{2\pi}{4}\right)k}$$

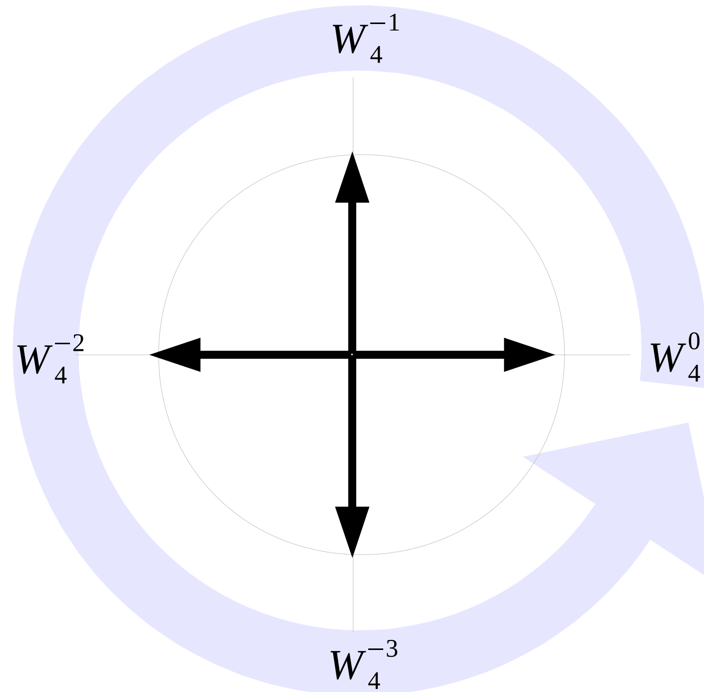
$$W_4^{-k} = e^{+j\left(\frac{2\pi}{4}\right)k}$$



$$W_4^1 = W_4^{-3}$$

$$W_4^2 = W_4^{-2}$$

$$W_4^3 = W_4^{-1}$$

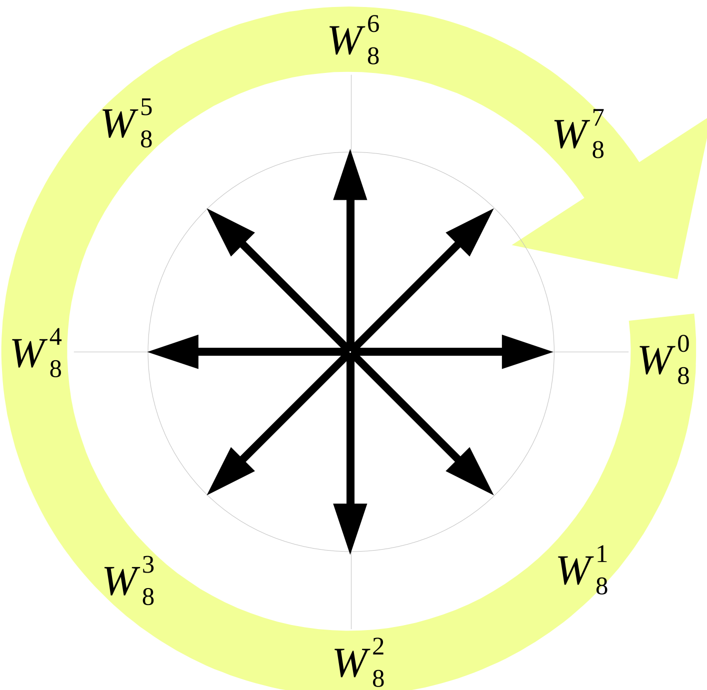


$$W_N^{k-N} = W_N^{k}$$

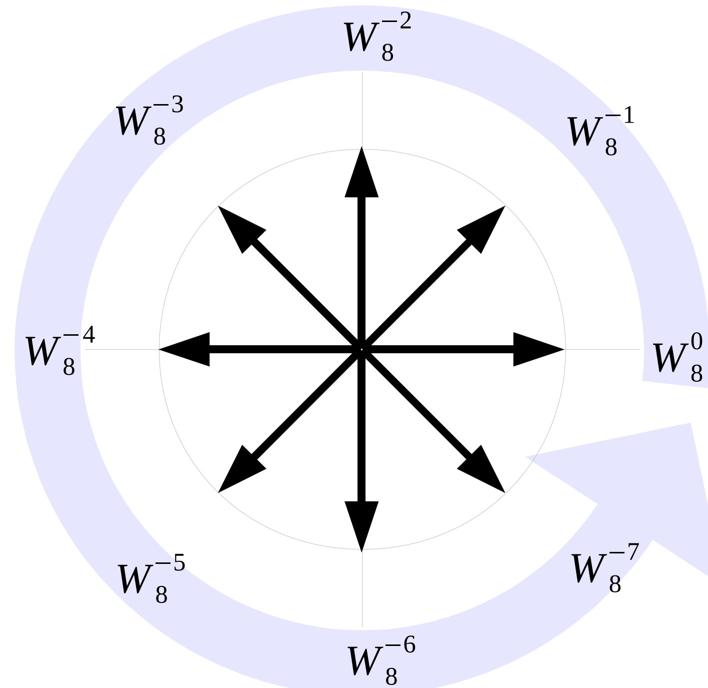
Complex Phase Factor (2)

$$W_8^{k} = e^{-j\left(\frac{2\pi}{8}\right)k}$$

$$W_8^{-k} = e^{+j\left(\frac{2\pi}{8}\right)k}$$



$$W_8^{1} = W_8^{-7}$$
$$W_8^{2} = W_8^{-6}$$
$$W_8^{3} = W_8^{-5}$$
$$W_8^{4} = W_8^{-4}$$
$$W_8^{5} = W_8^{-3}$$
$$W_8^{6} = W_8^{-2}$$
$$W_8^{7} = W_8^{-1}$$



$$W_N^{k-N} = W_N^{k}$$

Complex Phase Factor (3)

$$W_N^{k} = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{-k} = e^{+j\left(\frac{2\pi}{N}\right)k}$$

$$W_N^{k-N} = W_N^{k}$$

$$W_N^{k+N} = W_N^{k}$$

$$W_N^{k-N} = e^{-j\left(\frac{2\pi}{N}\right)(k-N)}$$

$$\frac{W_N^{k-N}}{W_N^{k}} = \frac{e^{-j\left(\frac{2\pi}{N}\right)(k-N)}}{e^{-j\left(\frac{2\pi}{N}\right)k}} = e^{j2\pi} = 1$$

$$W_N^{k+N} = e^{-j\left(\frac{2\pi}{N}\right)(k+N)}$$

$$\frac{W_N^{k+N}}{W_N^{k}} = \frac{e^{-j\left(\frac{2\pi}{N}\right)(k+N)}}{e^{-j\left(\frac{2\pi}{N}\right)k}} = e^{-j2\pi} = 1$$

$$W_N^{kN} = 1$$

$$W_N^{-kN} = 1$$

$$W_N^{kN} = e^{-j\left(\frac{2\pi}{N}\right)kN} = e^{-j2\pi k} = 1$$

$$W_N^{kN} = e^{+j\left(\frac{2\pi}{N}\right)kN} = e^{+j2\pi k} = 1$$

DFT Symmetry

$$X^*[k] = X[N-k]$$

$$X^*[k]$$

$$= \sum_{n=0}^{N-1} x[n]\, W_N^{-kn}$$

$$= \sum_{n=0}^{N-1} x[n]\, W_N^{nN}\, W_N^{-kn}$$

$$= \sum_{n=0}^{N-1} x[n]\, W_N^{n(N-k)}$$

$$= X[N-k]$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}$$

$$W_N^{nN} = 1$$

$$W_N^{k-N} = W_N^{k}$$

N=4 DFT

$$X[k] = \sum_{n=0}^{3} x[n]\, W_4^{kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

N=8 DFT

$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

N=8 DFT Matrix (1)

$n \cdot k$ ***mod*** **8**

$$W_N^{nk} = e^{-j(2\pi/N)nk}$$

N = 8

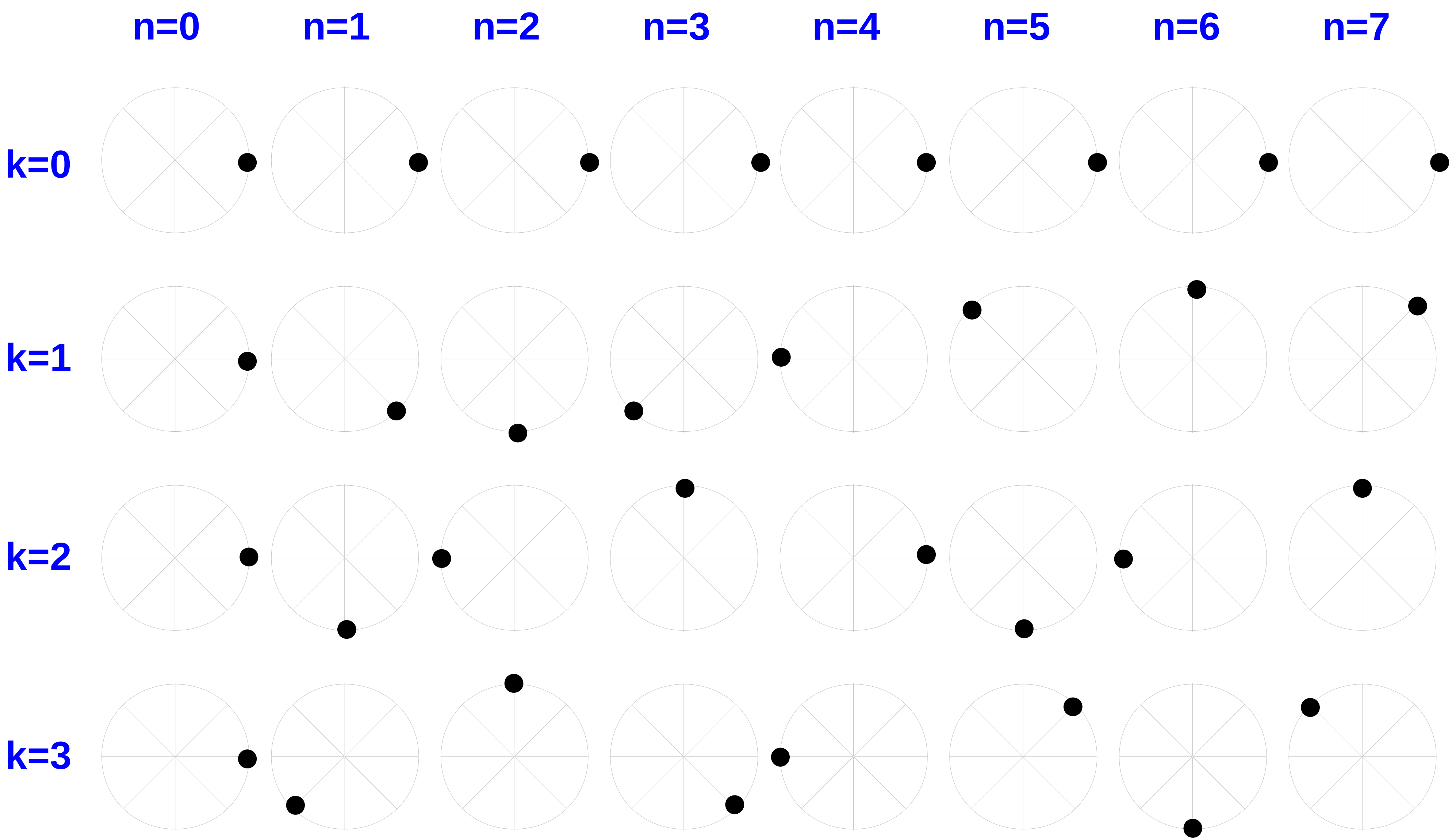
k \ n	0	1	2	3	4	5	6	7
0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1	0 0	-1 7	-2 6	-3 5	-4 4	-5 3	-6 2	-7 1
2	0 0	-2 6	-4 4	-6 2	-8 0	-10 6	-12 4	-14 2
3	0 0	-3 5	-6 2	-9 7	-12 4	-15 1	-18 6	-21 3
4	0 0	-4 4	-8 0	-12 4	-16 0	-20 4	-24 0	-28 4
5	0 0	-5 3	-10 6	-15 1	-20 4	-25 7	-30 2	-35 5
6	0 0	-6 2	-12 4	-18 6	-24 0	-30 2	-36 4	-42 6
7	0 0	-7 1	-14 2	-21 3	-28 4	-35 5	-42 6	-49 7

N=8 DFT Matrix (2)

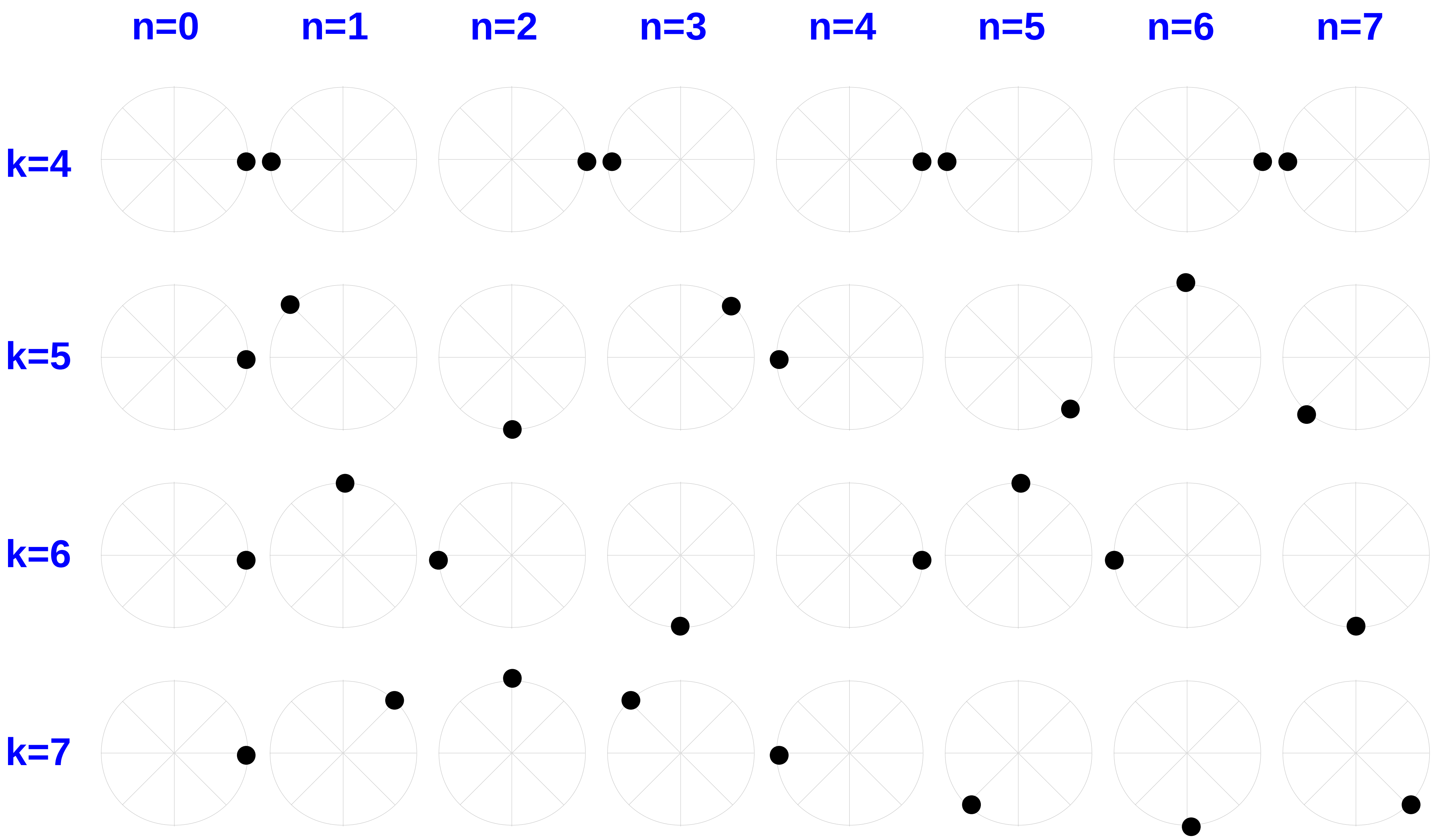
$$X[k] = \sum_{n=0}^{7} W_8^{kn}\, x[n] \qquad W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix}
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 1} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 2} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 6} \\
e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 7}
\end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix}$$

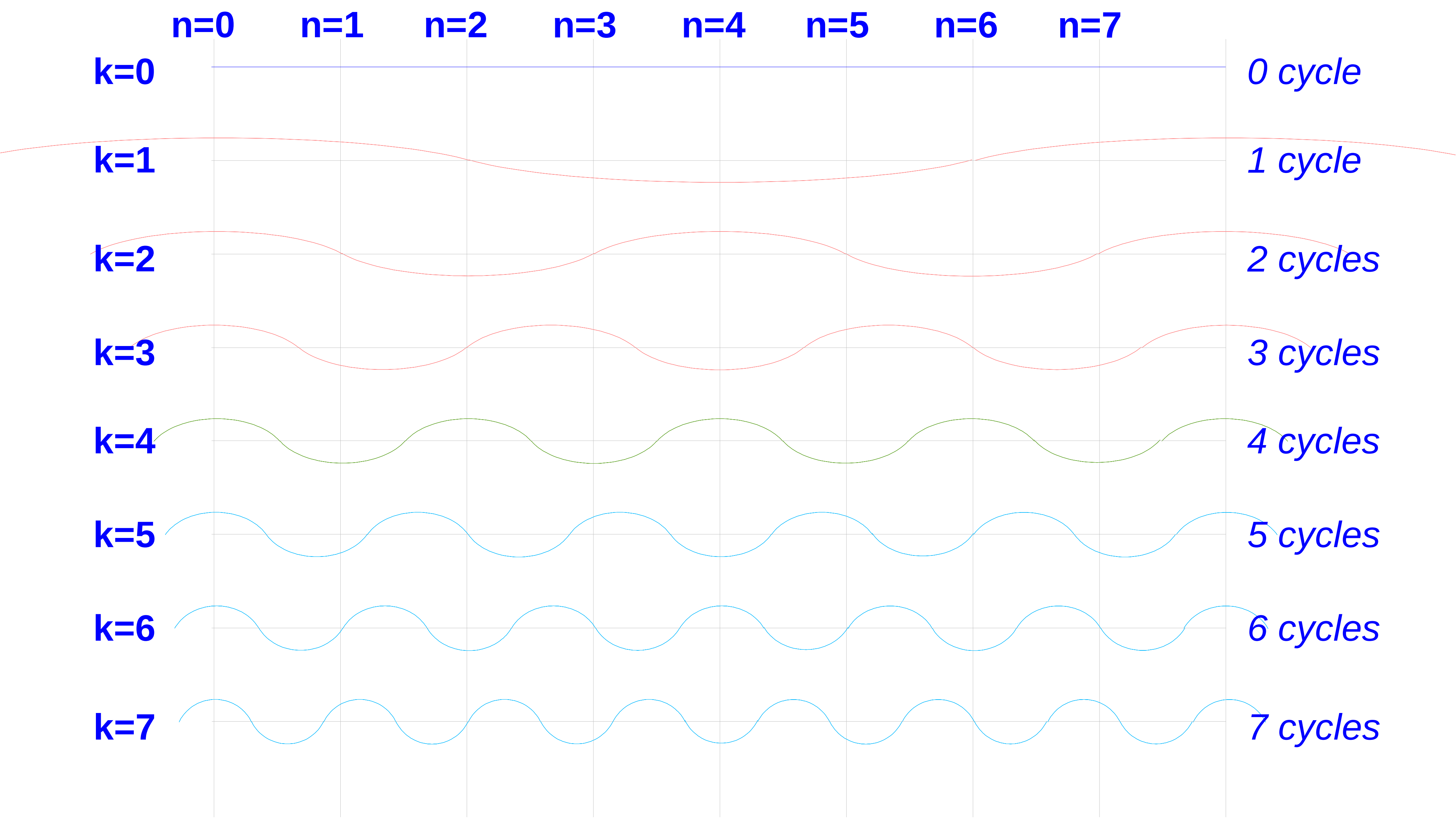
N=8 DFT Complex Factors (1)



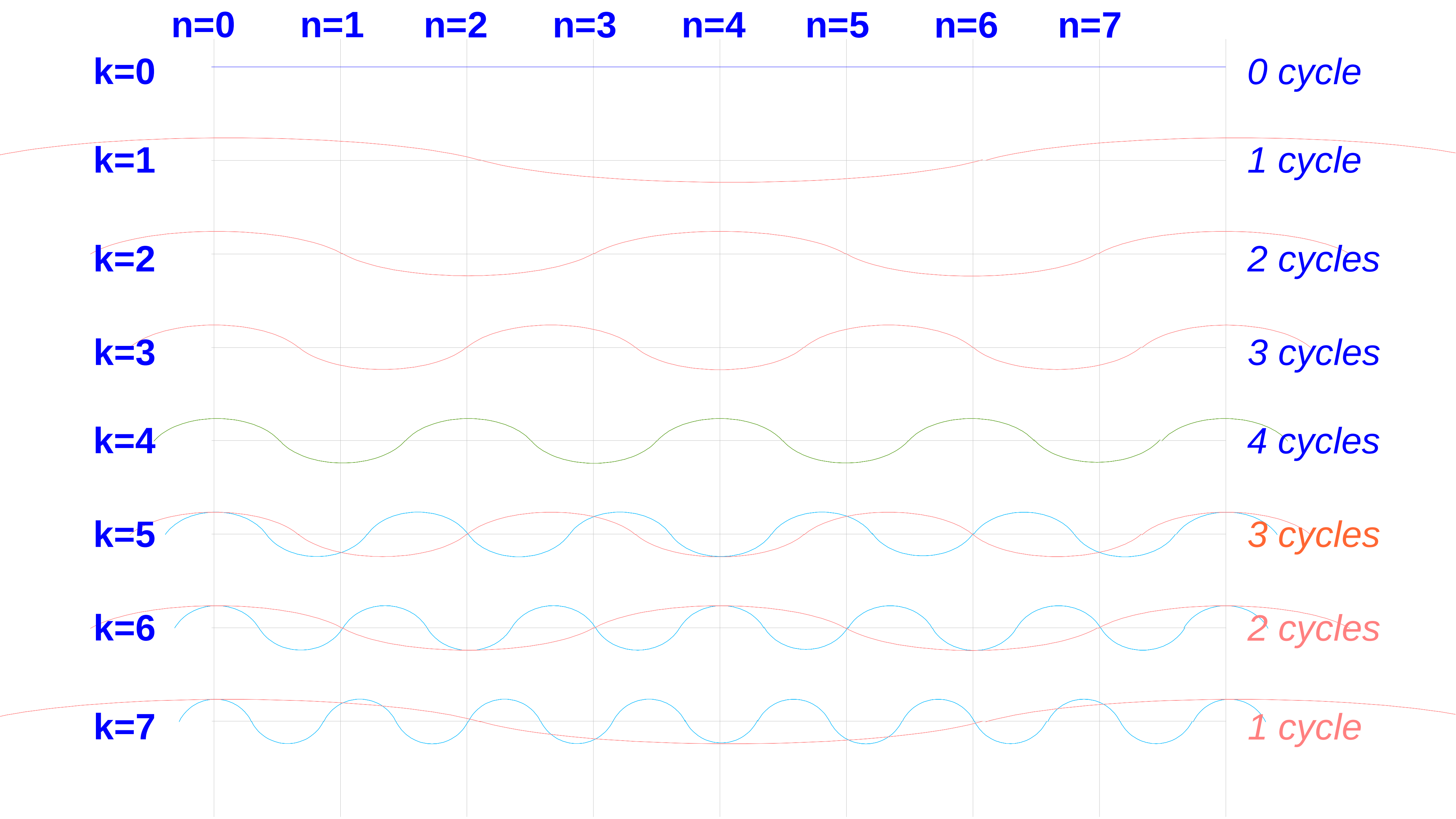
N=8 DFT Complex Factors (2)



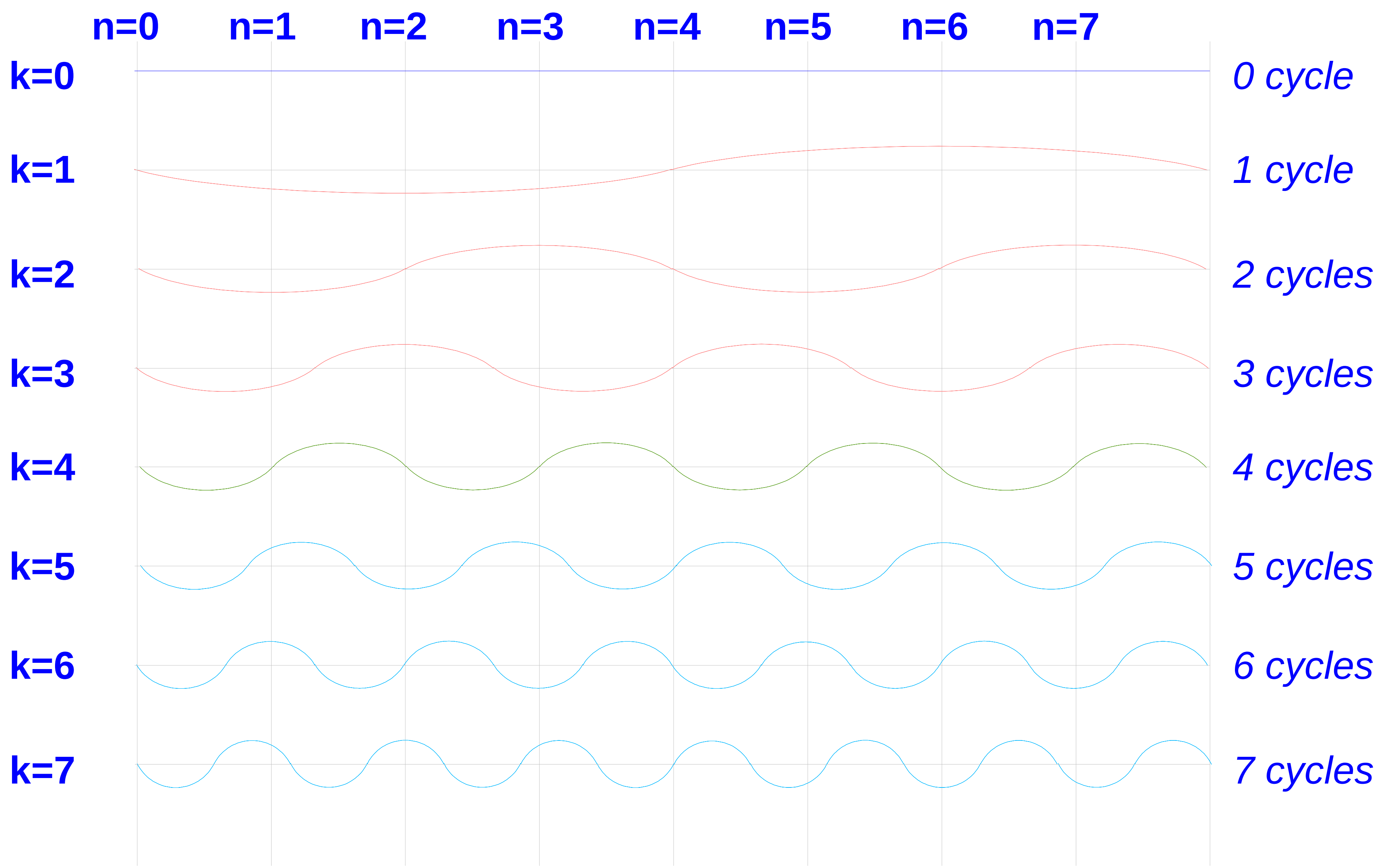
N=8 DFT Real Factors - (1)



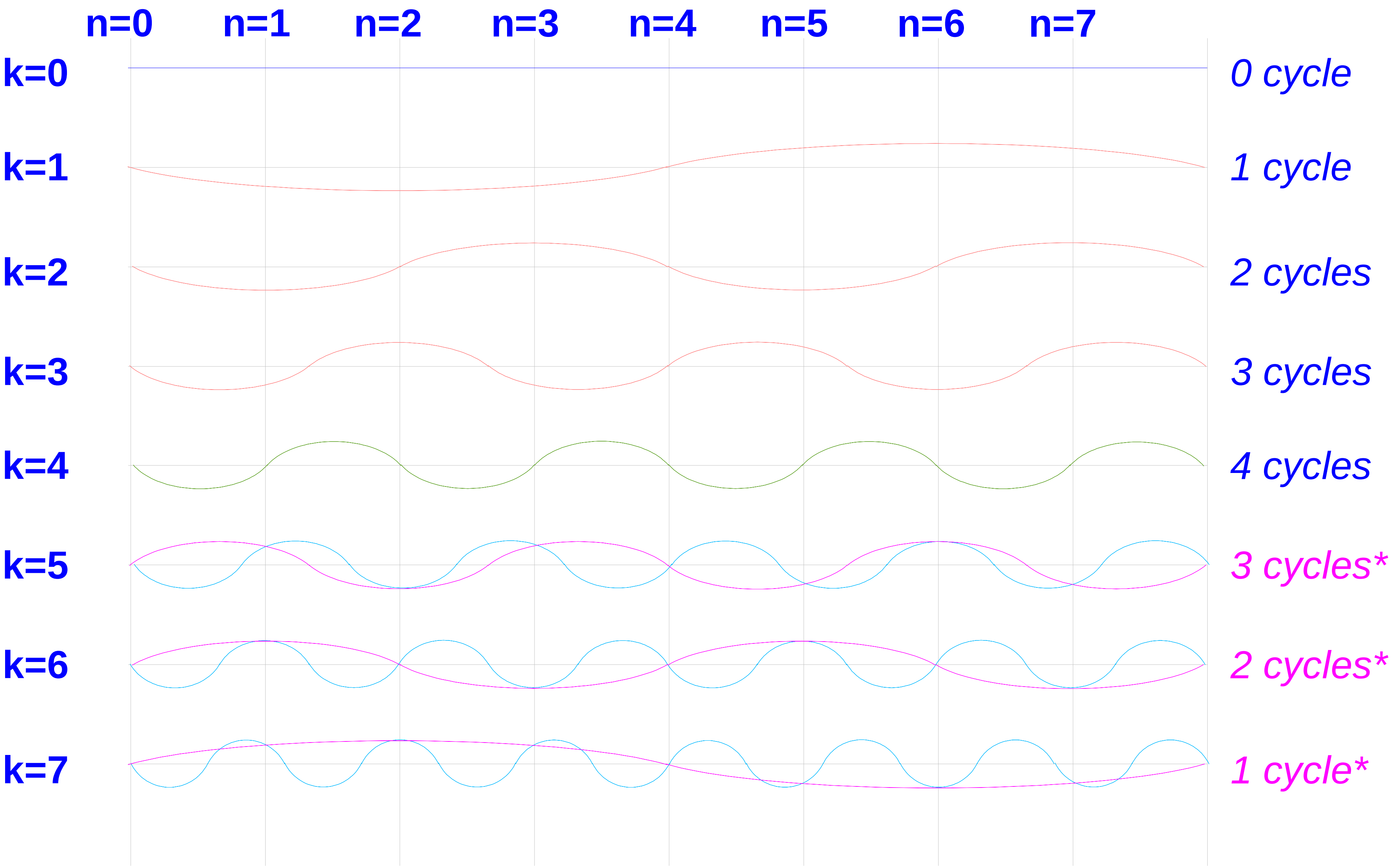
N=8 DFT Real Factors - (2)



N=8 DFT Imaginary Factors - (1)



N=8 DFT Imaginary Factors - (2)



N=8 IDFT

$$x[n] = \frac{1}{N}\sum_{k=0}^{7} W_8^{-kn}\, X[k] \qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} & W_8^{0} \\ W_8^{0} & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7} \\ W_8^{0} & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^{-8} & W_8^{-10} & W_8^{-12} & W_8^{-14} \\ W_8^{0} & W_8^{-3} & W_8^{-6} & W_8^{-9} & W_8^{-12} & W_8^{-15} & W_8^{-18} & W_8^{-21} \\ W_8^{0} & W_8^{-4} & W_8^{-8} & W_8^{-12} & W_8^{-16} & W_8^{-20} & W_8^{-24} & W_8^{-28} \\ W_8^{0} & W_8^{-5} & W_8^{-10} & W_8^{-15} & W_8^{-20} & W_8^{-25} & W_8^{-30} & W_8^{-35} \\ W_8^{0} & W_8^{-6} & W_8^{-12} & W_8^{-18} & W_8^{-24} & W_8^{-30} & W_8^{-36} & W_8^{-42} \\ W_8^{0} & W_8^{-7} & W_8^{-14} & W_8^{-21} & W_8^{-28} & W_8^{-35} & W_8^{-42} & W_8^{-49} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix}$$

N=8 IDFT Matrix (1)

$n \cdot k \;\; mod \;\; 8$

$W_N^{-nk} =$

$e^{+j(2\pi/N)nk}$

N = 8

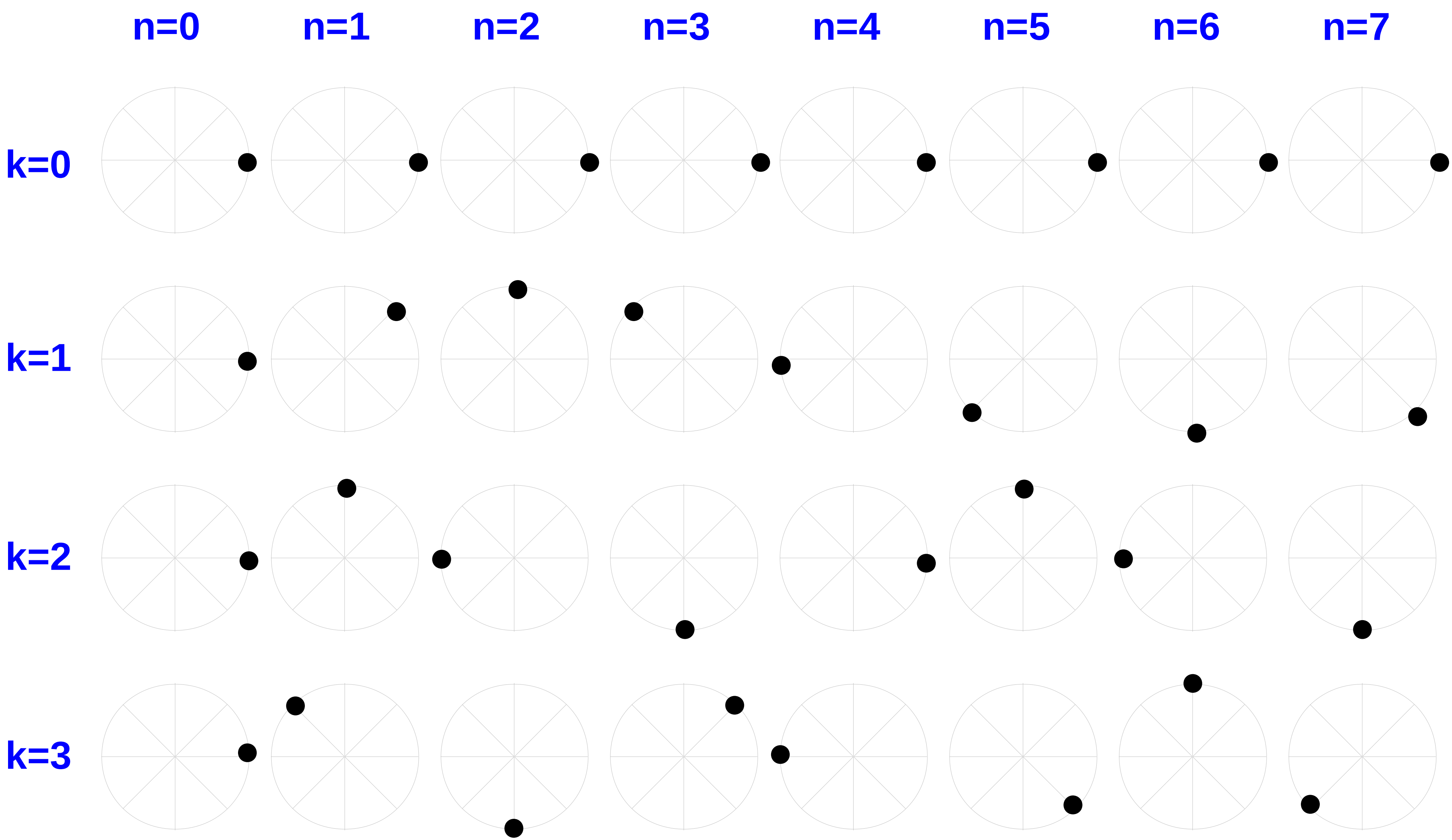
k \ n	0	1	2	3	4	5	6	7
0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1	0 0	1 1	2 2	3 3	4 4	5 5	6 6	7 7
2	0 0	2 2	4 4	6 6	8 0	10 2	12 4	14 6
3	0 0	3 3	6 6	9 1	12 4	15 7	18 2	21 5
4	0 0	4 4	8 0	12 4	16 0	20 4	24 0	28 4
5	0 0	5 5	10 2	15 7	20 4	25 1	30 6	35 3
6	0 0	6 6	12 4	18 2	24 0	30 6	36 4	42 2
7	0 0	7 7	14 6	21 5	28 4	35 3	42 2	49 1

N=8 IDFT Matrix (2)

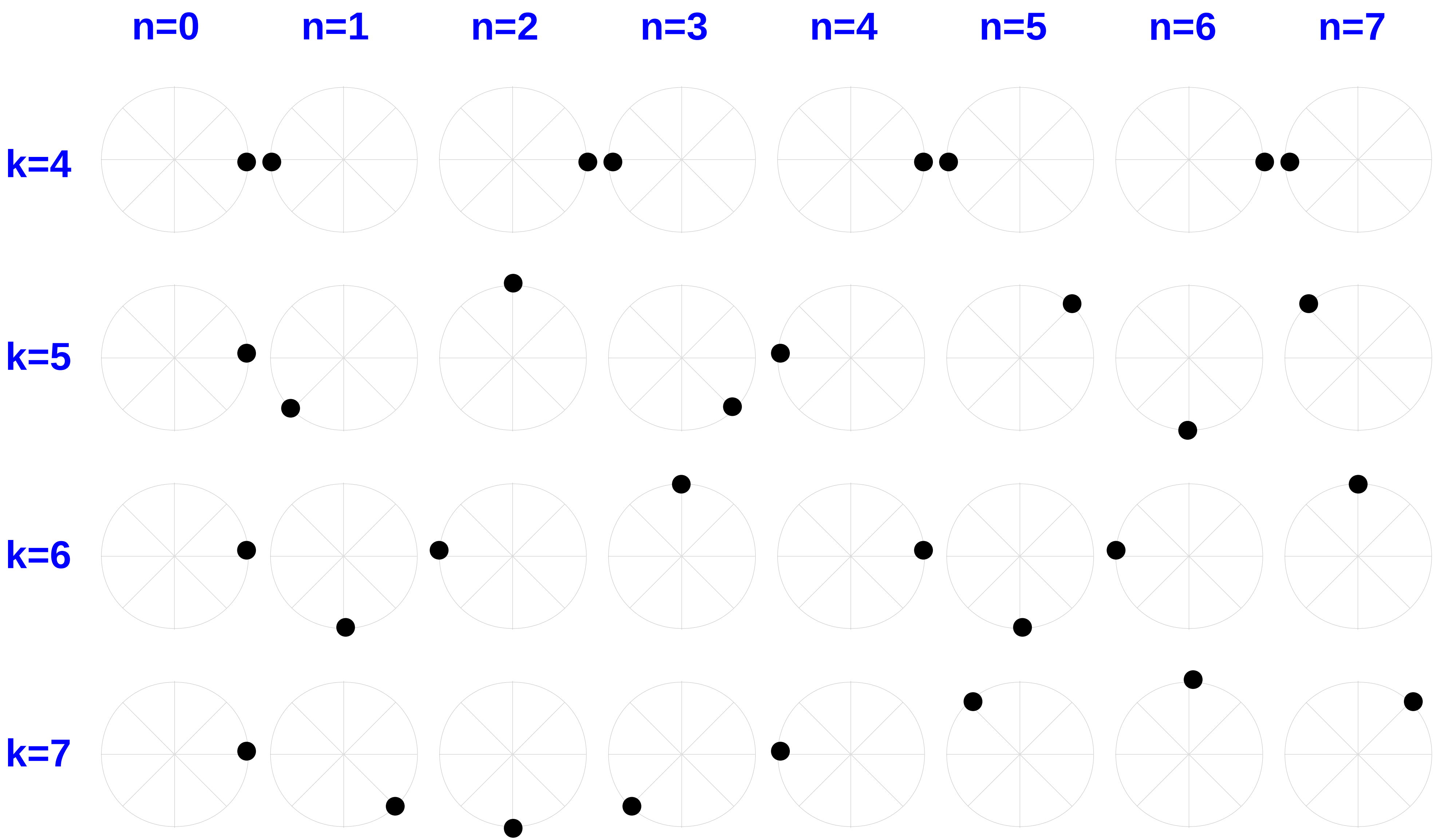
$$x[n] = \frac{1}{N}\sum_{k=0}^{7} W_8^{-kn}\, X[k] \qquad W_8^{-kn} = e^{+j(\frac{2\pi}{8})kn}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \begin{bmatrix}
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 0} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 7} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 6} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 5} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 4} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 1} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 3} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 2} \\
e^{+j\cdot\frac{\pi}{4}\cdot 0} & e^{+j\cdot\frac{\pi}{4}\cdot 7} & e^{+j\cdot\frac{\pi}{4}\cdot 6} & e^{+j\cdot\frac{\pi}{4}\cdot 5} & e^{+j\cdot\frac{\pi}{4}\cdot 4} & e^{+j\cdot\frac{\pi}{4}\cdot 3} & e^{+j\cdot\frac{\pi}{4}\cdot 2} & e^{+j\cdot\frac{\pi}{4}\cdot 1}
\end{bmatrix} \begin{bmatrix} \frac{X[0]}{N} \\ \frac{X[1]}{N} \\ \frac{X[2]}{N} \\ \frac{X[3]}{N} \\ \frac{X[4]}{N} \\ \frac{X[5]}{N} \\ \frac{X[6]}{N} \\ \frac{X[7]}{N} \end{bmatrix}$$

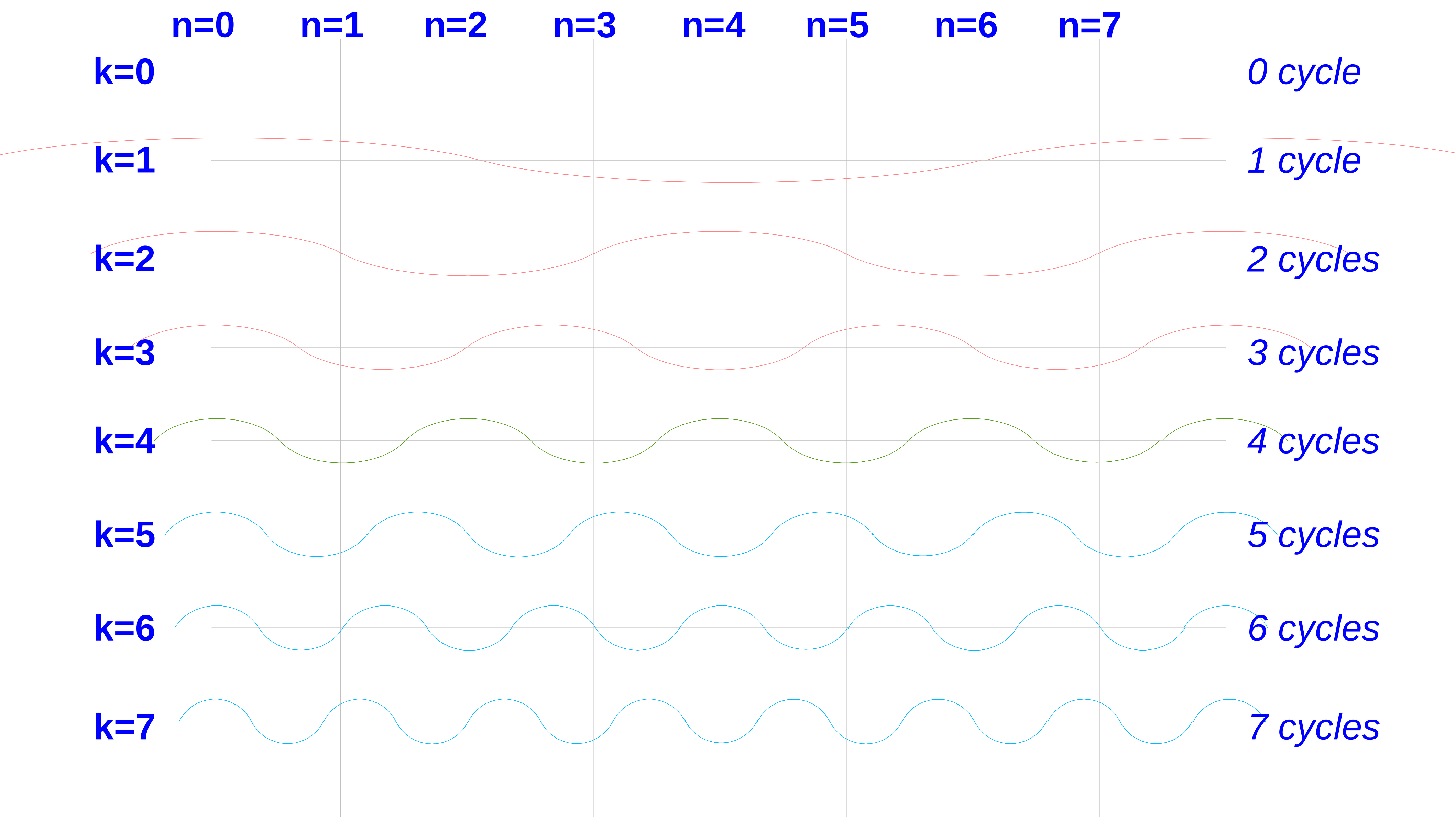
N=8 IDFT Complex Factors (1)



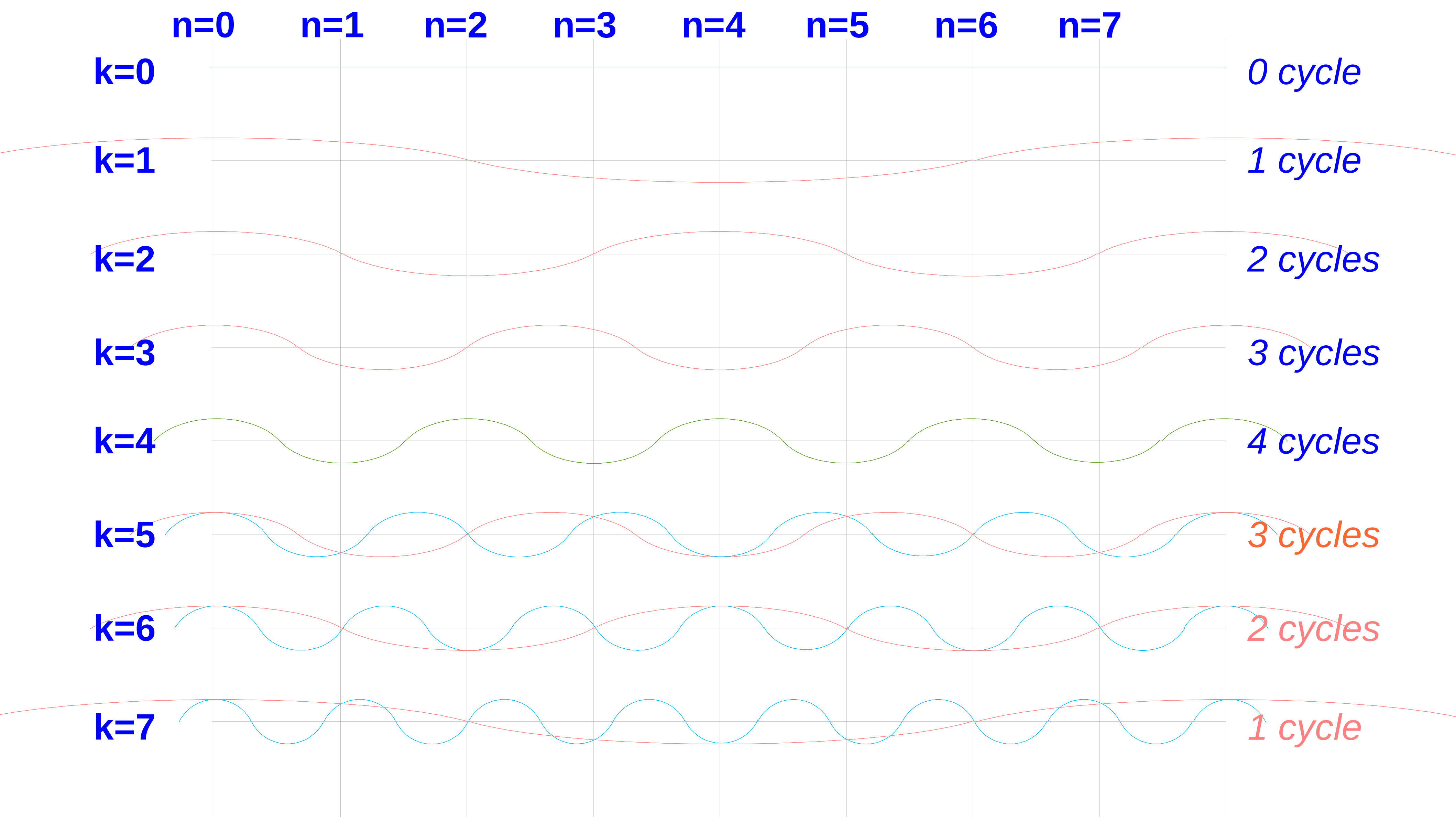
N=8 IDFT Complex Factors (2)



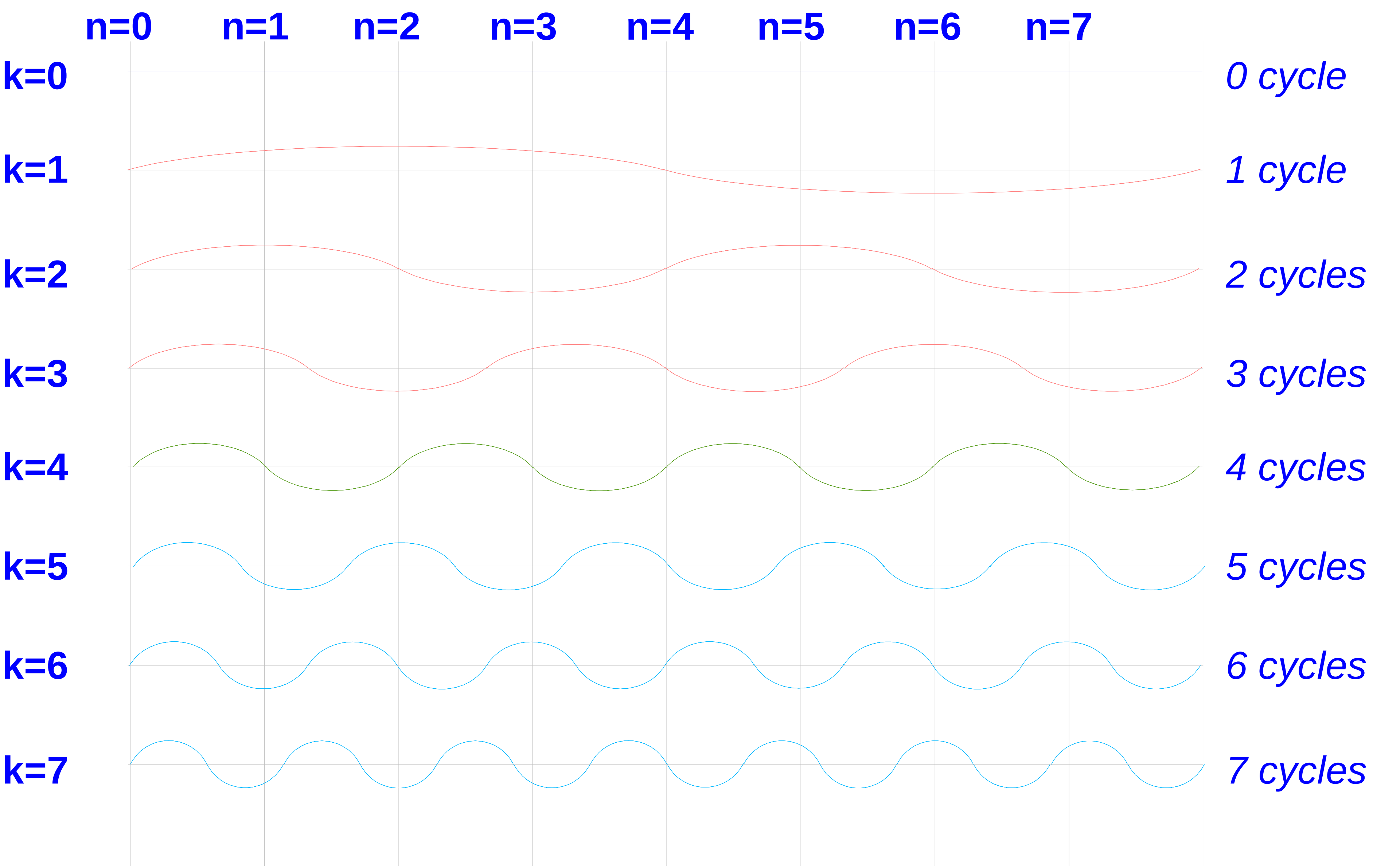
N=8 IDFT Real Factors - (1)



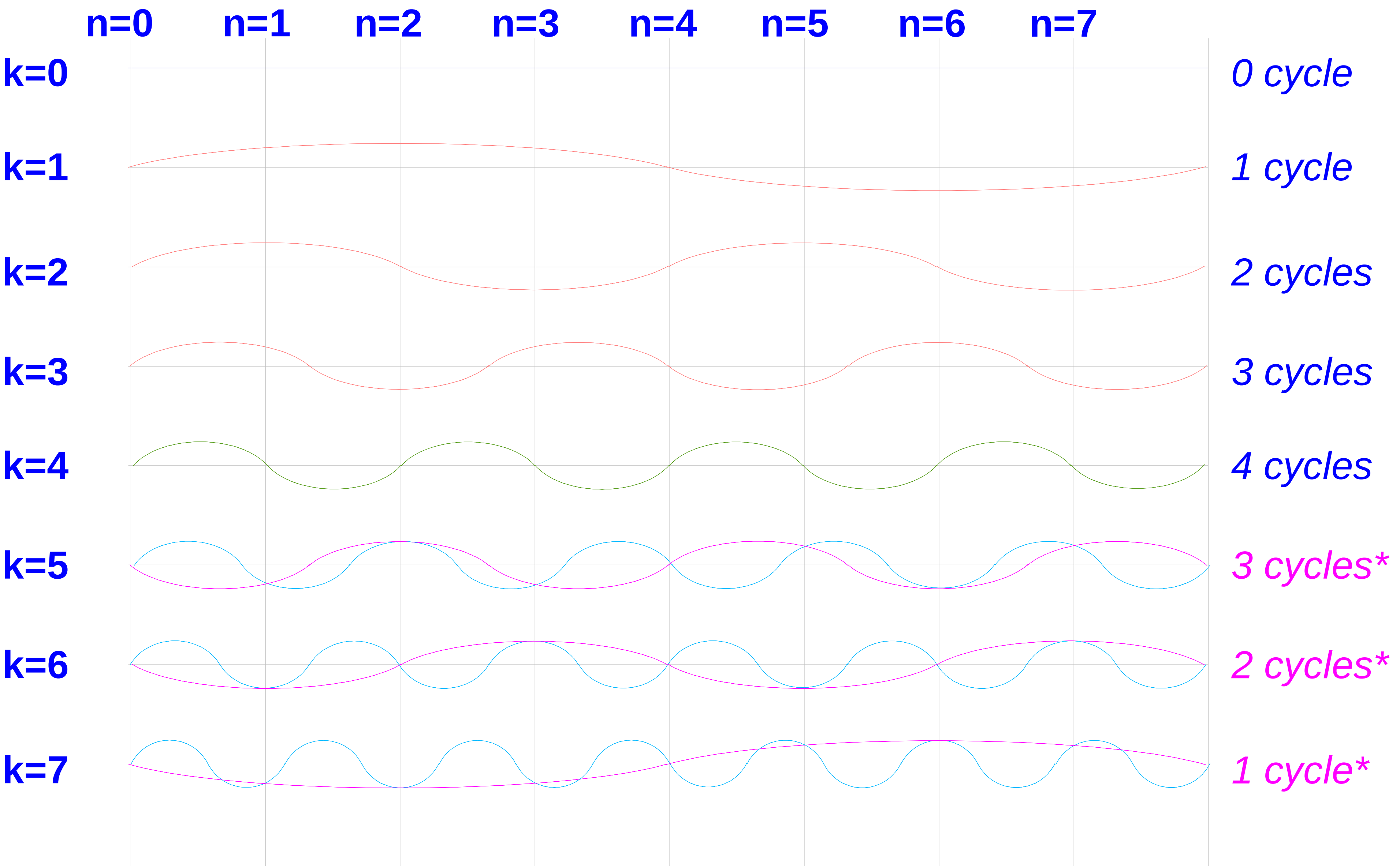
N=8 IDFT Real Factors - (2)



N=8 IDFT Imaginary Factors - (1)



N=8 IDFT Imaginary Factors - (2)



N=8 DFT & IDFT Matrix (1)

	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7
k=0	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$
k=1	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$
k=2	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$
k=3	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$

N=8 DFT & IDFT Matrix (2)

	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7
k=0	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$
k=1	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$
k=2	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$
k=3	$e^{-j\cdot\frac{\pi}{4}\cdot 0}$	$e^{-j\cdot\frac{\pi}{4}\cdot 1}$	$e^{-j\cdot\frac{\pi}{4}\cdot 2}$	$e^{-j\cdot\frac{\pi}{4}\cdot 3}$	$e^{-j\cdot\frac{\pi}{4}\cdot 4}$	$e^{-j\cdot\frac{\pi}{4}\cdot 5}$	$e^{-j\cdot\frac{\pi}{4}\cdot 6}$	$e^{-j\cdot\frac{\pi}{4}\cdot 7}$
	$e^{+j\cdot\frac{\pi}{4}\cdot 0}$	$e^{+j\cdot\frac{\pi}{4}\cdot 7}$	$e^{+j\cdot\frac{\pi}{4}\cdot 6}$	$e^{+j\cdot\frac{\pi}{4}\cdot 5}$	$e^{+j\cdot\frac{\pi}{4}\cdot 4}$	$e^{+j\cdot\frac{\pi}{4}\cdot 3}$	$e^{+j\cdot\frac{\pi}{4}\cdot 2}$	$e^{+j\cdot\frac{\pi}{4}\cdot 1}$

References

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