

3.4.a

$$I = \int_0^{\pi} e^x \cdot \cos x \, dx = \int_0^{\pi} f(x) \, dx$$

$$f'(x) = \frac{d}{dx} e^x \cdot \cos x = e^x \cdot \cos x - e^x \sin x \\ = e^x (\cos x - \sin x)$$

$$f'(0) = 1 \quad f'(\pi) = -e^{\pi} \quad f(0) = 1 \quad f(\pi) = -e^{\pi}$$

$$I = \int_0^{\pi} e^x \cos x \, dx = \int_0^{\pi} e^x \sin(x) \, dx + e^x \cos x$$

$$dv = e^x dx \rightarrow v = \int e^x dx = e^x \quad \int u dv = uv - \int v du$$

$$\int_0^{\pi} e^x \sin x \, dx = e^x \sin x - \int_0^{\pi} e^x \cos x \, dx$$

$$\rightarrow \int_0^{\pi} e^x \cos x \, dx = e^x (\sin x + \cos x) - \int_0^{\pi} e^x \cos x \, dx$$

$$\rightarrow 2 \int_0^{\pi} e^x \cos x \, dx = e^x (\sin x + \cos x)$$

$$I = \int_0^{\pi} e^x \cos x \, dx = \frac{e^x}{2} (\sin x + \cos x) + \text{const} \Big|_0^{\pi}$$

$$= -\frac{1}{2} - \frac{e^{\pi}}{2} = -\frac{1}{2} (e^{\pi} + 1) //$$