

# Convolution (1A)

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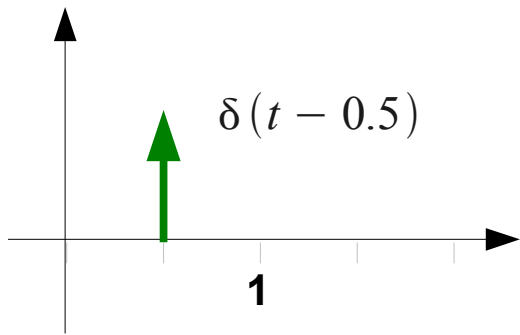
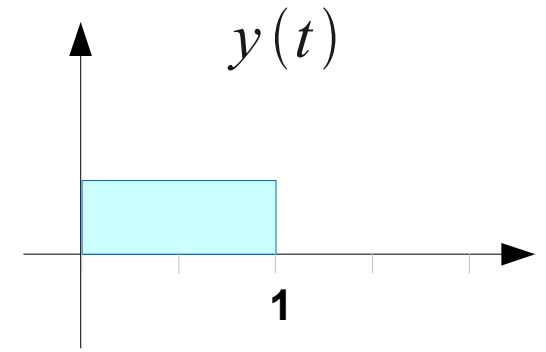
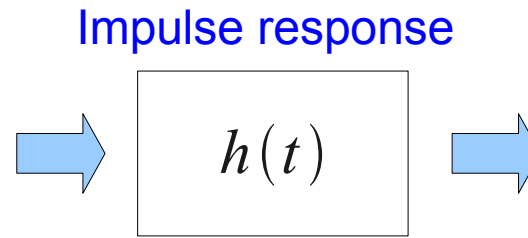
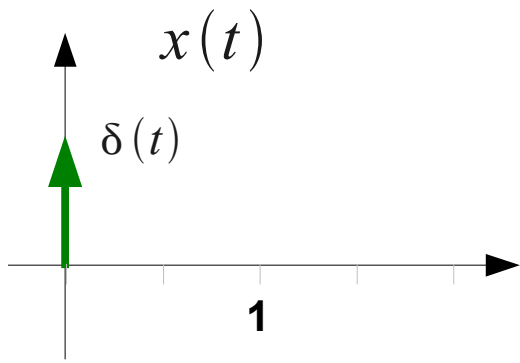
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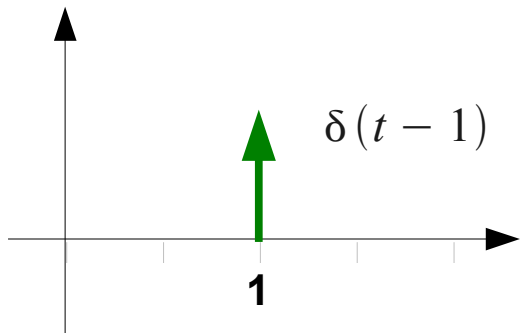
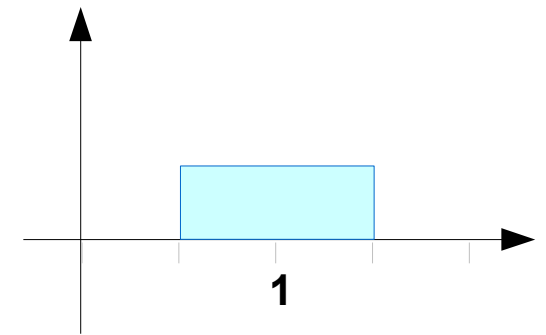
Please send corrections (or suggestions) to [youngwlim@hotmail.com](mailto:youngwlim@hotmail.com).

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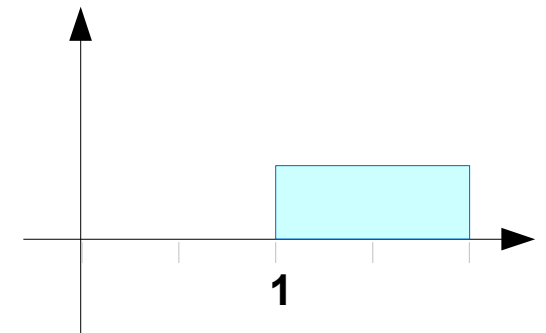
# Impulse Response



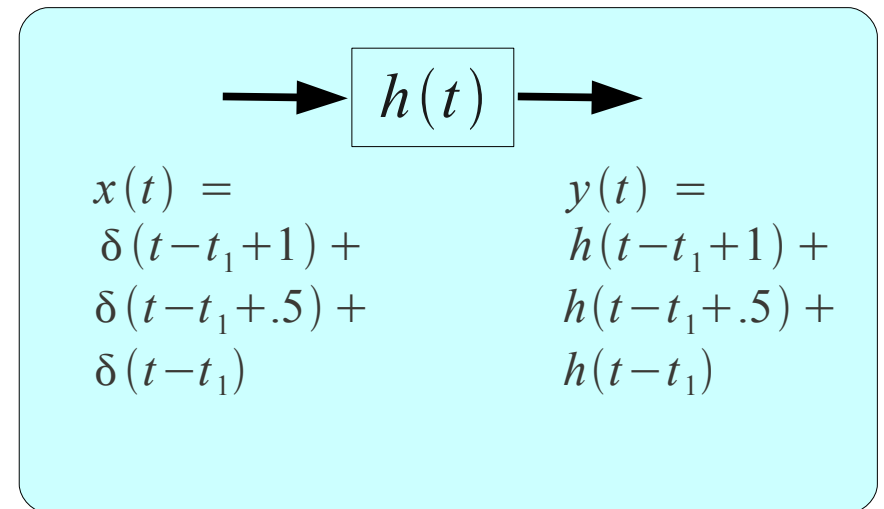
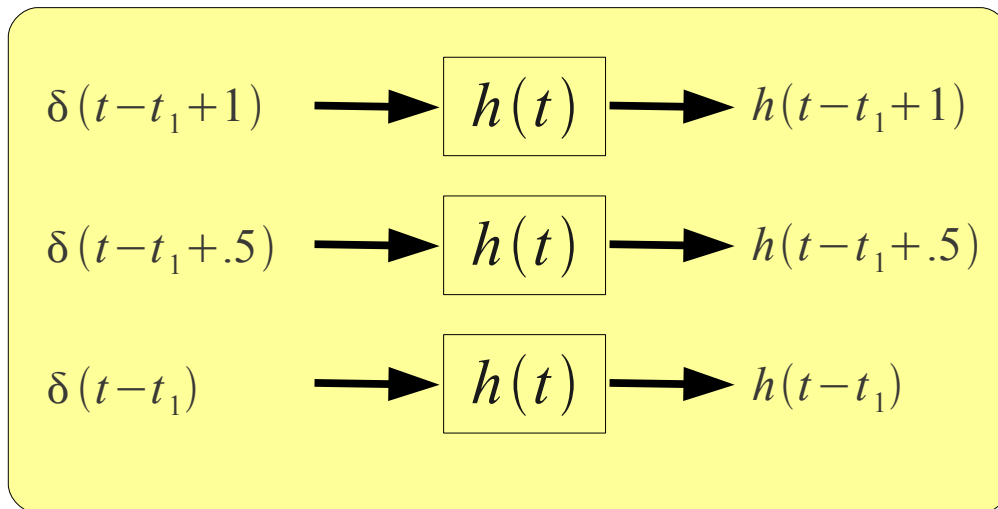
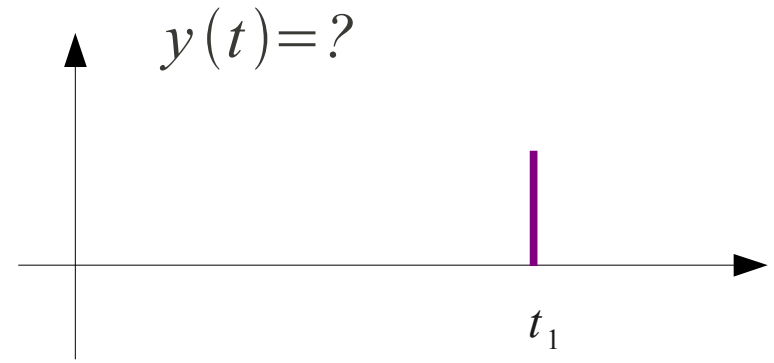
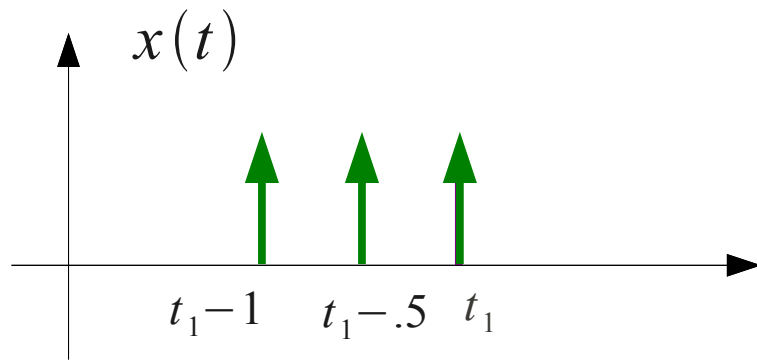
delayed response  
by 0.5



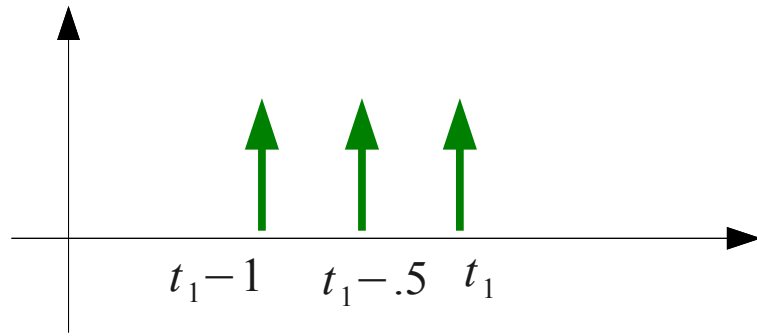
delayed response  
by 1



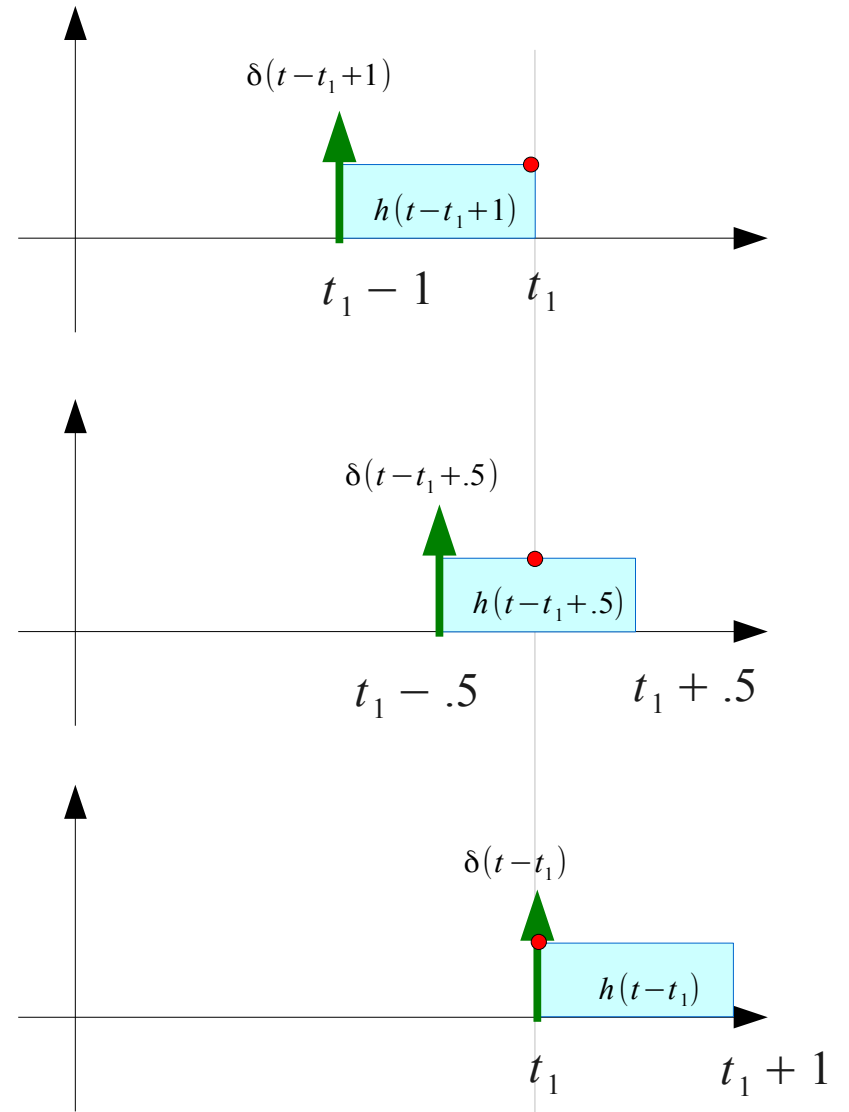
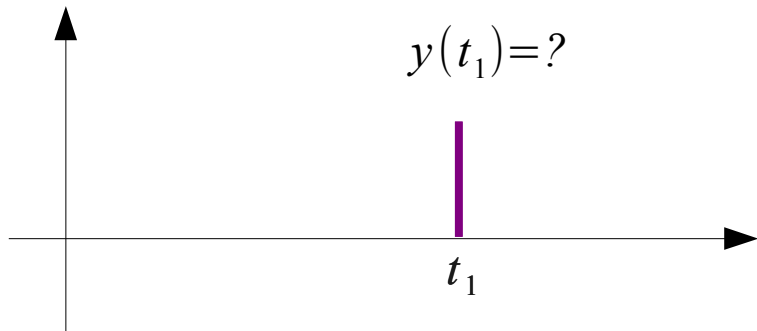
# LTI System



# Output at $t = t_1$



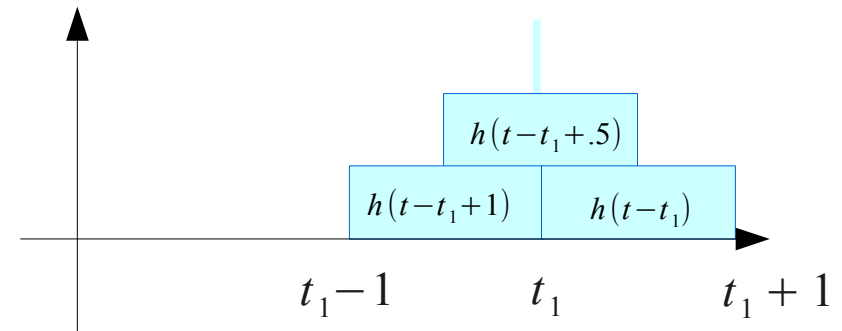
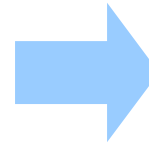
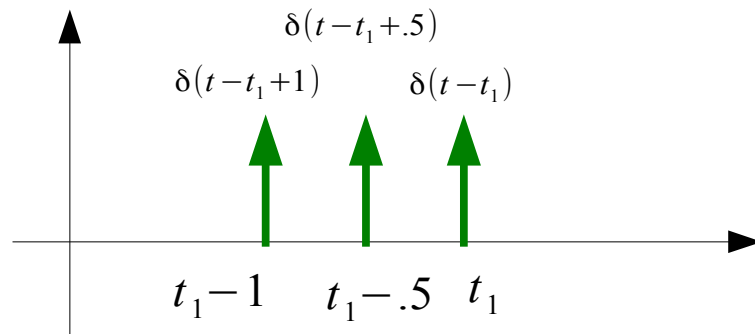
$$x(t) = \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1)$$



# Using Convolution

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$



$$y(t) = \int x(v)h(t-v) dv$$

$$= \int \delta(v-t_1+1)h(t-v) dv \quad \rightarrow \quad h(t-t_1+1) \quad \text{impulse response delayed by } t_1-1$$

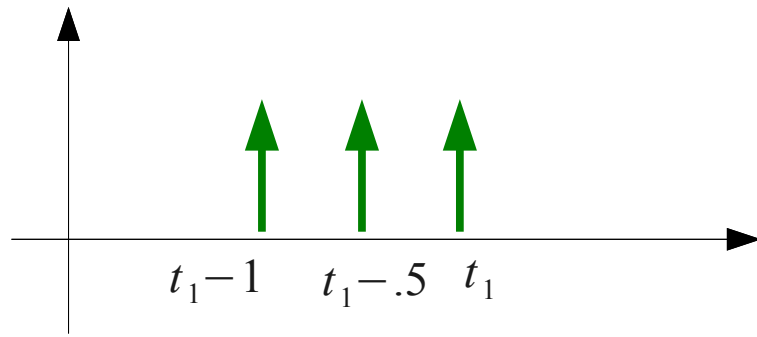
$$+ \int \delta(v-t_1+.5)h(t-v) dv \quad \rightarrow \quad h(t-t_1+.5) \quad \text{impulse response delayed by } t_1-.5$$

$$+ \int \delta(v-t_1)h(t-v) dv \quad \rightarrow \quad h(t-t_1) \quad \text{impulse response delayed by } t_1$$

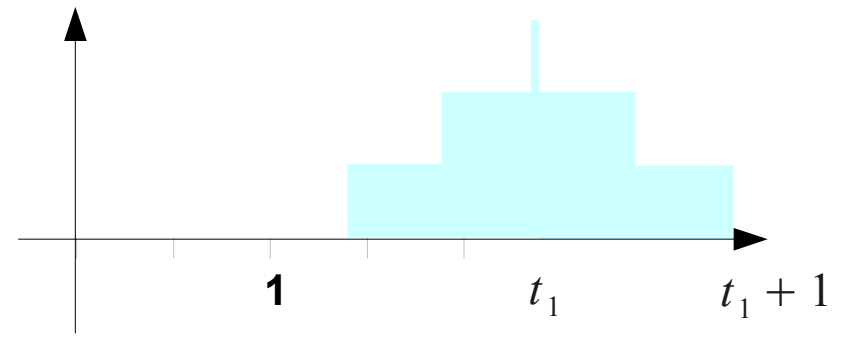
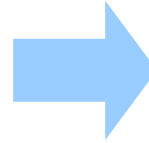
$$y(t_1) = h(t_1-t_1+1) + h(t_1-t_1+.5) + h(t_1-t_1)$$

$$= h(1) + h(.5) + h(0)$$

# N=8 DFT

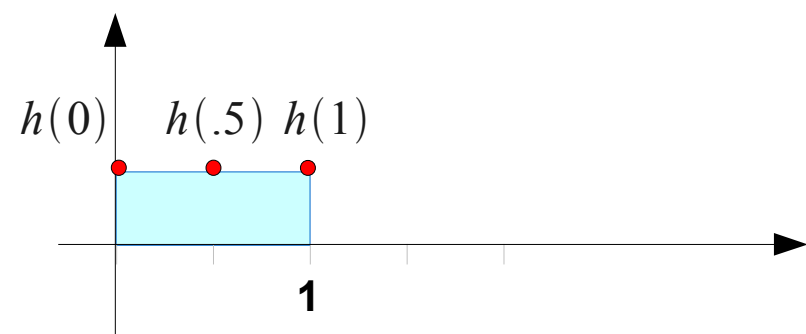
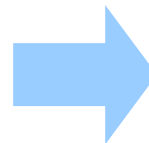
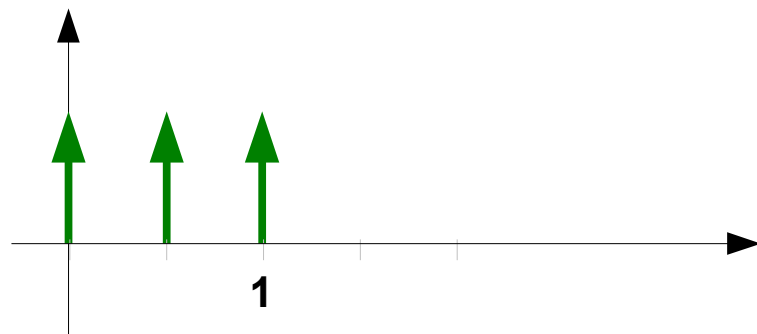


$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$



$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$x(t) = \delta(t) + \delta(t-.5) + \delta(t-1)$$



$$y(t_1) = h(1) + h(.5) + h(0)$$

# The Computation of Convolution (1)

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

↓ Change of variables  $t \rightarrow v$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

↓ Flip around y axis and then shift to the right by  $t$   $v \rightarrow t-v$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

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$$y(t) = \int x(t-v) h(v) dv$$

$$= \int \delta(t-v-t_1+1) h(v) dv \quad \rightarrow h(t-t_1+1) \text{ impulse response delayed by } t_1-1$$

$$+ \int \delta(t-v-t_1+.5) h(v) dv \quad \rightarrow h(t-t_1+.5) \text{ impulse response delayed by } t_1-.5$$

$$+ \int \delta(t-v-t_1) h(v) dv \quad \rightarrow h(t-t_1) \text{ impulse response delayed by } t_1$$

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$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\rightarrow y(t_1) = h(1) + h(.5) + h(0)$$

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# The Computation of Convolution (2)

$$h(t)$$

↓ Change of variables  $t \rightarrow v$

$$h(v)$$

↓ Flip around y axis and then shift to the right by  $t$   $v \rightarrow t-v$

$$h(t-v)$$

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$$y(t) = \int x(v)h(t-v) dv$$

$$= \int \delta(v-t_1+1)h(t-v) dv \rightarrow h(t-t_1+1) \text{ impulse response delayed by } t_1-1$$

$$+ \int \delta(v-t_1+.5)h(t-v) dv \rightarrow h(t-t_1+.5) \text{ impulse response delayed by } t_1-.5$$

$$+ \int \delta(v-t_1)h(t-v) dv \rightarrow h(t-t_1) \text{ impulse response delayed by } t_1$$

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$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\Rightarrow y(t_1) = h(1) + h(.5) + h(0)$$

# The Commutativity of Convolution (1)

$$y(t) = \int x(v)h(t-v) dv$$

$$= \int \delta(v-t_1+1)h(t-v) dv$$

$$+ \int \delta(v-t_1+.5)h(t-v) dv$$

$$+ \int \delta(v-t_1)h(t-v) dv$$

$$\rightarrow h(t-t_1+1) \leftarrow$$

$$\rightarrow h(t-t_1+.5) \leftarrow$$

$$\rightarrow h(t-t_1) \leftarrow$$

$$y(t) = \int x(t-v)h(v) dv$$

$$= \int \delta(t-v-t_1+1)h(v) dv$$

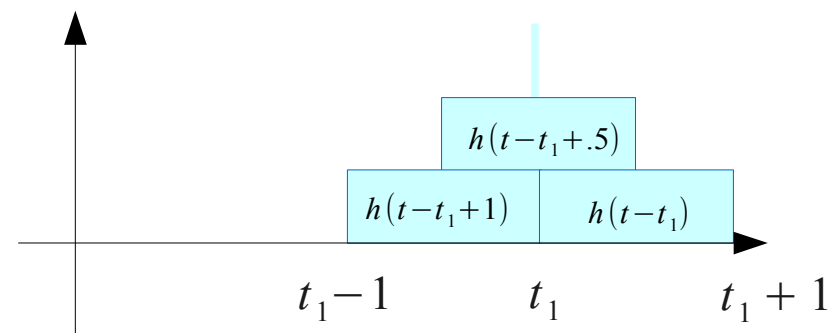
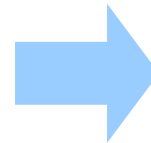
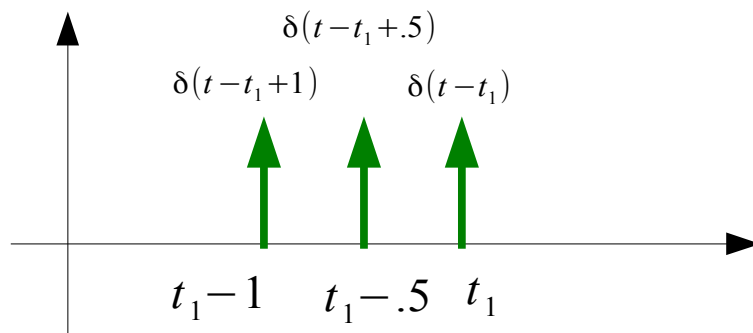
$$+ \int \delta(t-v-t_1+.5)h(v) dv$$

$$+ \int \delta(t-v-t_1)h(v) dv$$

## Sum of delayed impulse response

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$



# The Commutativity of Convolution (2)

$$y(t) = \int x(v)h(t-v) dv$$

$$= \int \delta(v-t_1+1)h(t-v) dv \quad \rightarrow \quad h(t-t_1+1) \quad \leftarrow$$

$$+ \int \delta(v-t_1+.5)h(t-v) dv \quad \rightarrow \quad h(t-t_1+.5) \quad \leftarrow$$

$$+ \int \delta(v-t_1)h(t-v) dv \quad \rightarrow \quad h(t-t_1) \quad \leftarrow$$

$$y(t) = \int x(t-v)h(v) dv$$

$$= \int \delta(t-v-t_1+1)h(v) dv$$

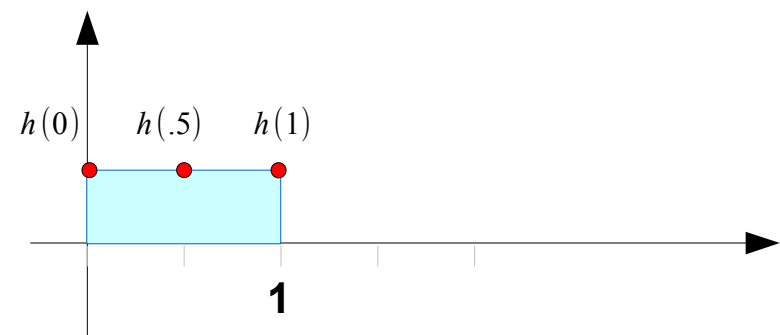
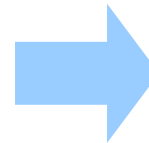
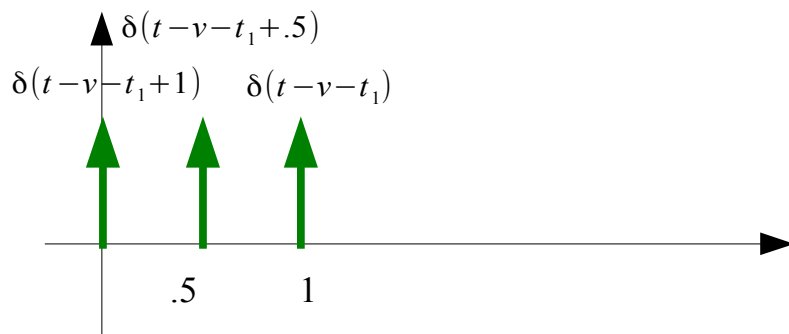
$$+ \int \delta(t-v-t_1+.5)h(v) dv$$

$$+ \int \delta(t-v-t_1)h(v) dv$$

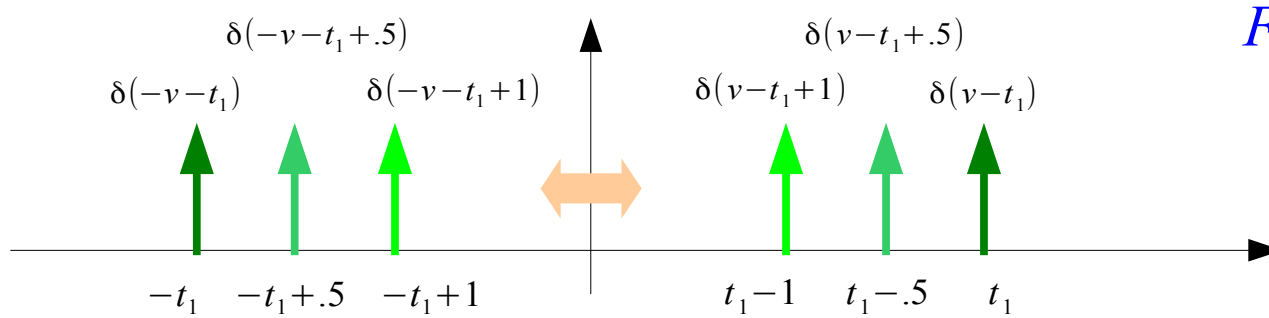
## Flip and shift input

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

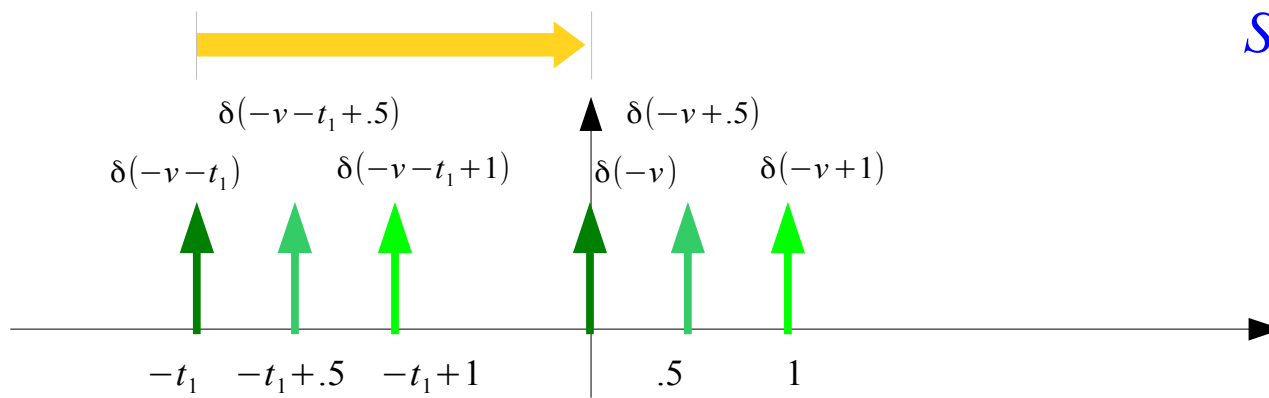
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$



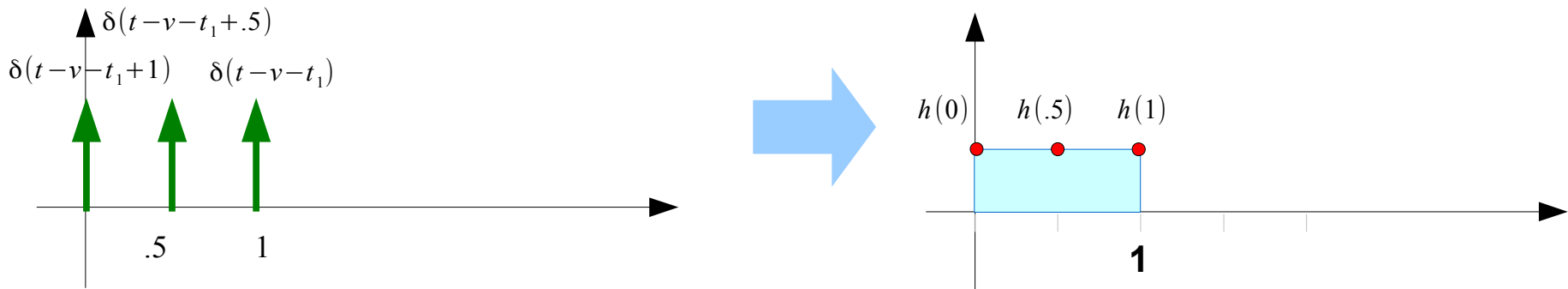
# The Commutativity of Convolution (3)



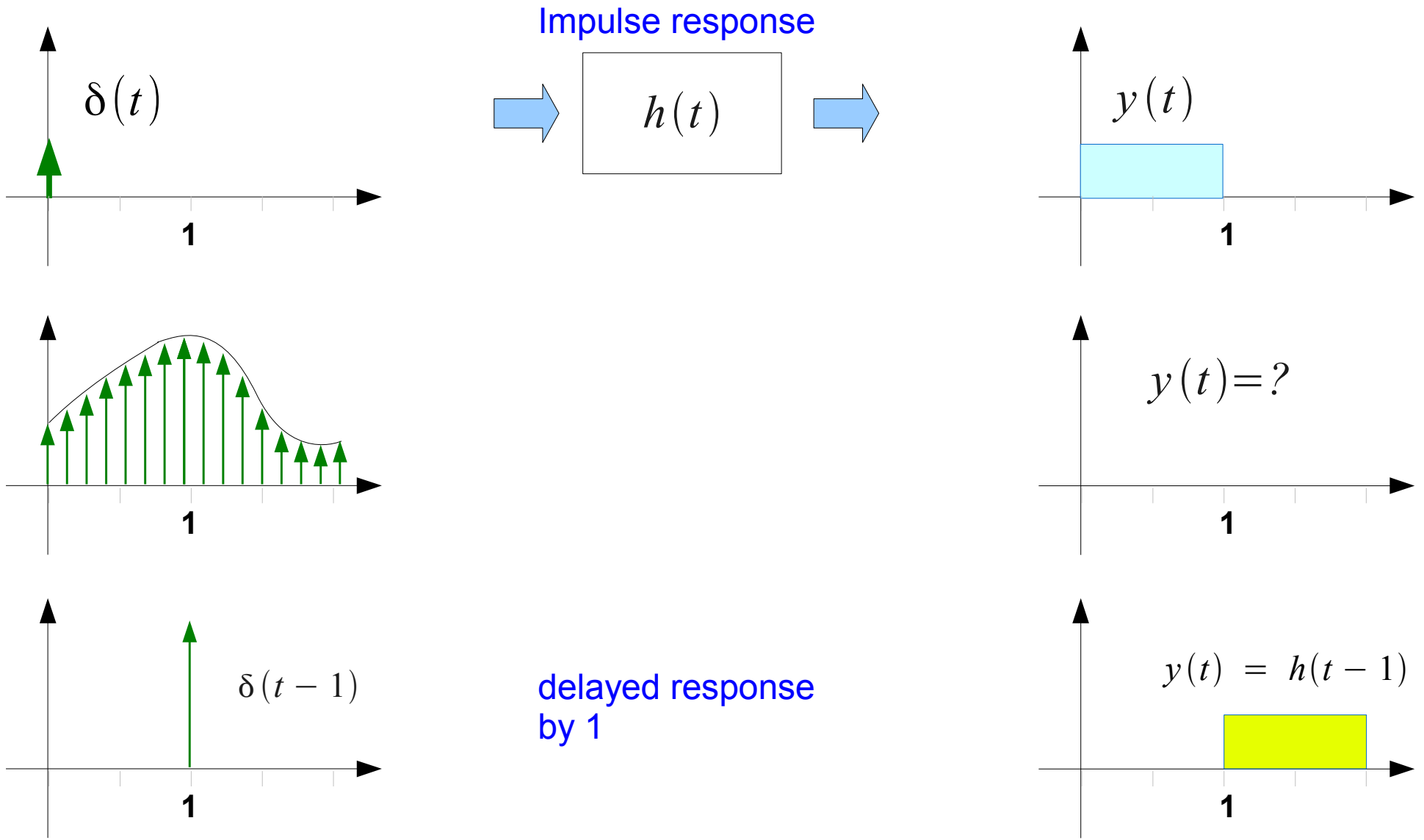
*Flip around y-axis*



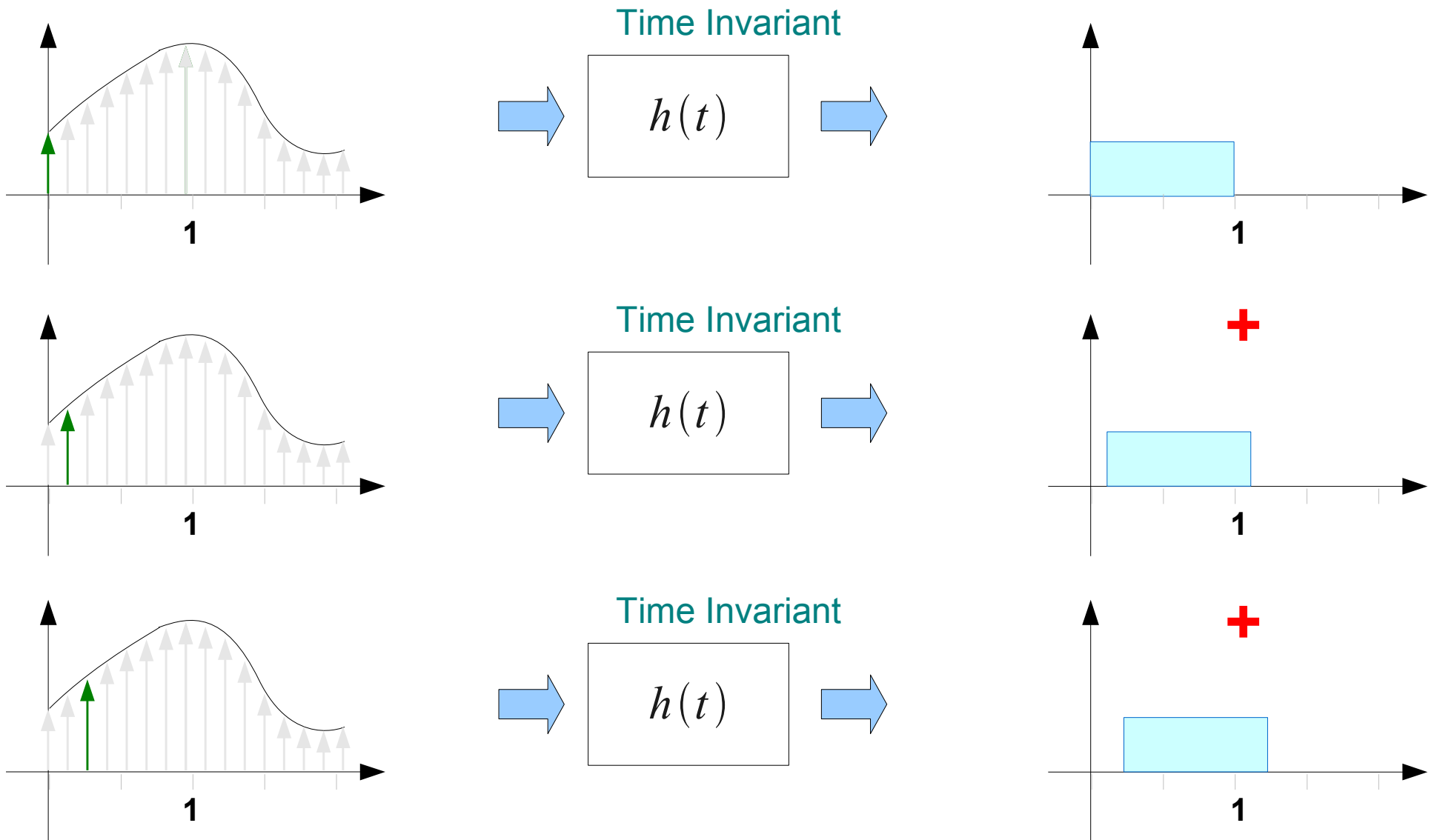
*Shift to the right by  $t_1$*



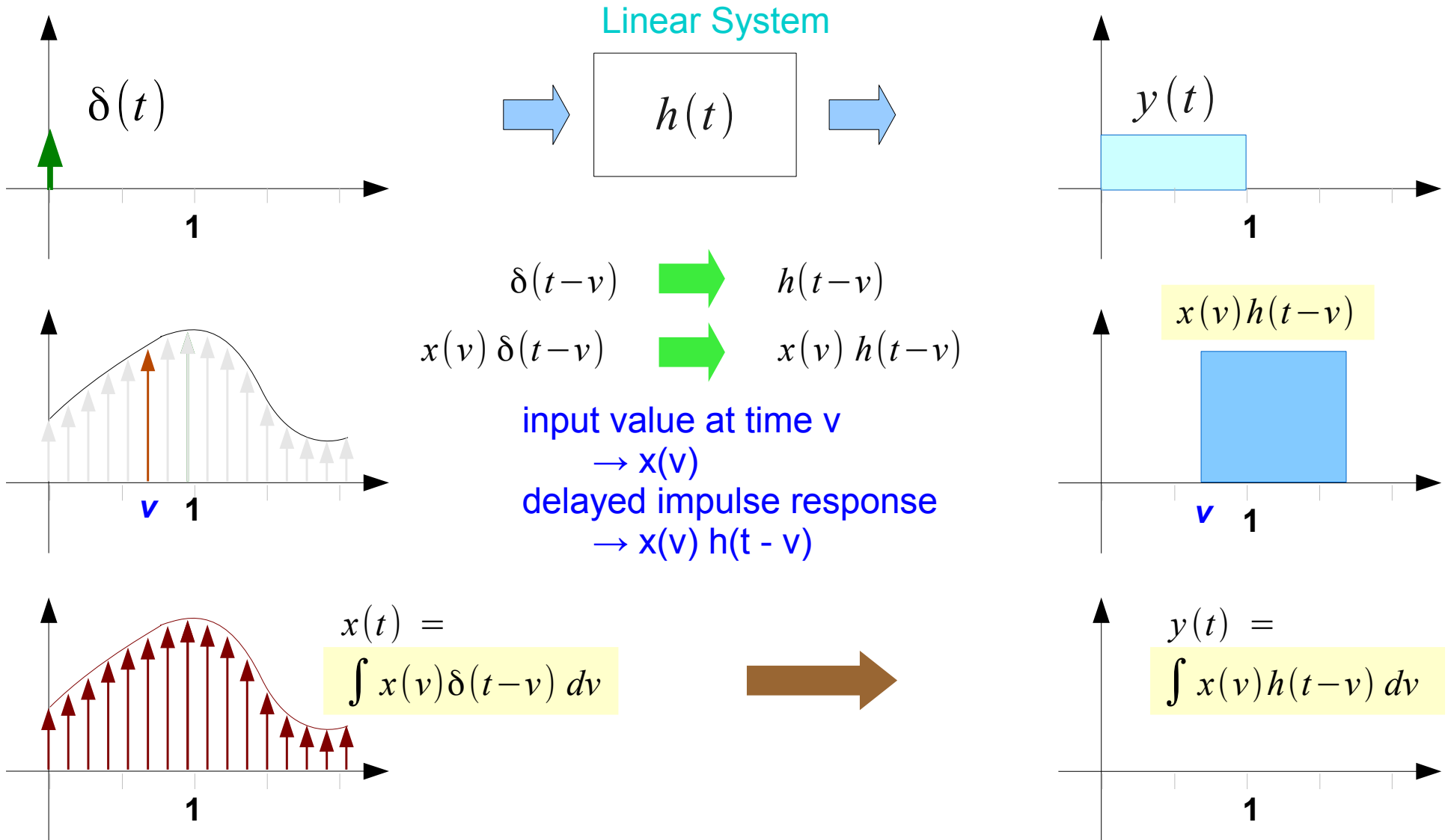
# Convolution: delayed response of $h(t)$ (1)



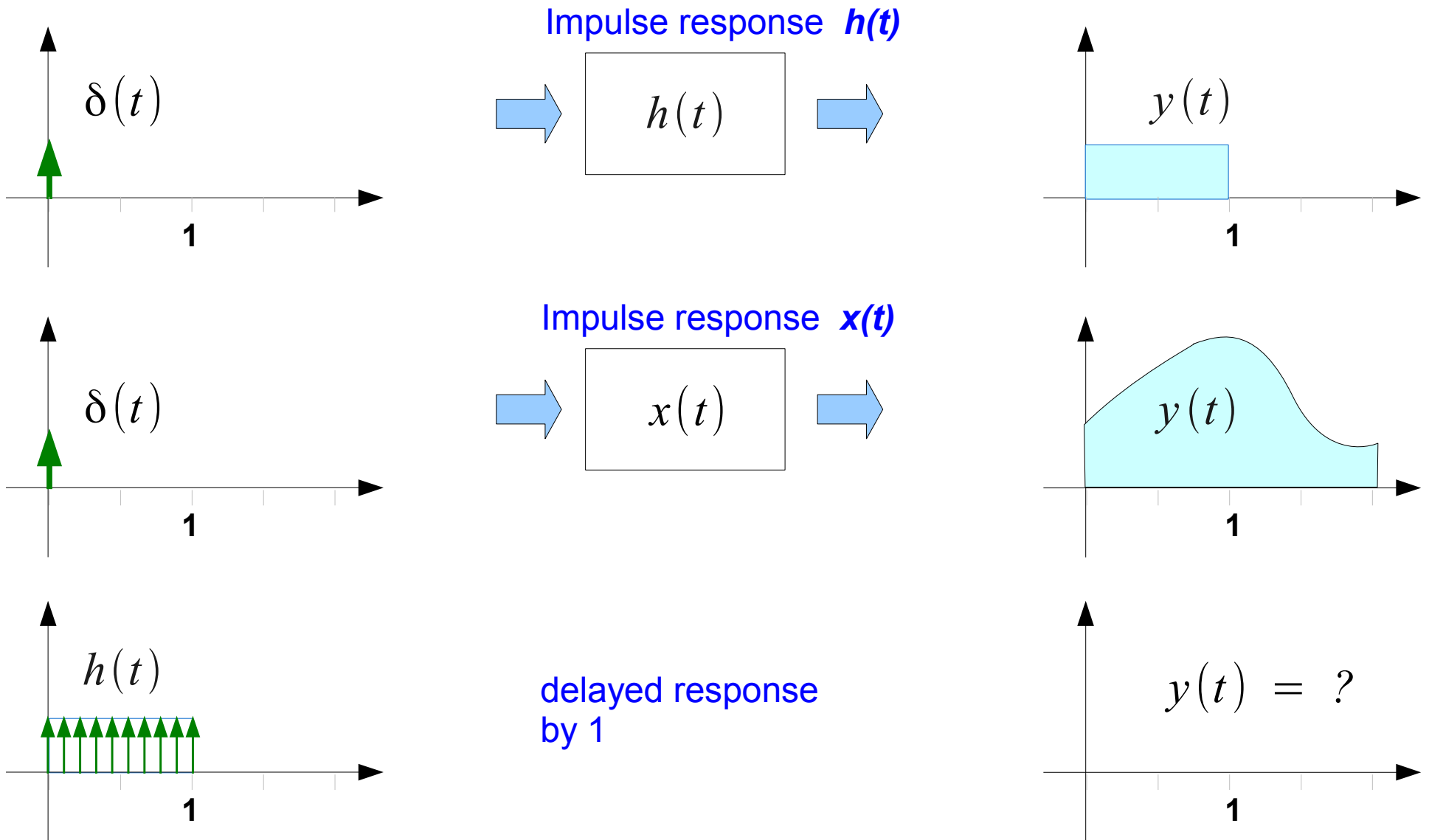
# Convolution: delayed response of $h(t)$ (2)



# Convolution: delayed response of $h(t)$ (3)

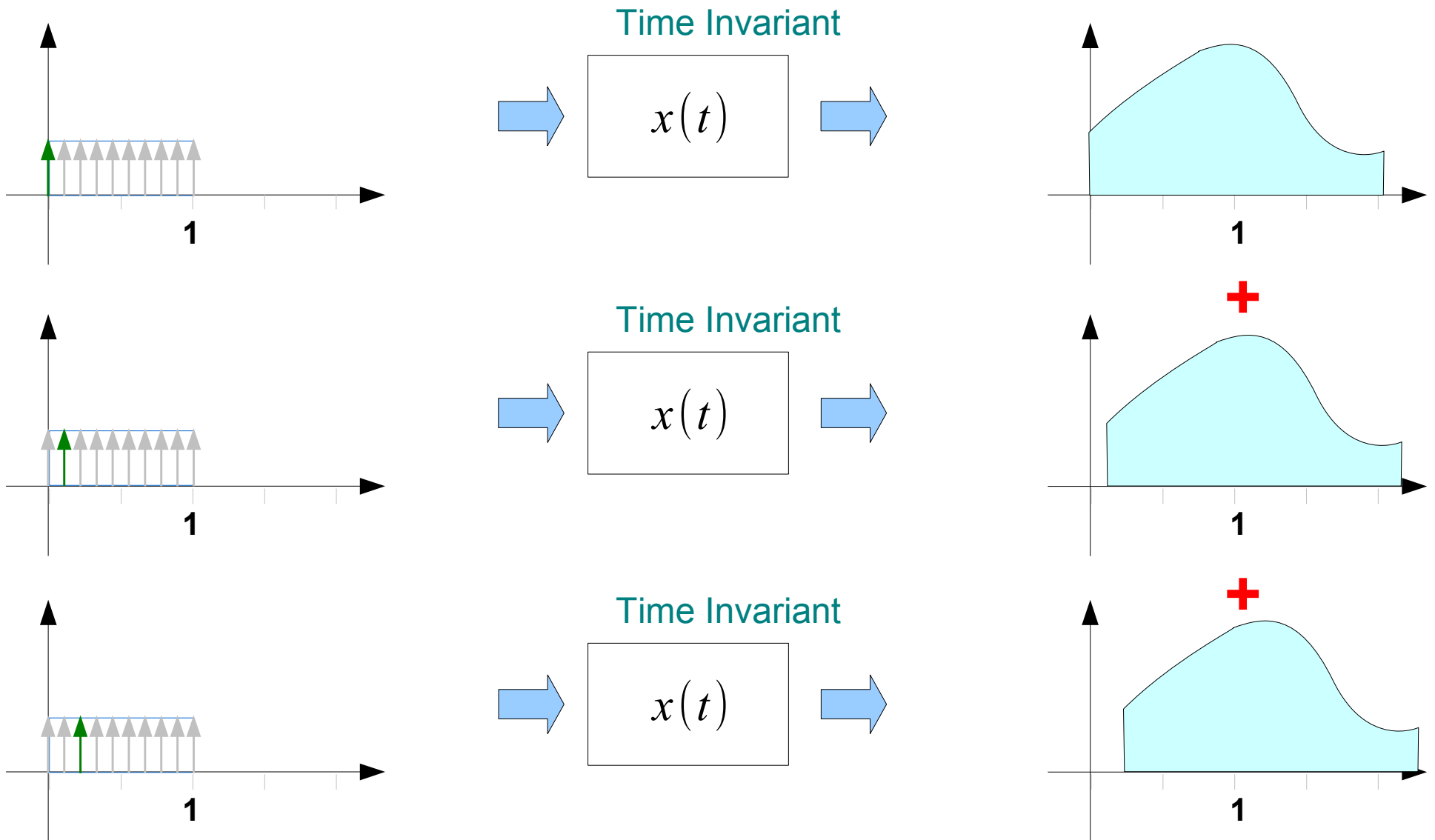


# Convolution: delayed response of $x(t)$ (1)

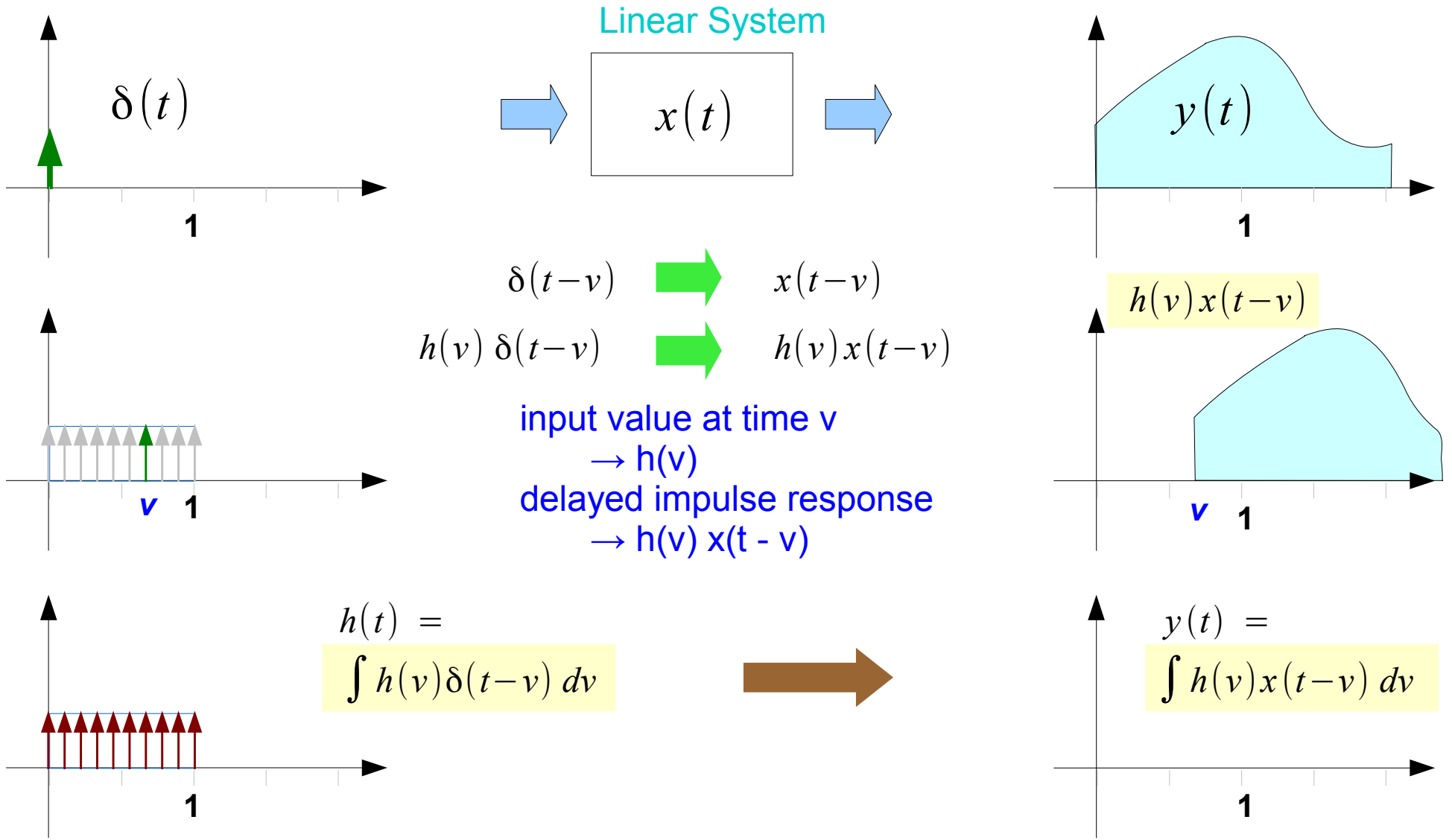




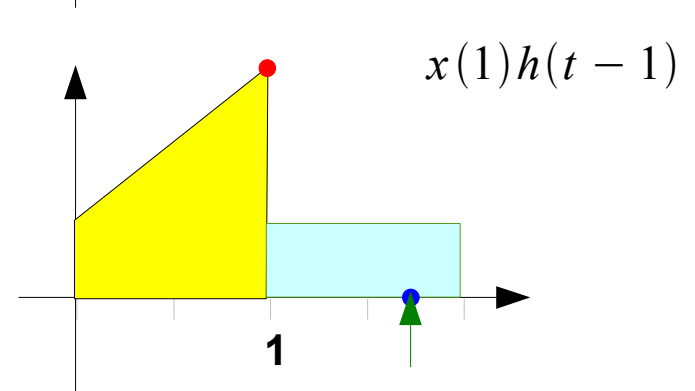
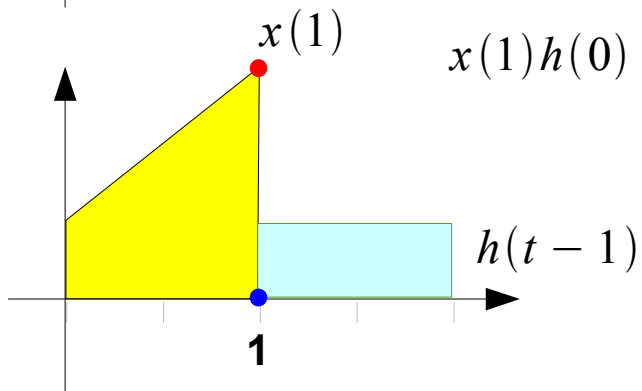
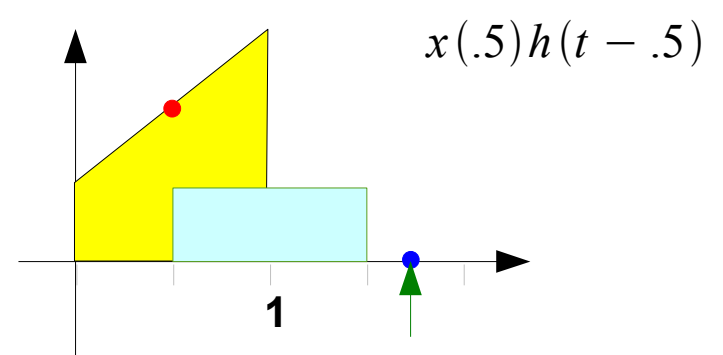
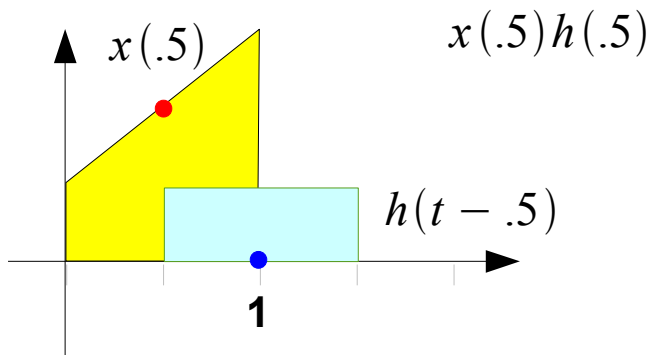
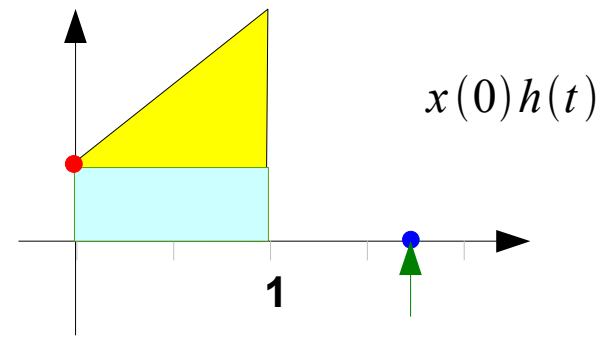
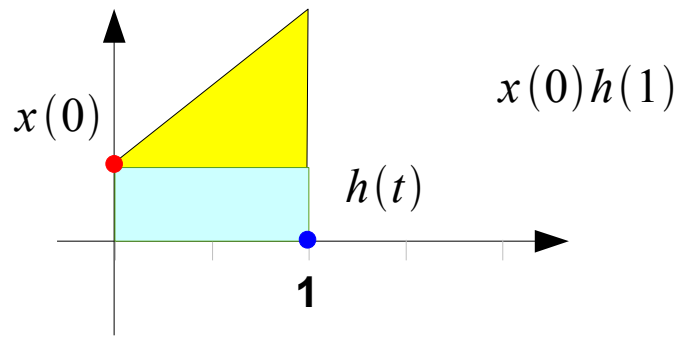
# Convolution: delayed response of $x(t)$ (2)



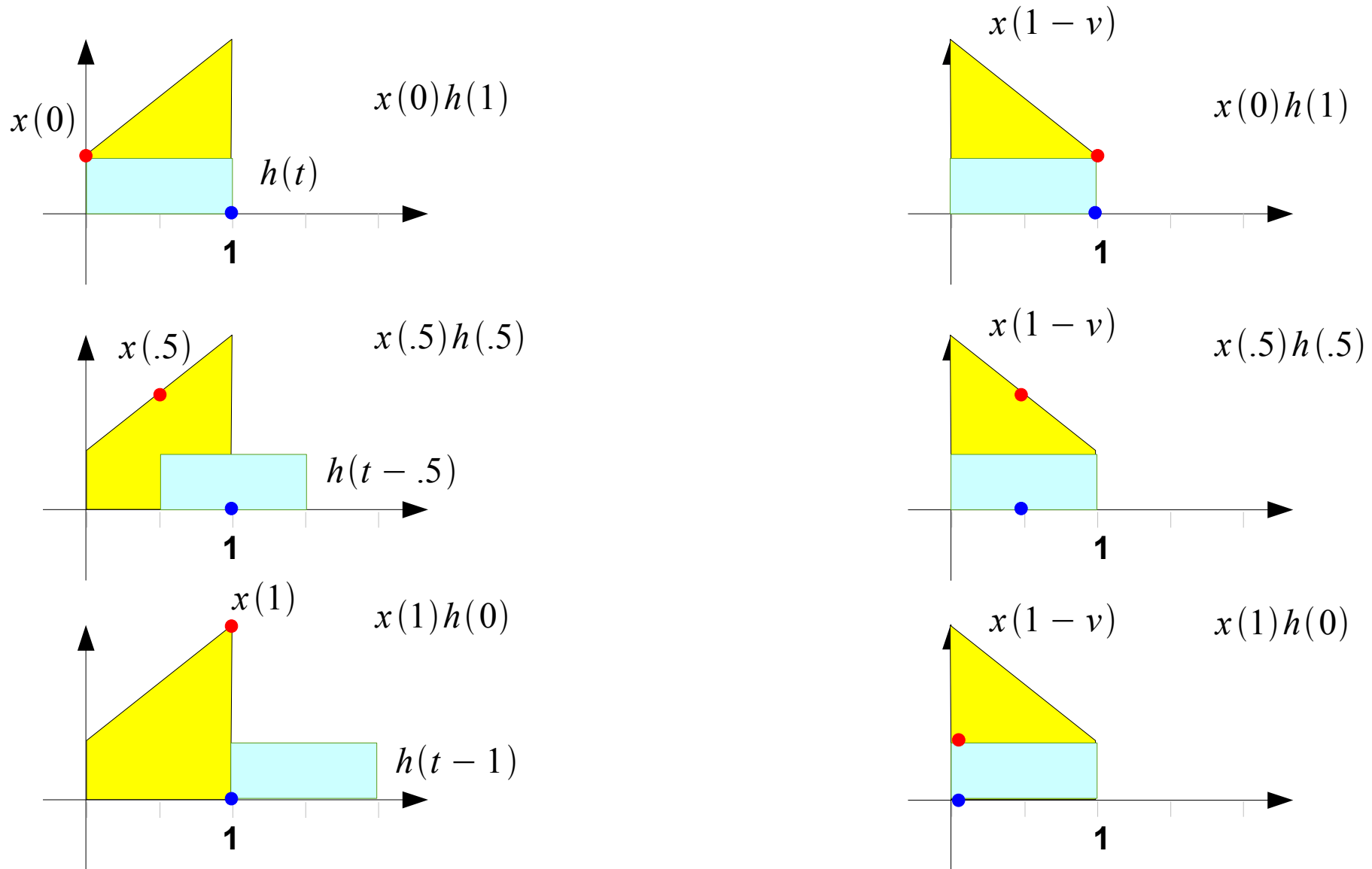
# Convolution: delayed response of $x(t)$ (3)



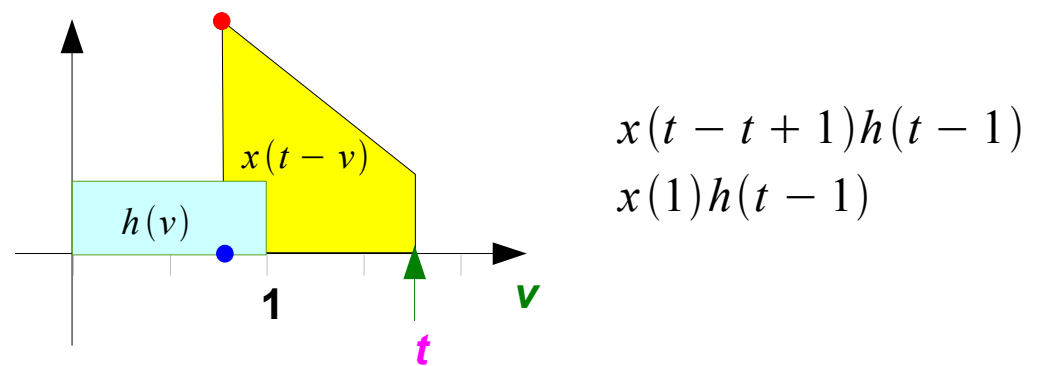
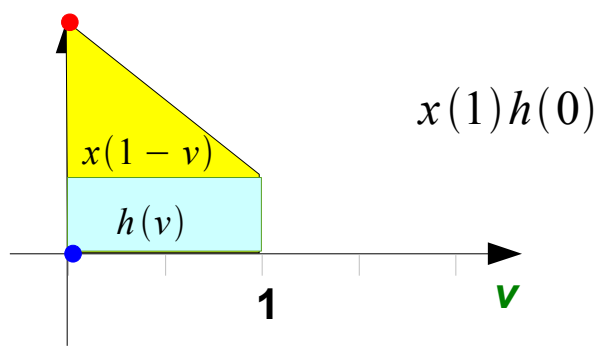
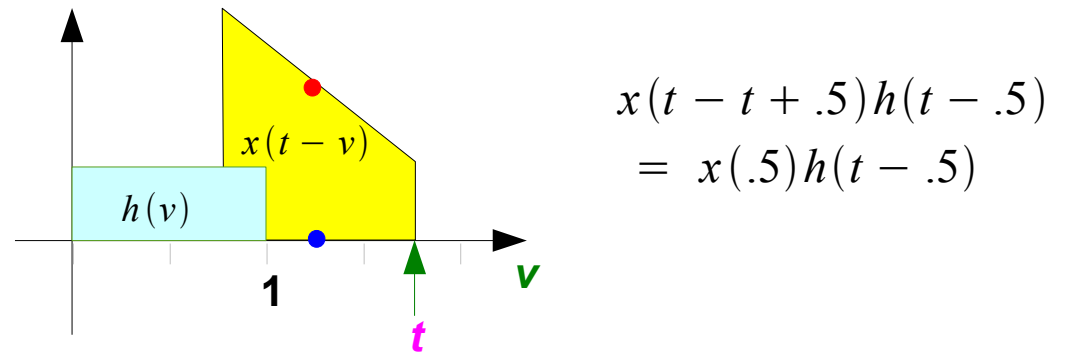
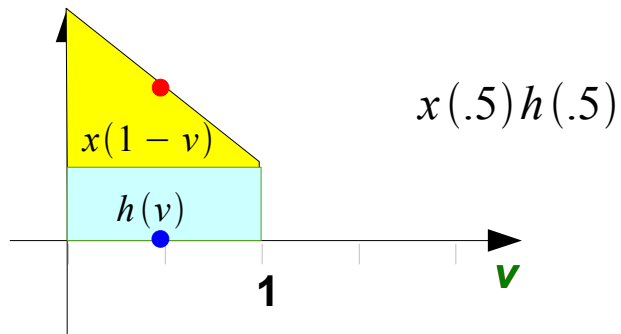
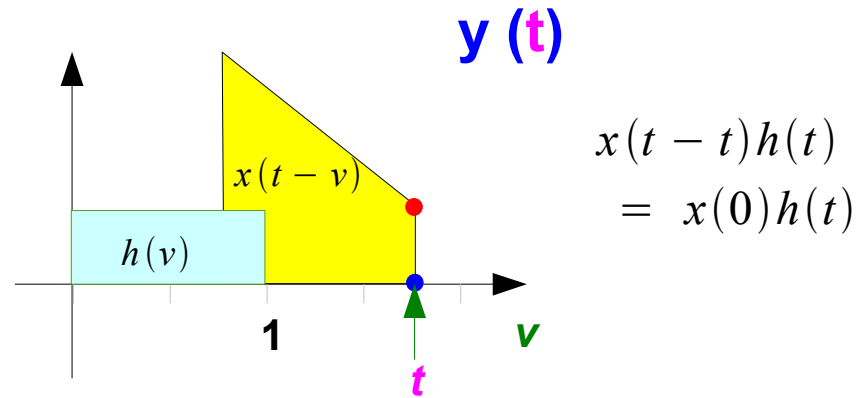
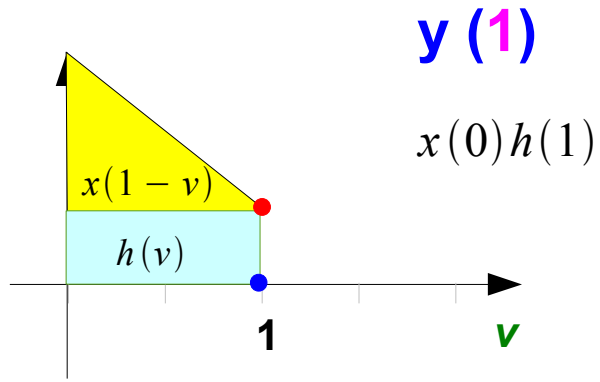
# Impulse Response



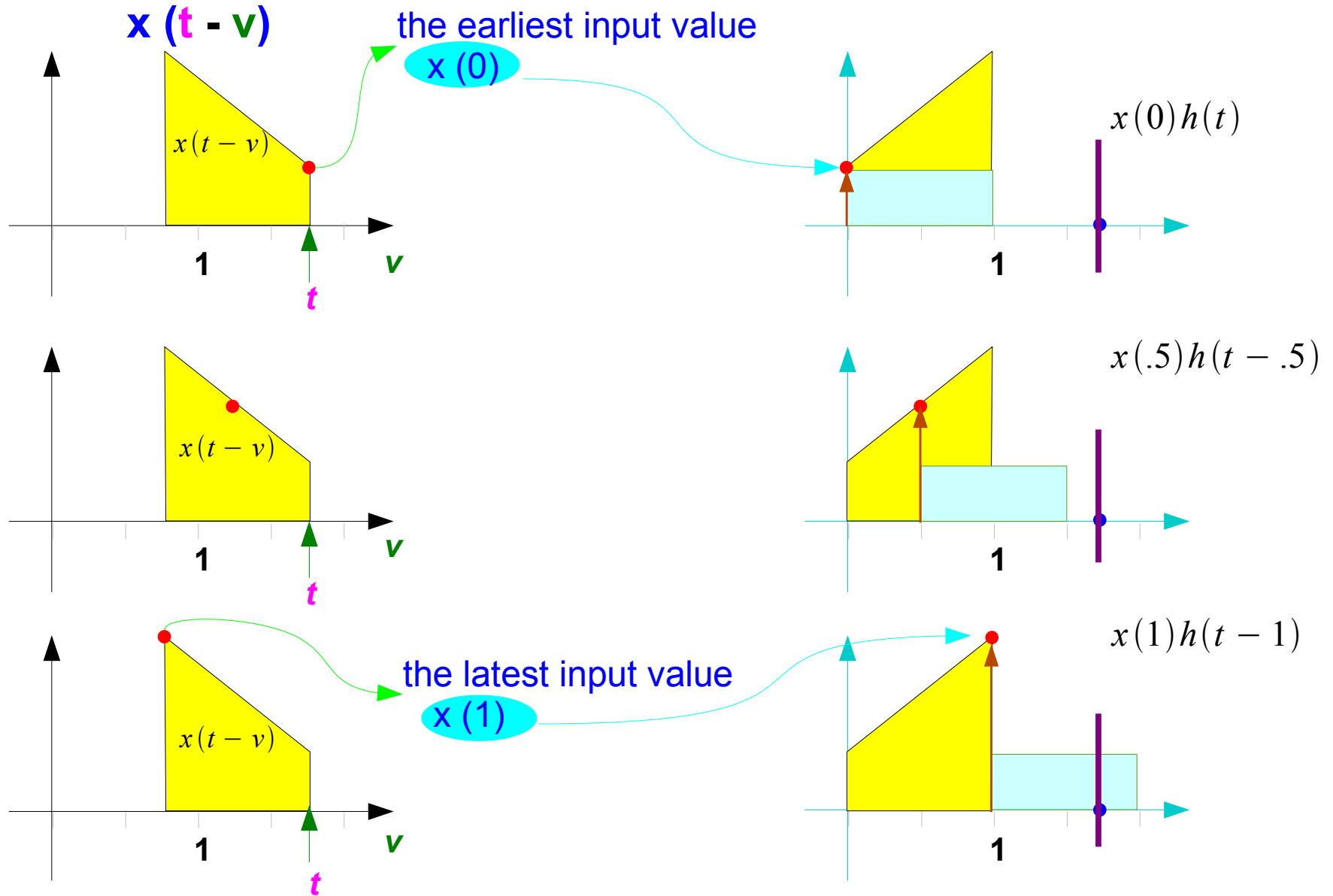
# Impulse Response



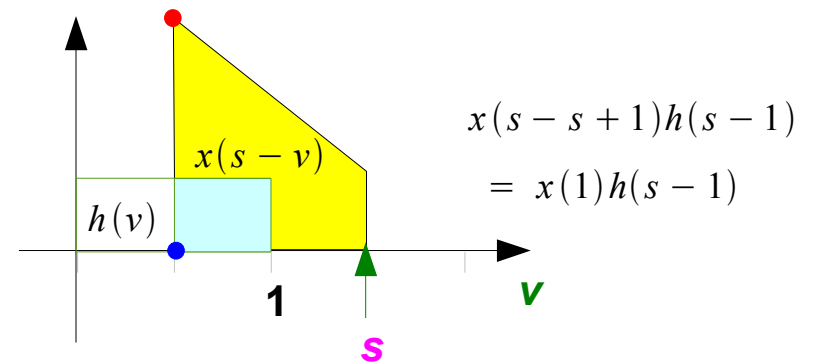
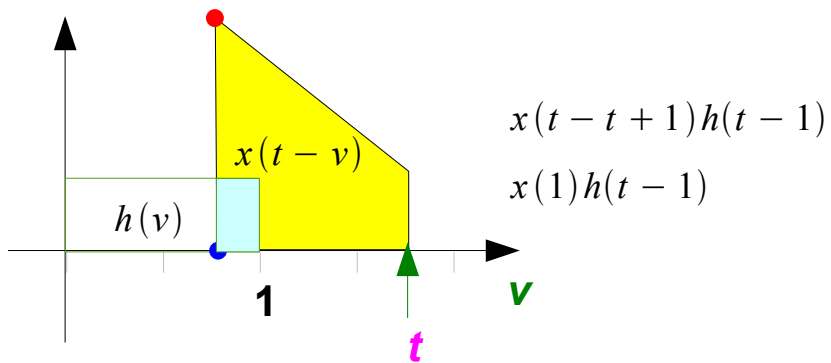
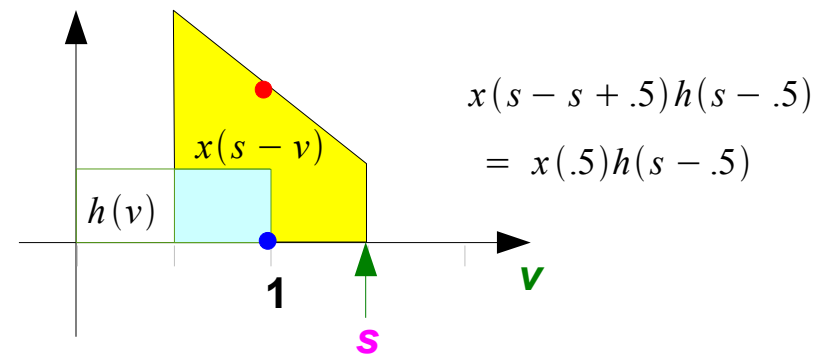
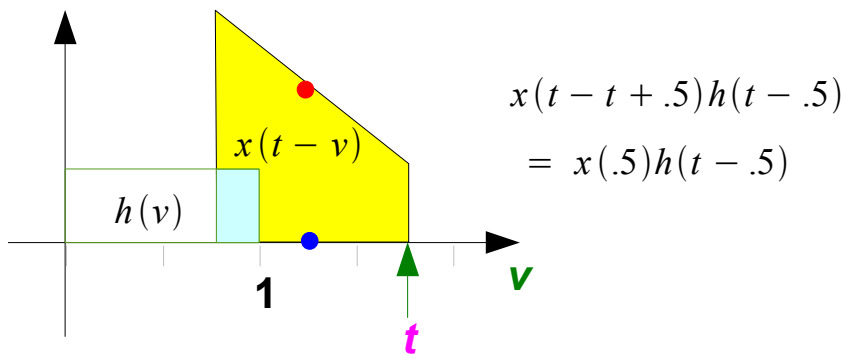
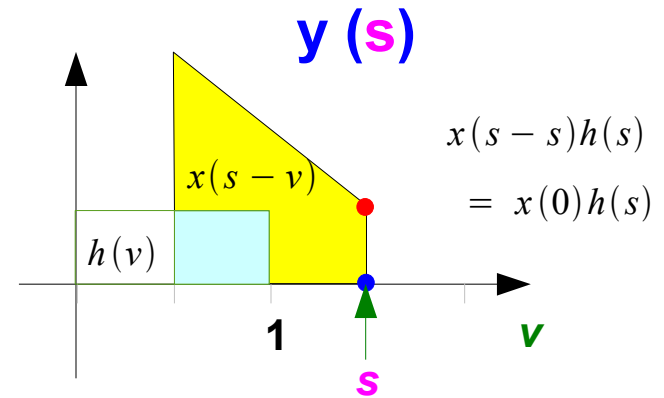
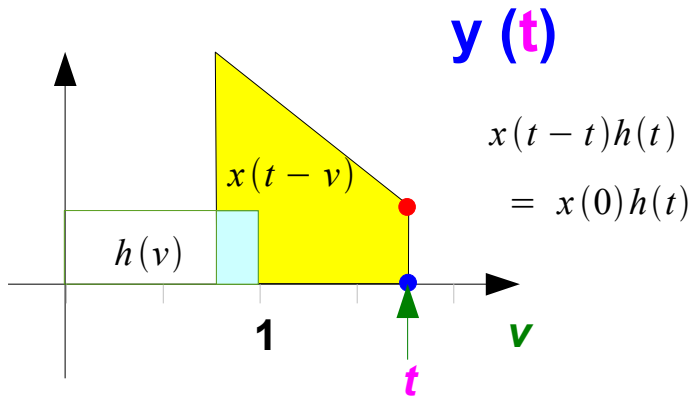
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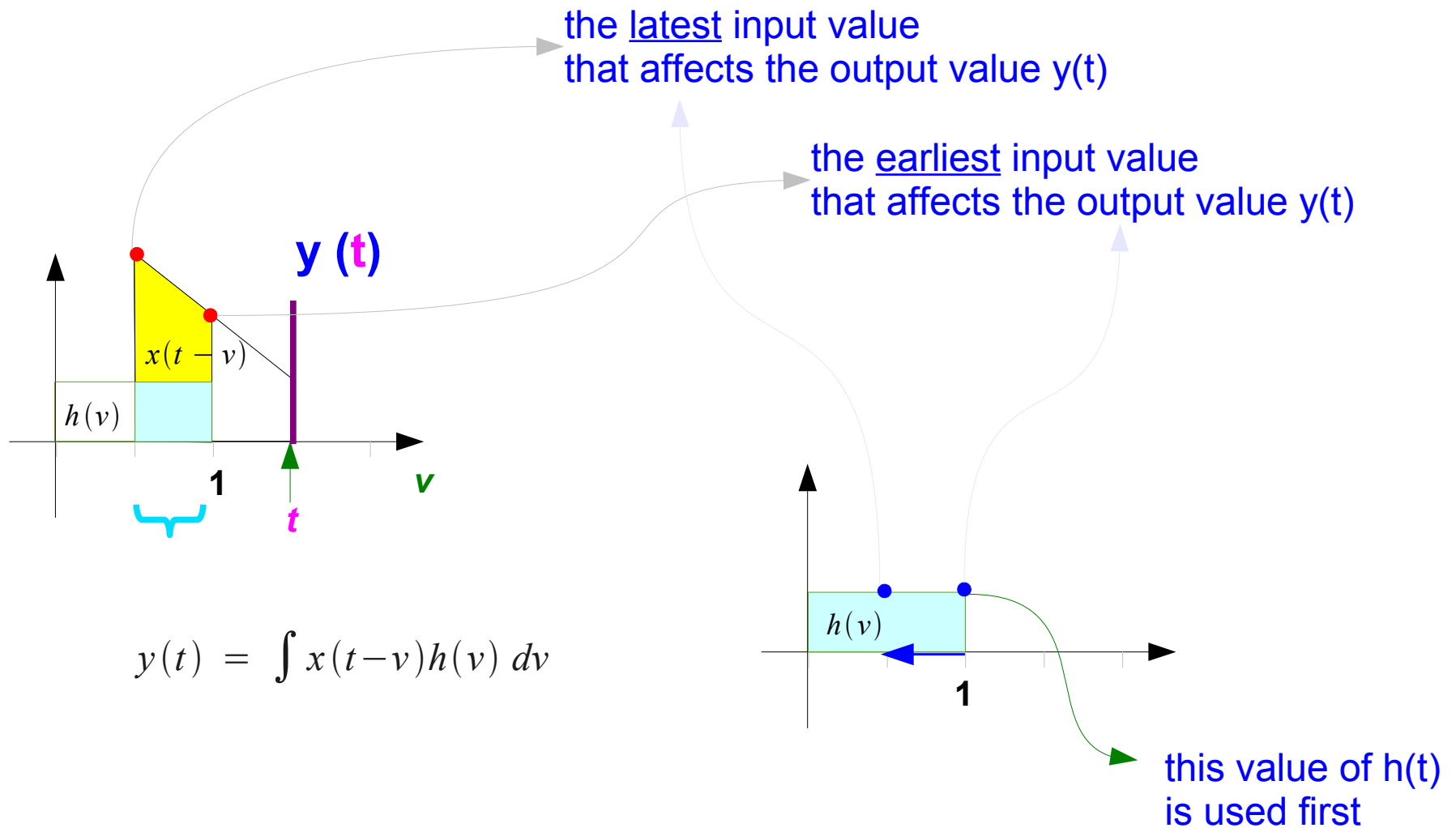
# Impulse Response



# Impulse Response

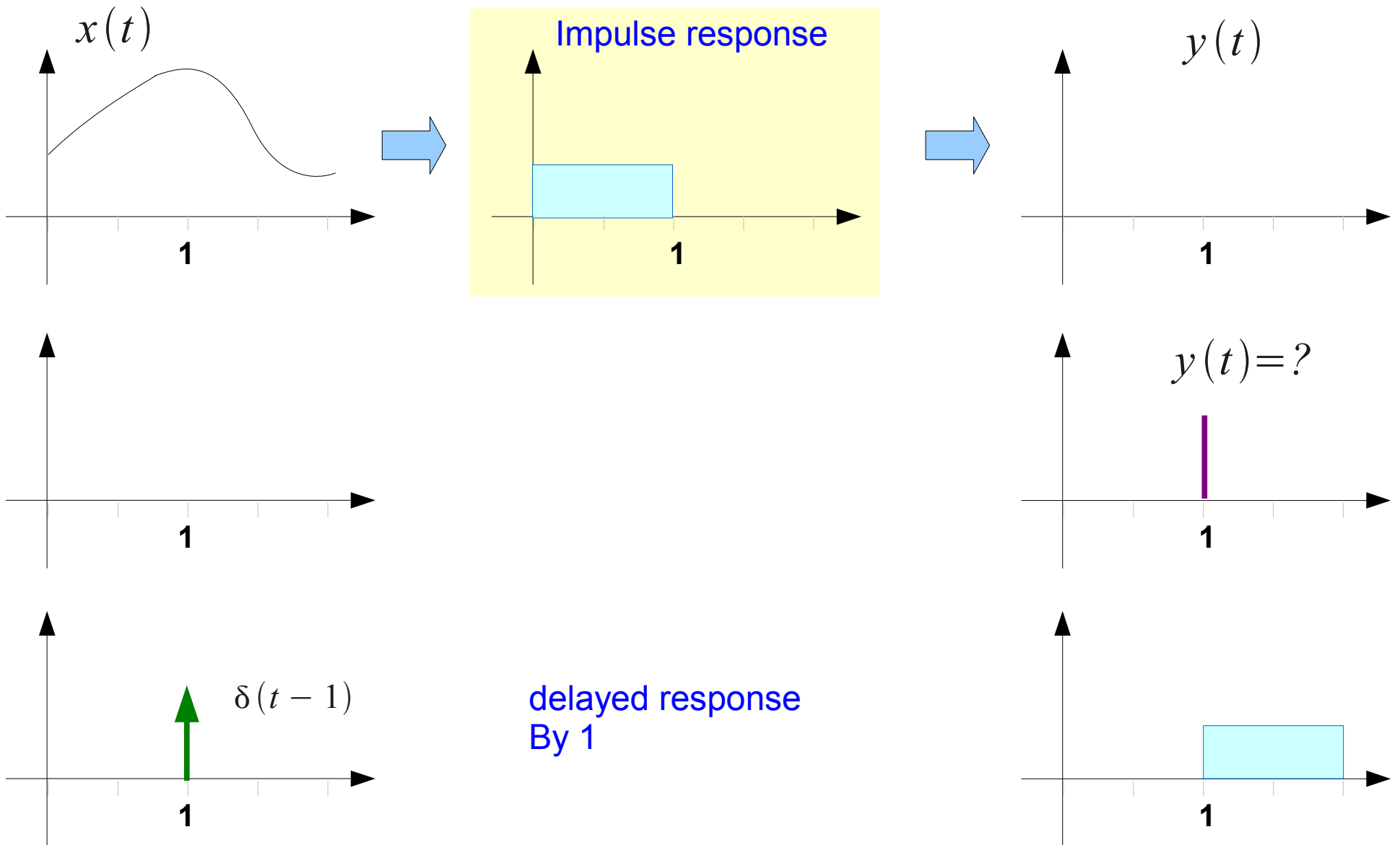


# Impulse Response

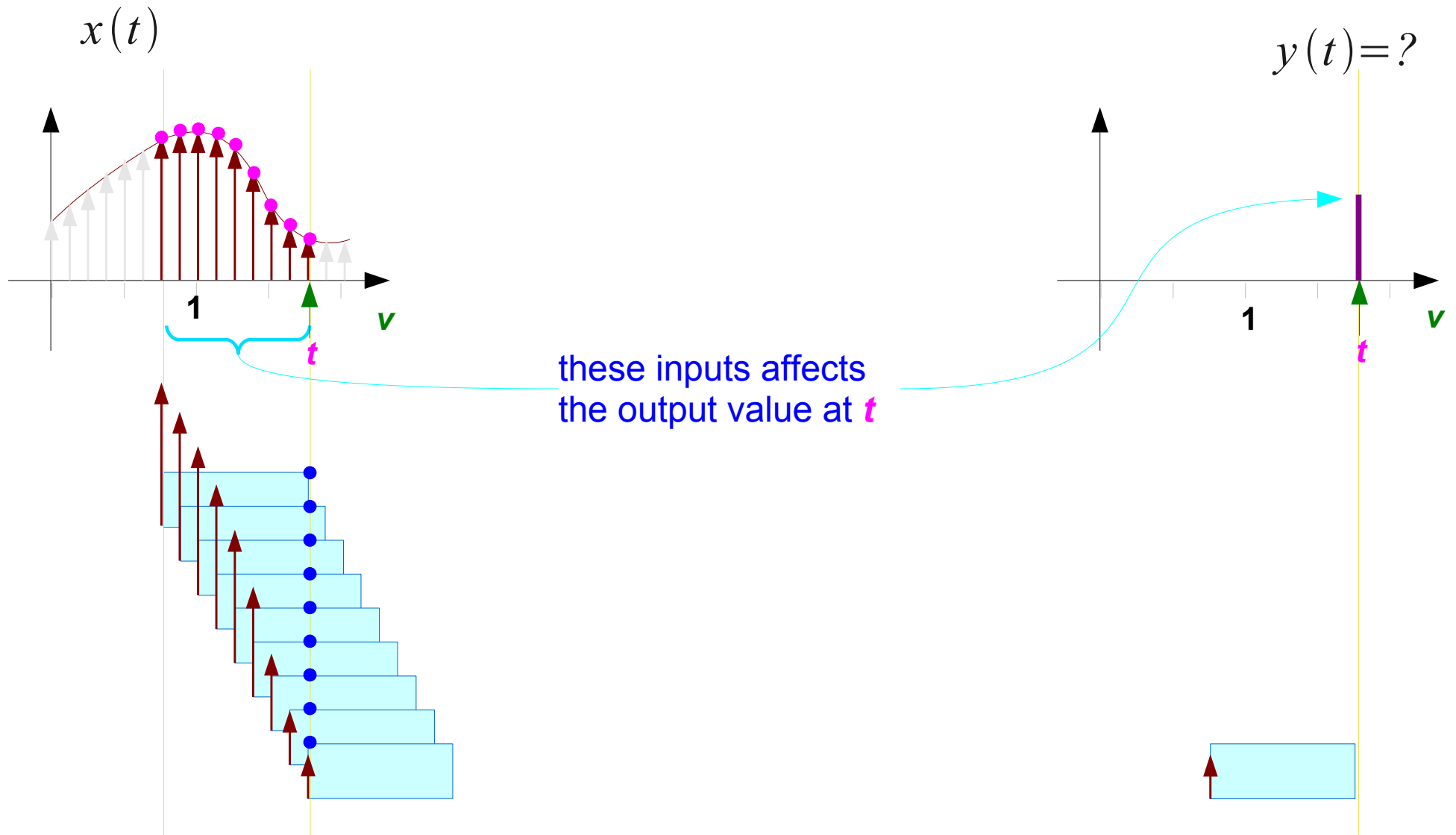




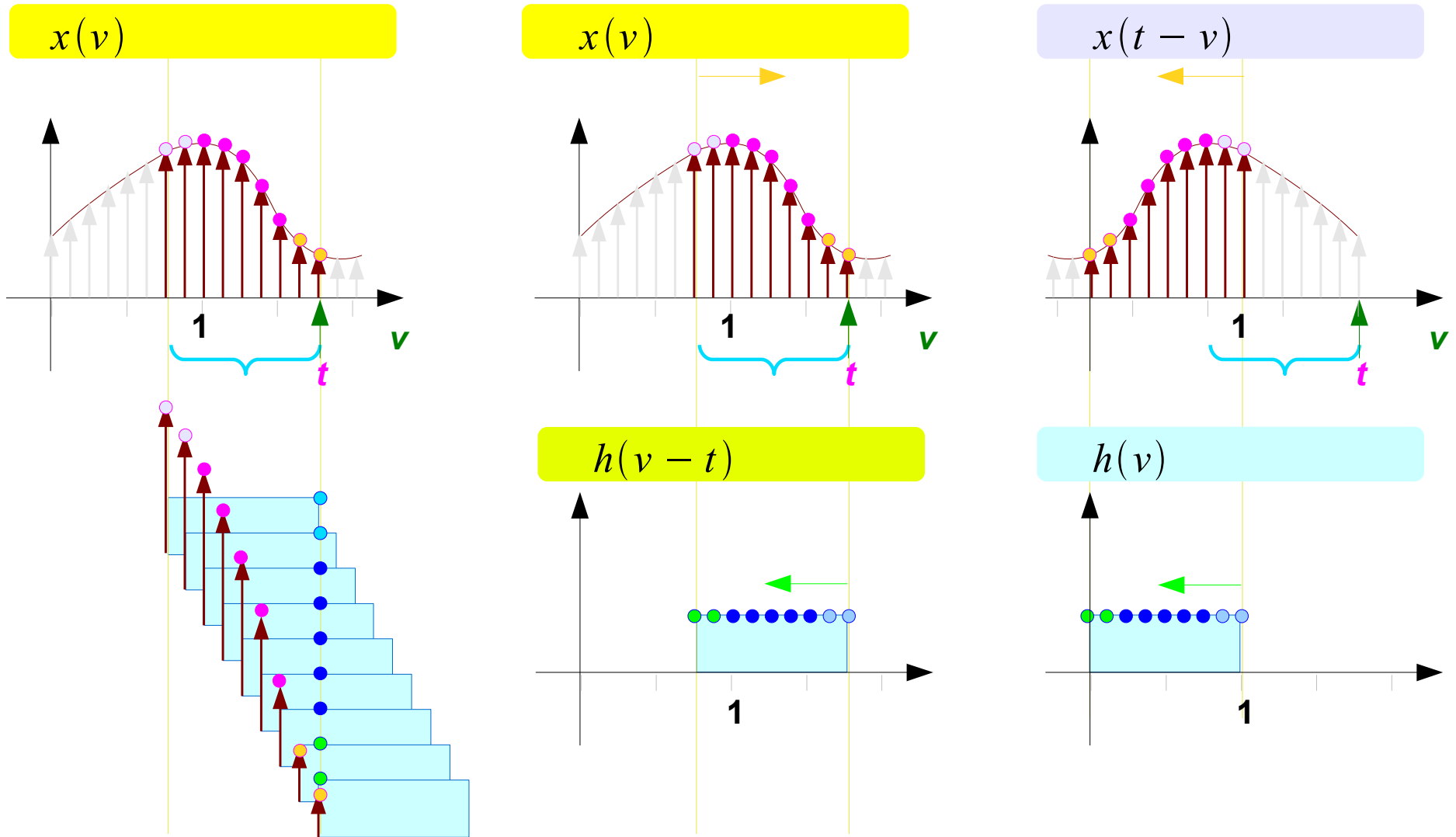
# Impulse Response



# Impulse Response



# Impulse Response



## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003