Convolution (1A)

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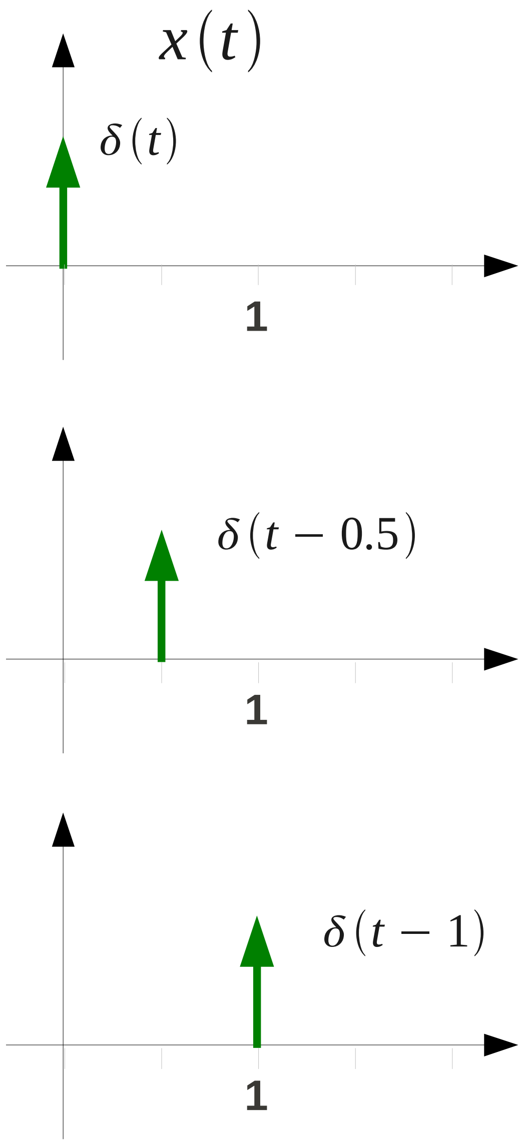
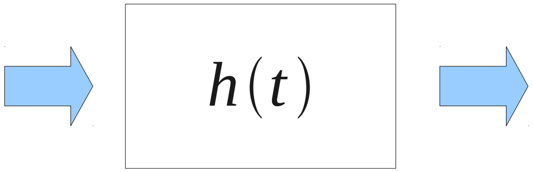
Impulse Response

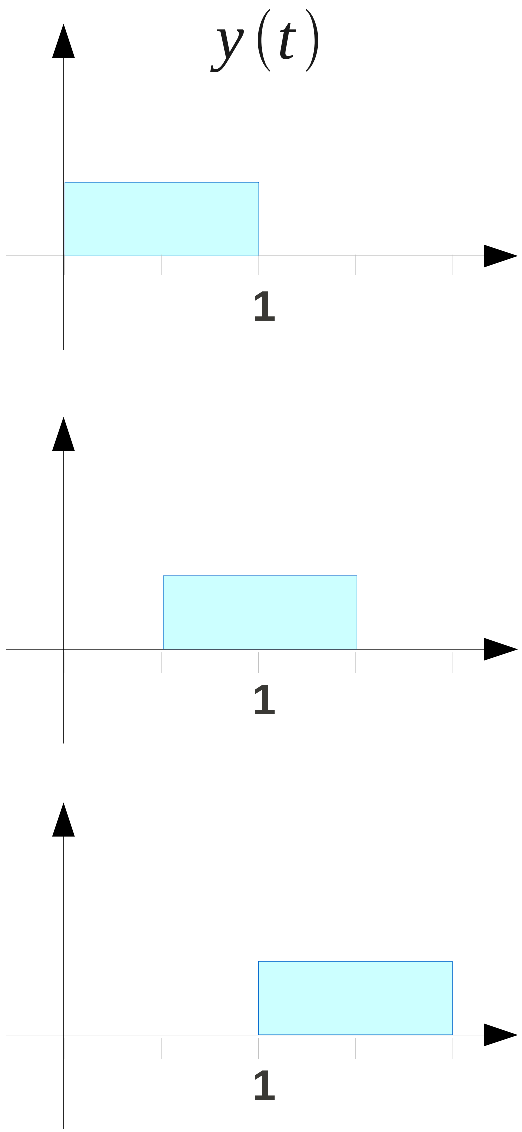


Impulse response

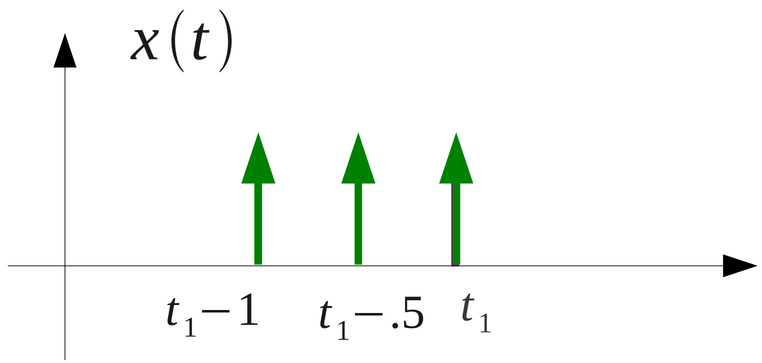


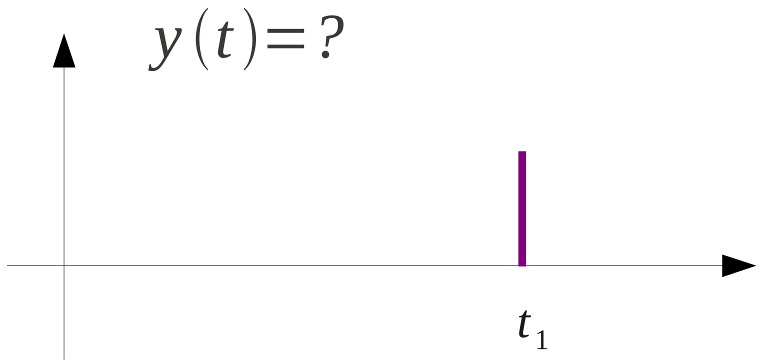
delayed response
by 0.5

delayed response
by 1



LTI System





$\delta(t-t_1+1) \rightarrow \boxed{h(t)} \rightarrow h(t-t_1+1)$

$\delta(t-t_1+.5) \rightarrow \boxed{h(t)} \rightarrow h(t-t_1+.5)$

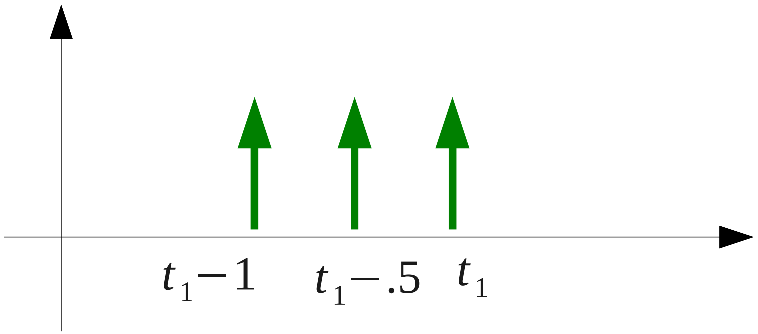
$\delta(t-t_1) \rightarrow \boxed{h(t)} \rightarrow h(t-t_1)$

$\rightarrow \boxed{h(t)} \rightarrow$

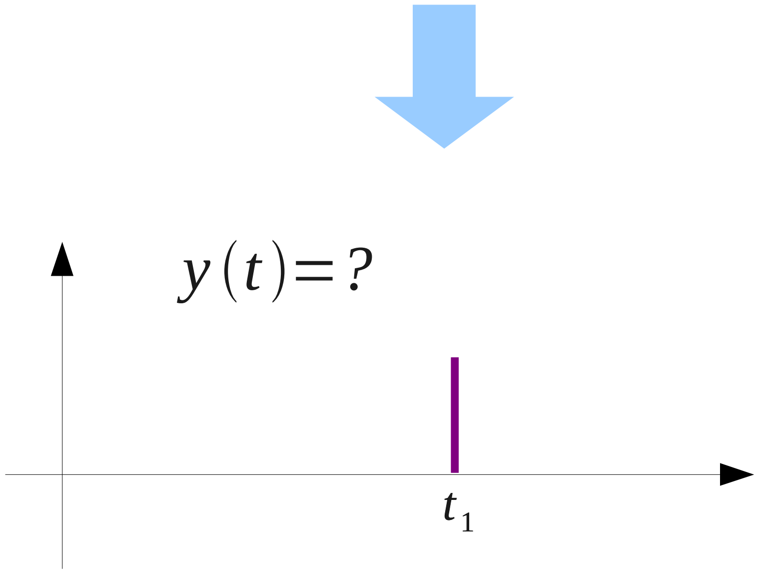
$x(t) =$
$\delta(t-t_1+1) +$
$\delta(t-t_1+.5) +$
$\delta(t-t_1)$

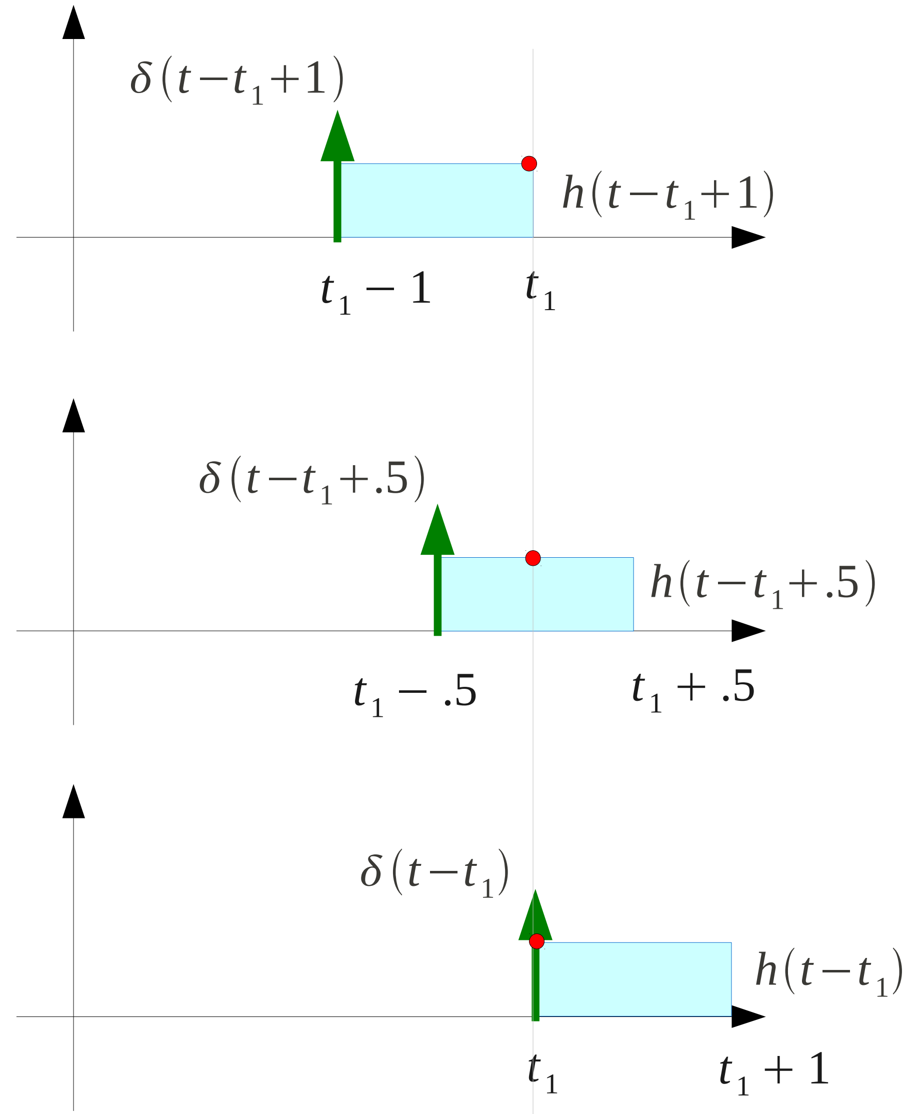
$y(t) =$
$h(t-t_1+1) +$
$h(t-t_1+.5) +$
$h(t-t_1)$

Output at t = t1



$$x(t) \; = \; \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1)$$

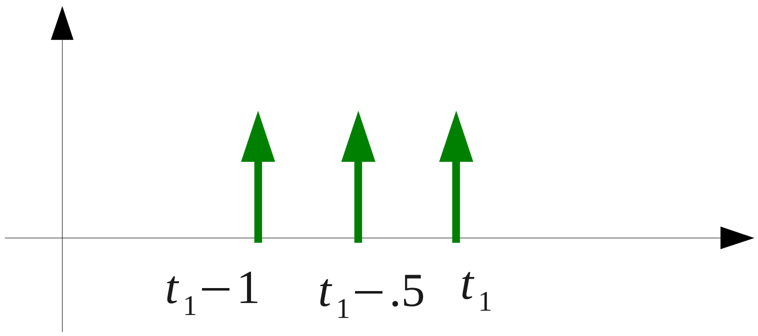


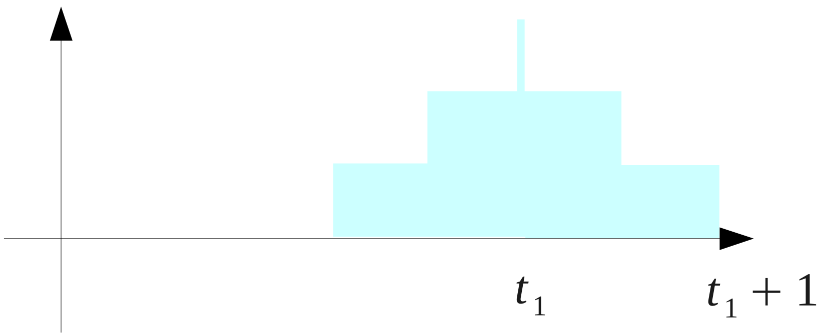


Using Convolution

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$





$$y(t) = \int x(v)h(t-v)\,dv$$

$$= \int \delta(v-t_1+1)h(t-v)\,dv \quad \rightarrow \quad h(t-t_1+1)$$

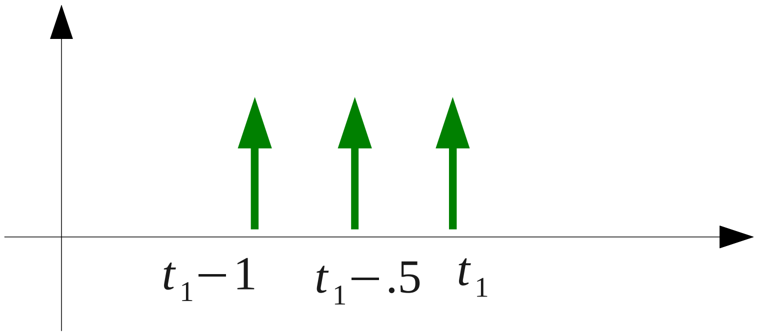
$$+ \int \delta(v-t_1+.5)h(t-v)\,dv \quad \rightarrow \quad h(t-t_1+.5)$$

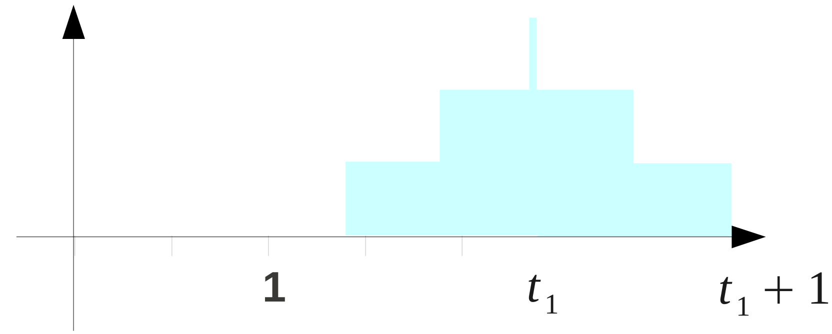
$$+ \int \delta(v-t_1)h(t-v)\,dv \quad \rightarrow \quad h(t-t_1)$$

$$y(t_1) = h(t_1-t_1+1) + h(t_1-t_1+.5) + h(t_1-t_1)$$

$$= h(1) + h(.5) + h(0)$$

N=8 DFT



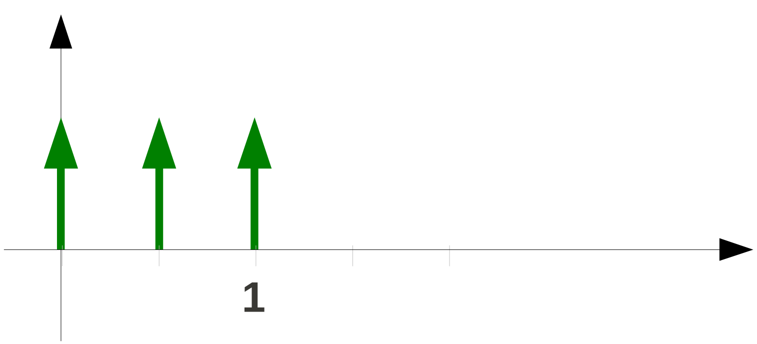


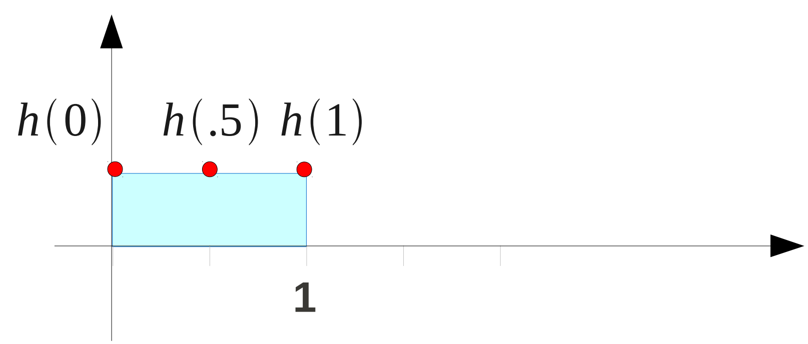
$$x(t) = \delta(t - t_1 + 1) + \delta(t - t_1 + .5) + \delta(t - t_1)$$

$$y(t) = h(t - t_1 + 1) + h(t - t_1 + .5) + h(t - t_1)$$

$$x(t) = \delta(t) + \delta(t - .5) + \delta(t - 1)$$

$$y(t_1) = h(1) + h(.5) + h(0)$$





The Meaning of Convolution

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

Change of variables $t \rightarrow v$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

Flip around y axis and shift to the right by *t* $v \rightarrow t-v$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

$$\begin{aligned} y(t) &= \int x(t-v)\, h(v)\, dv \\ &= \int \delta(t-v-t_1+1)\, h(v)\, dv && \rightarrow h(t-t_1+1) \\ &+ \int \delta(t-v-t_1+.5)\, h(v)\, dv && \rightarrow h(t-t_1+.5) \\ &+ \int \delta(t-v-t_1)\, h(v)\, dv && \rightarrow h(t-t_1) \end{aligned}$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\Rightarrow \quad y(t_1) = h(1) + h(.5) + h(0)$$

The Meaning of Convolution

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

Change of variables $t \rightarrow v$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

Flip around y axis and shift to the right by *t* $v \rightarrow t-v$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

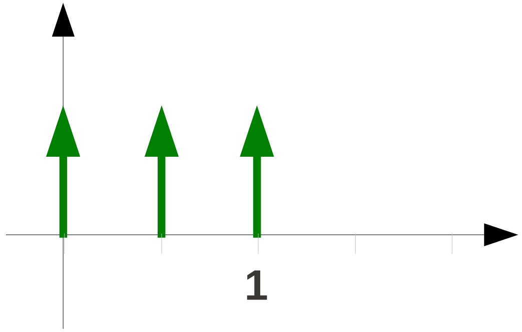
$$\begin{aligned} y(t) &= \int x(t-v)\, h(v)\, dv \\ &= \int \delta(t-v-t_1+1)\, h(v)\, dv && \rightarrow h(t-t_1+1) \\ &+ \int \delta(t-v-t_1+.5)\, h(v)\, dv && \rightarrow h(t-t_1+.5) \\ &+ \int \delta(t-v-t_1)\, h(v)\, dv && \rightarrow h(t-t_1) \end{aligned}$$

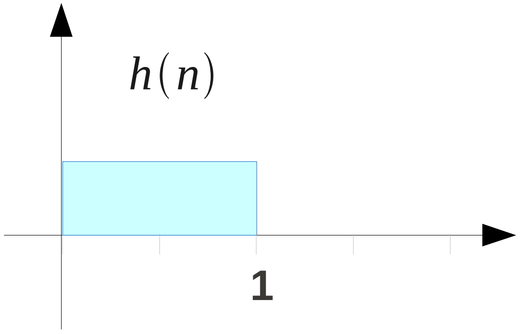
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

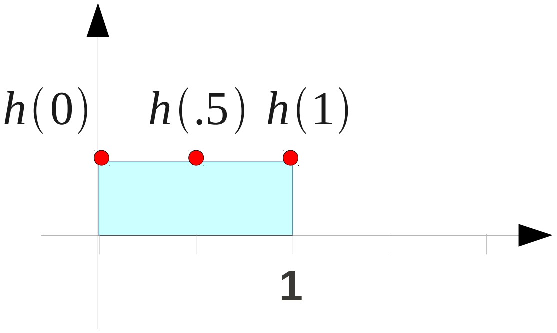
$$\Rightarrow \quad y(t_1) = h(1) + h(.5) + h(0)$$

The Meaning of Convolution

$\delta(t) + \delta(t-.5) + \delta(t-1)$

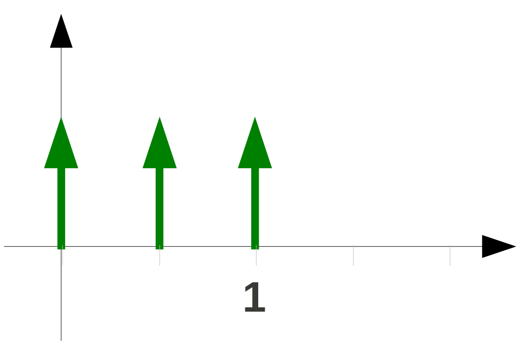


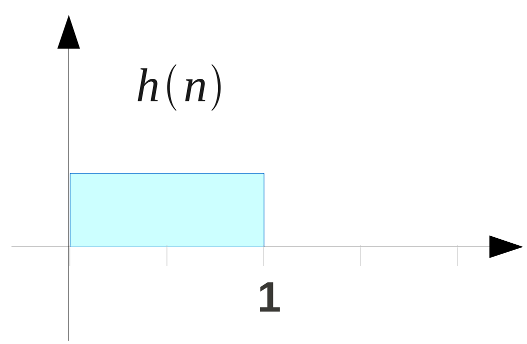


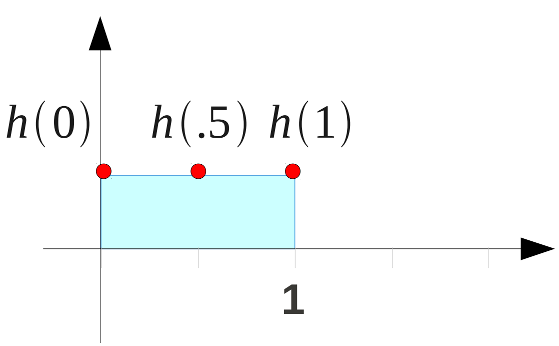


$$y(t) = \int x(v)h(t-v)\,dv$$
$$= \int \delta(v-t_1+1)h(t-v)\,dv \quad \rightarrow \quad h(t-t_1+1)$$
$$+ \int \delta(v-t_1+.5)h(t-v)\,dv \quad \rightarrow \quad h(t-t_1+.5)$$
$$+ \int \delta(v-t_1)h(t-v)\,dv \quad \rightarrow \quad h(t-t_1)$$

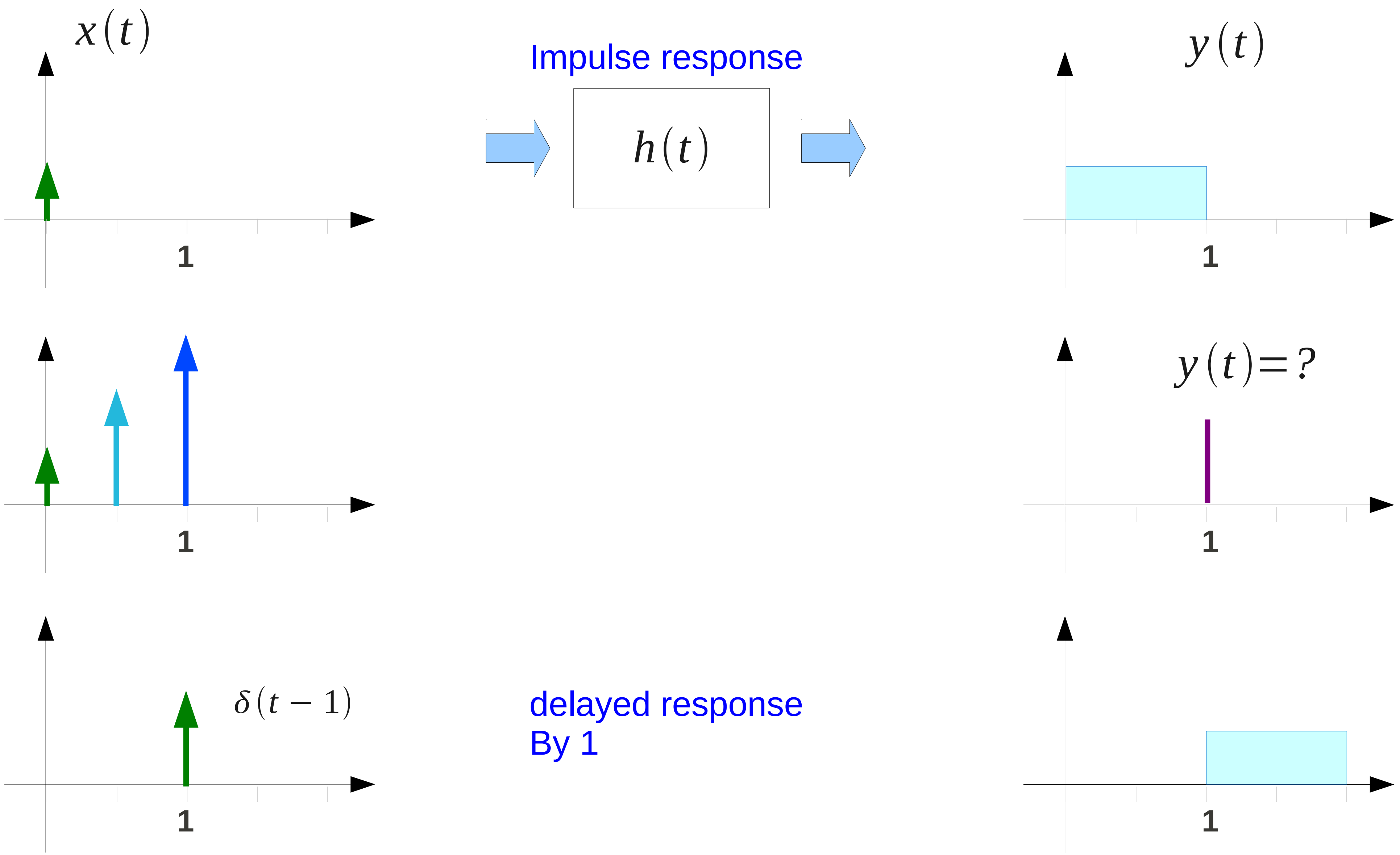
$$y(t) = \int x(t-v)h(v)\,dv$$
$$= \int \delta(t-v-t_1+1)h(v)\,dv \quad \rightarrow \quad h(t-t_1+1)$$
$$+ \int \delta(t-v-t_1+.5)h(v)\,dv \quad \rightarrow \quad h(t-t_1+.5)$$
$$+ \int \delta(t-v-t_1)h(v)\,dv \quad \rightarrow \quad h(t-t_1)$$



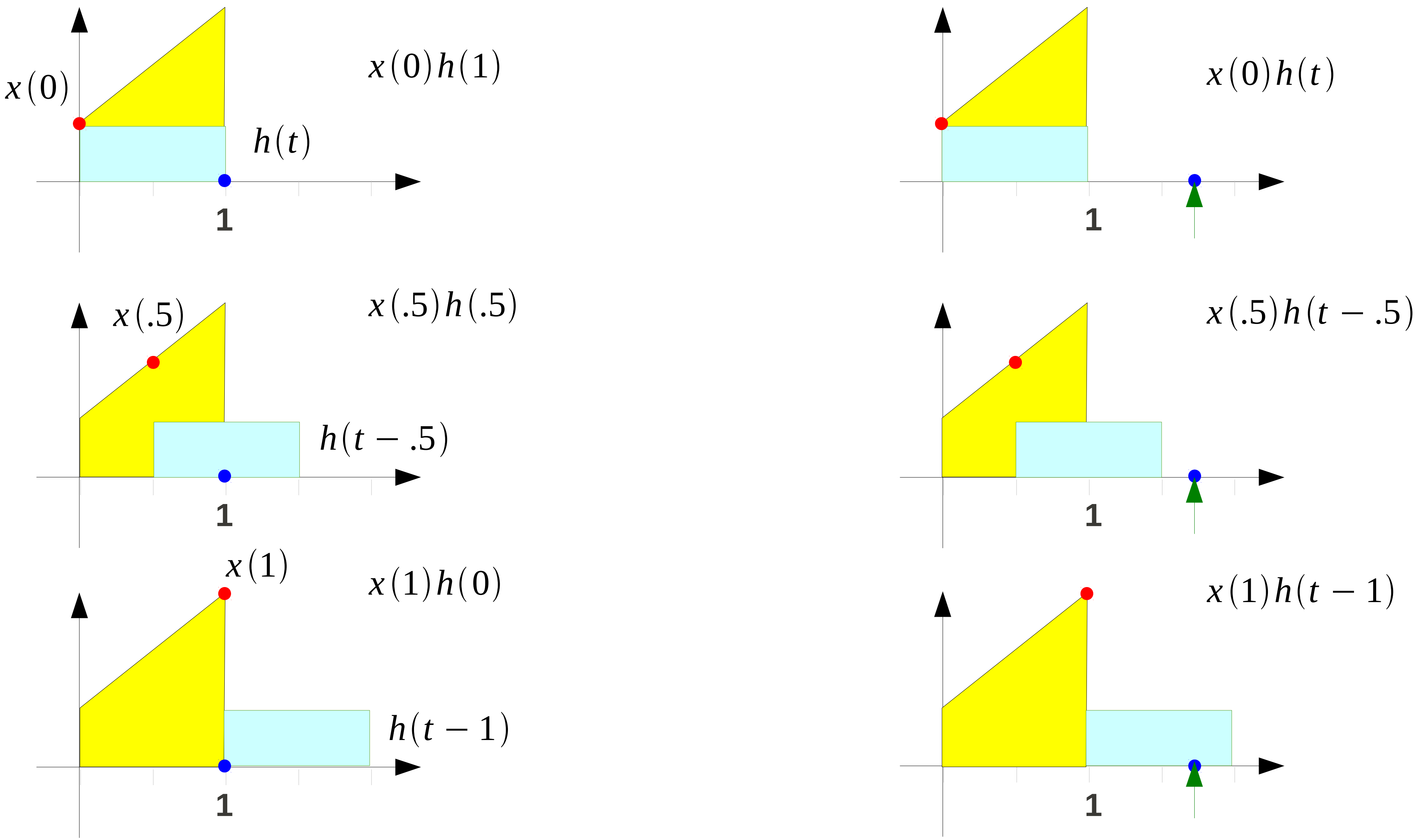




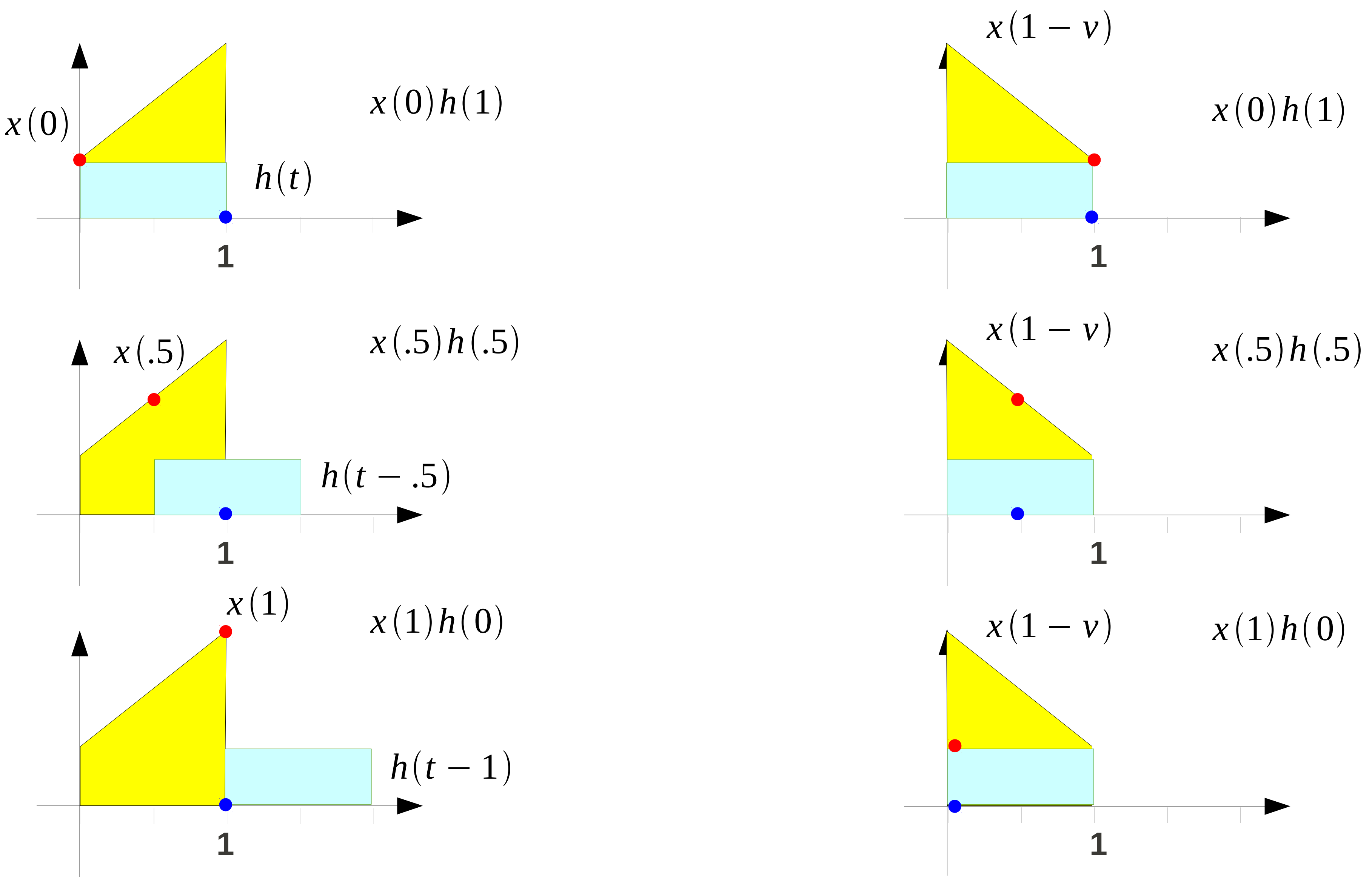
Impulse Response



Impulse Response

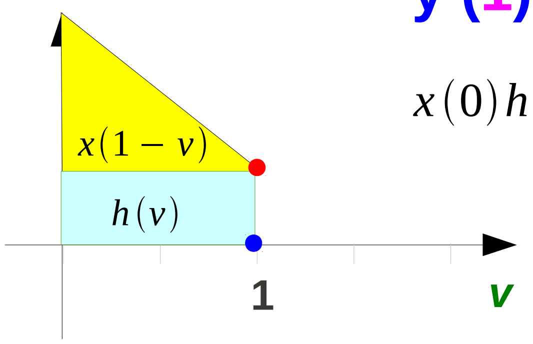


Impulse Response

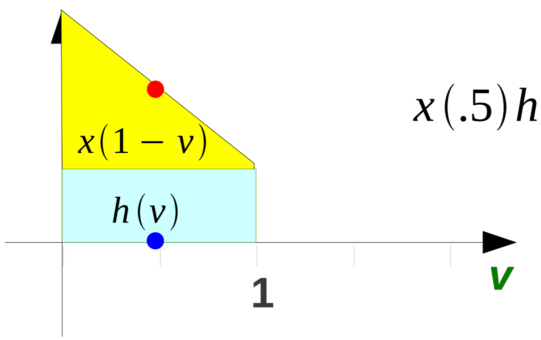


Impulse Response

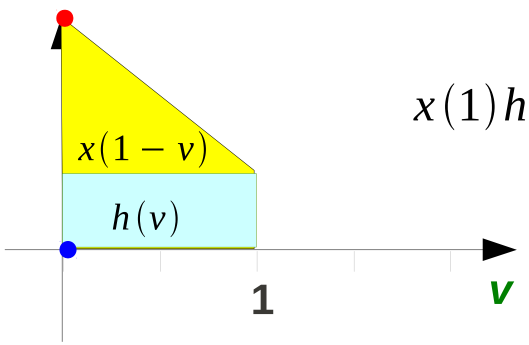
y (1)



$x(0)h(1)$

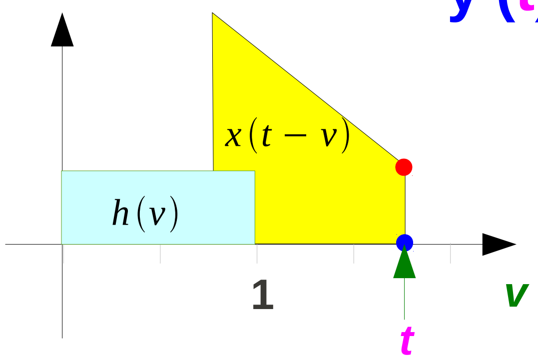


$x(.5)h(.5)$

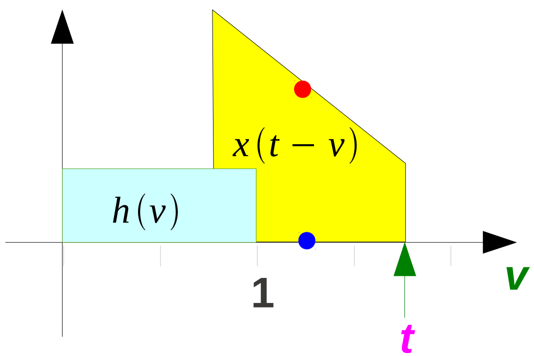


$x(1)h(0)$

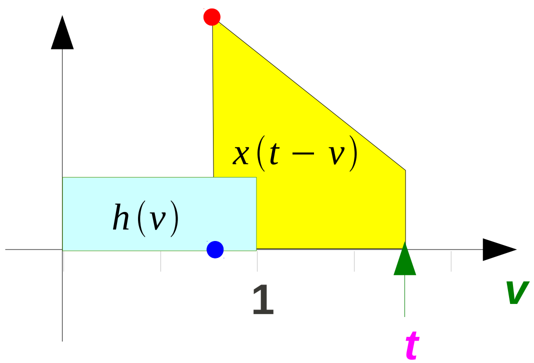
y (t)



$$x(t-t)h(t)$$
$$= \; x(0)h(t)$$

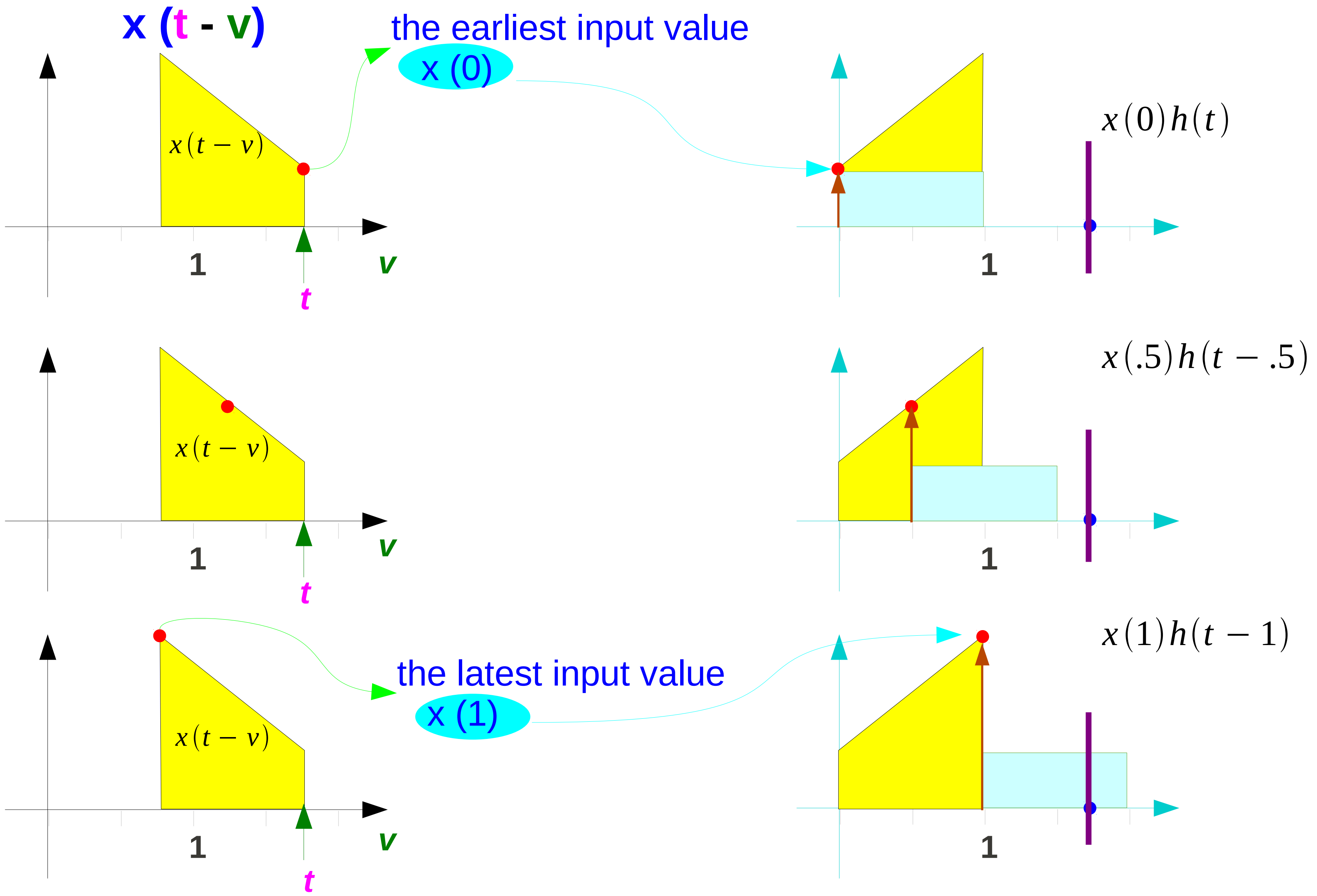


$$x(t-t+.5)h(t-.5)$$
$$= \; x(.5)h(t-.5)$$



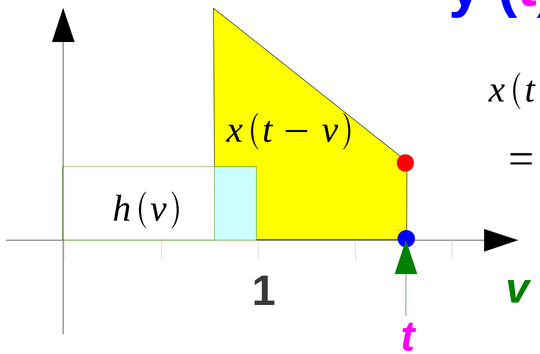
$$x(t-t+1)h(t-1)$$
$$x(1)h(t-1)$$

Impulse Response

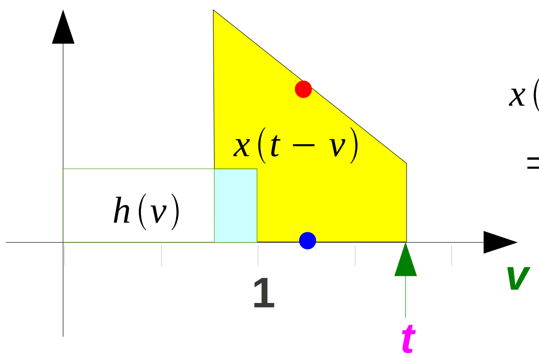


Impulse Response

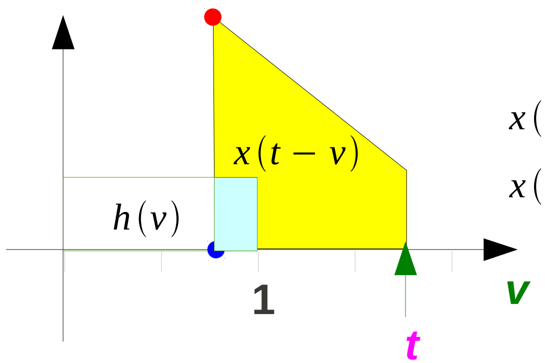
y (t)



$$x(t-t)h(t)$$
$$= x(0)h(t)$$

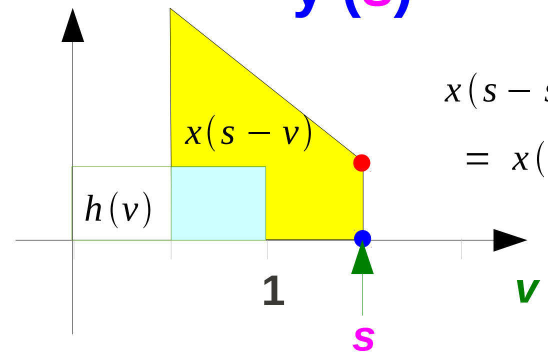


$$x(t-t+.5)h(t-.5)$$
$$= x(.5)h(t-.5)$$

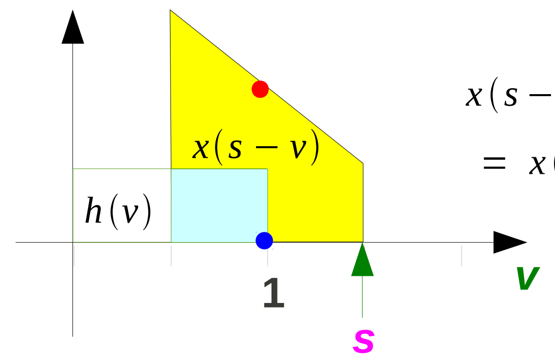


$$x(t-t+1)h(t-1)$$
$$x(1)h(t-1)$$

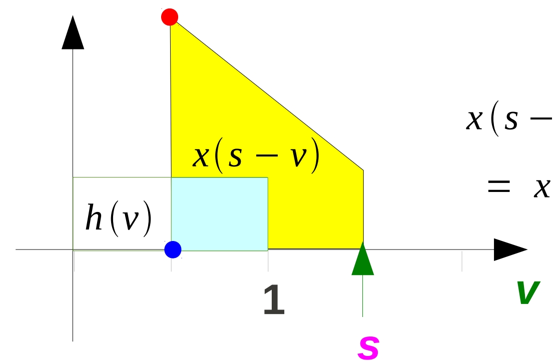
y (s)



$$x(s-s)h(s)$$
$$= x(0)h(s)$$

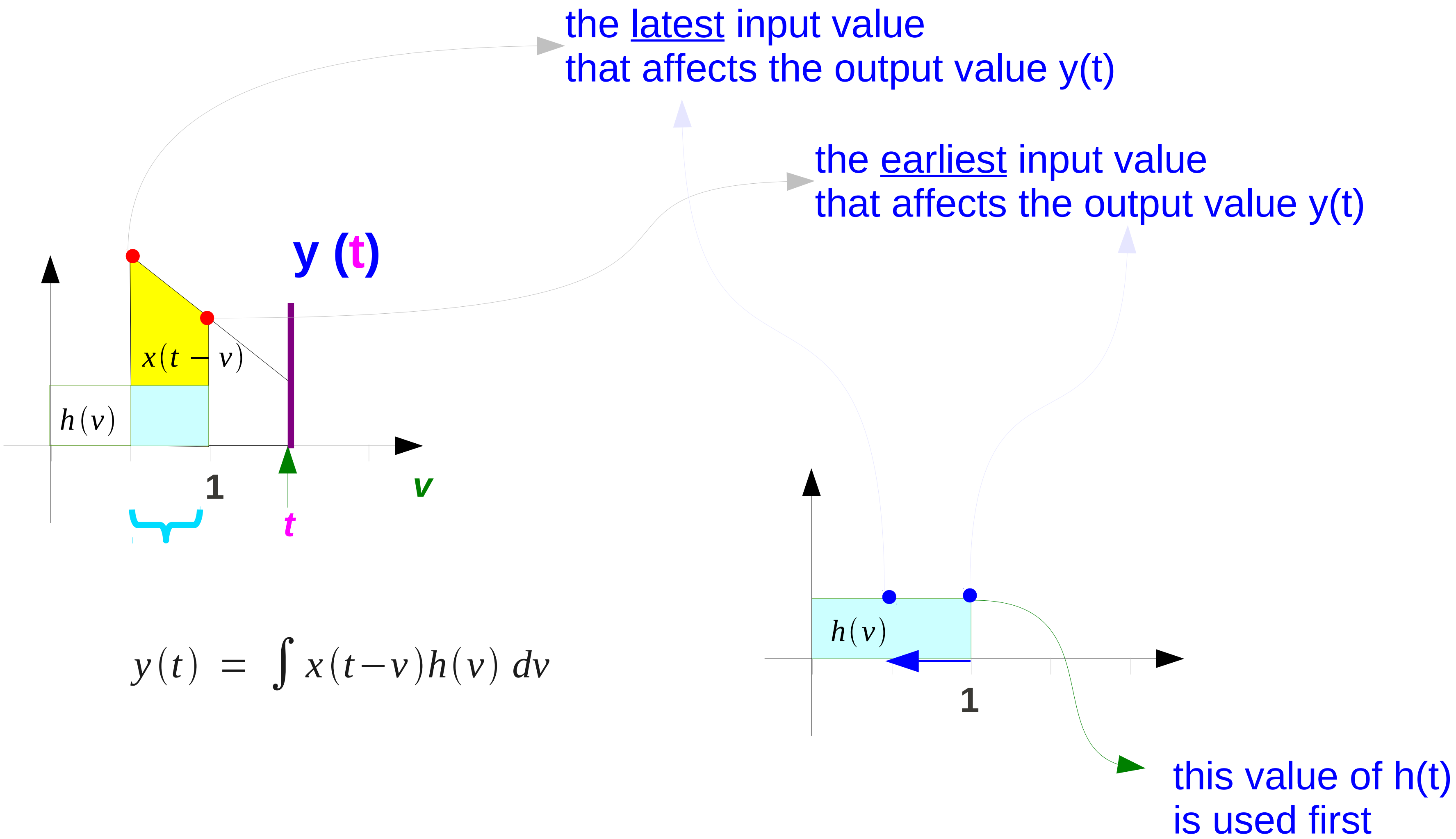


$$x(s-s+.5)h(s-.5)$$
$$= x(.5)h(s-.5)$$



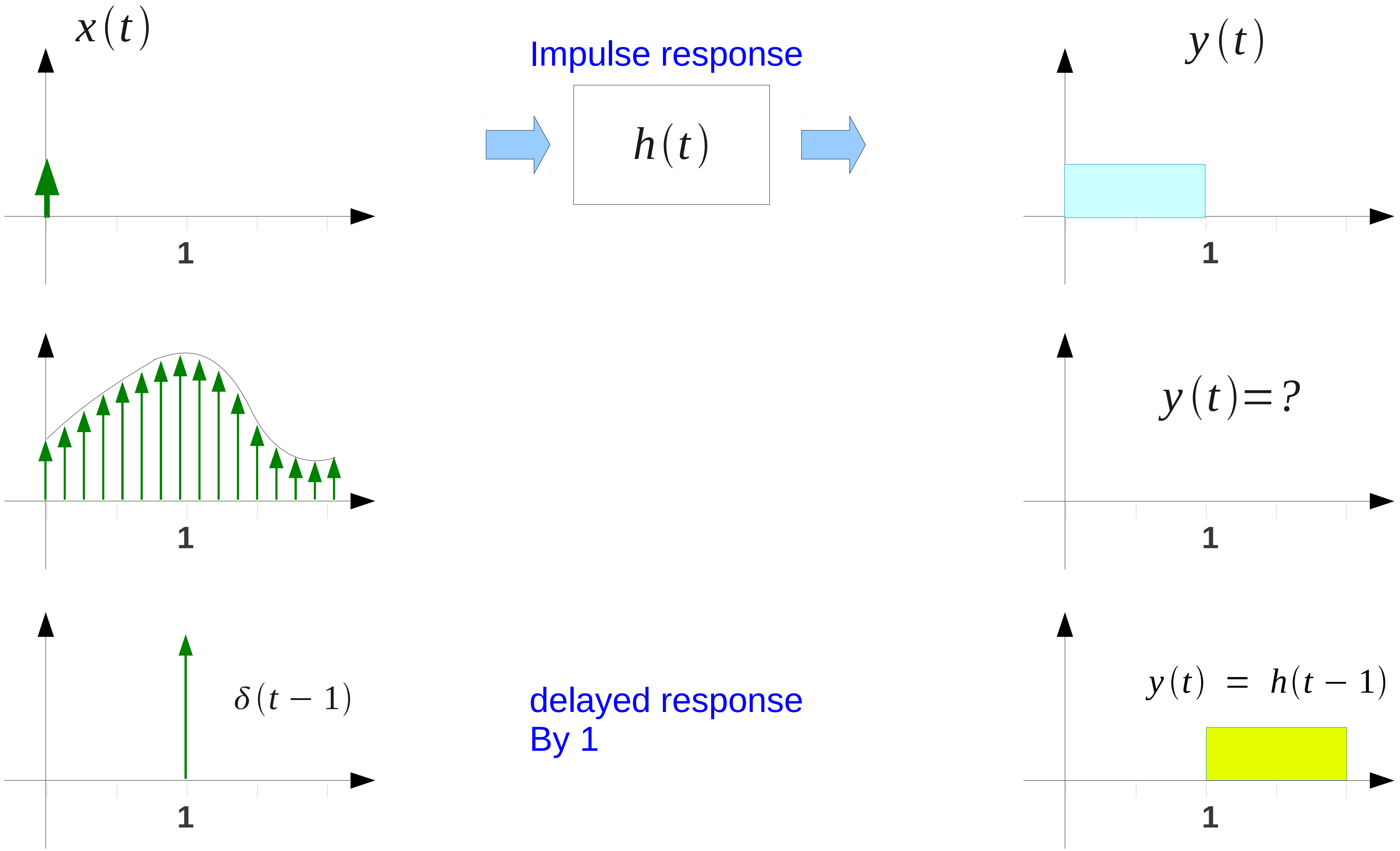
$$x(s-s+1)h(s-1)$$
$$= x(1)h(s-1)$$

Impulse Response

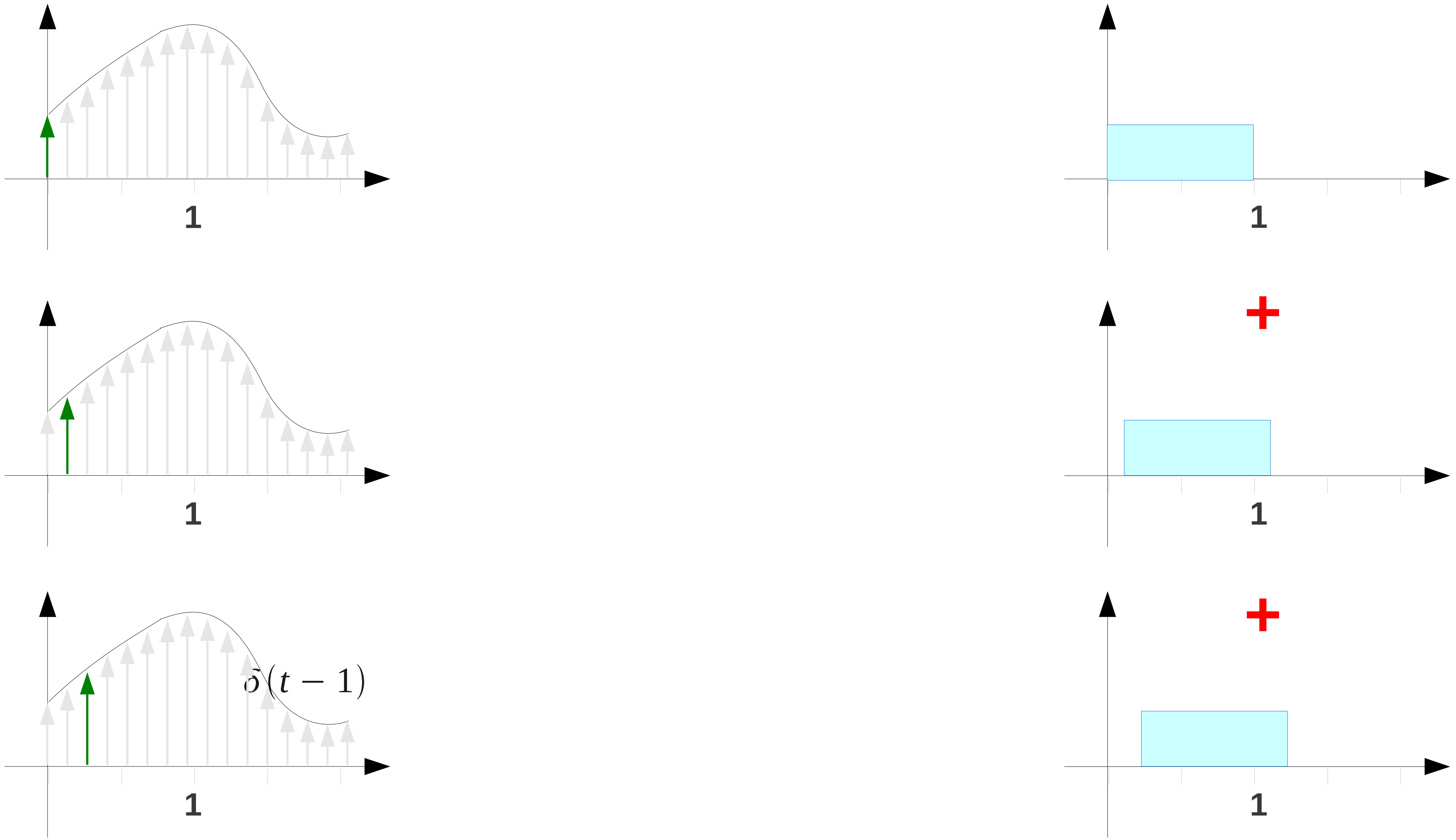


$$y(t) = \int x(t-v)h(v)\,dv$$

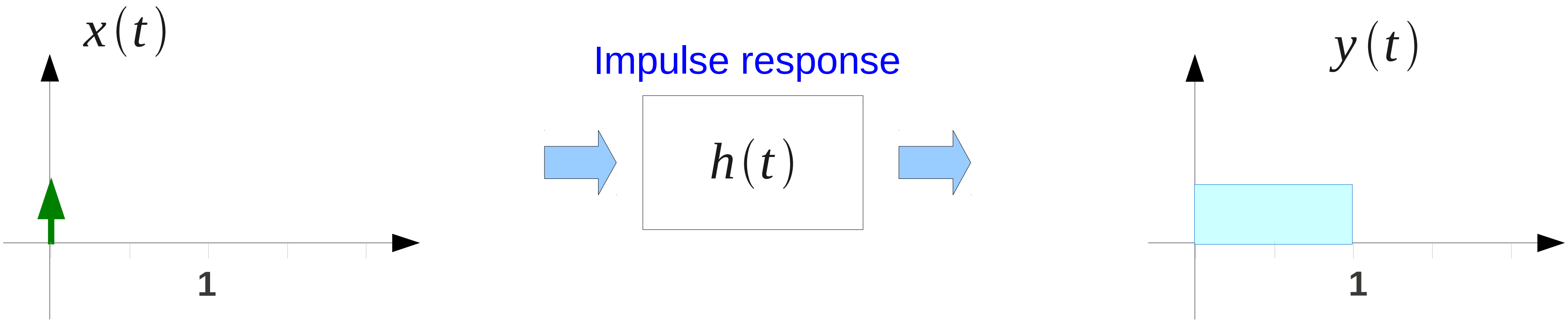
Impulse Response

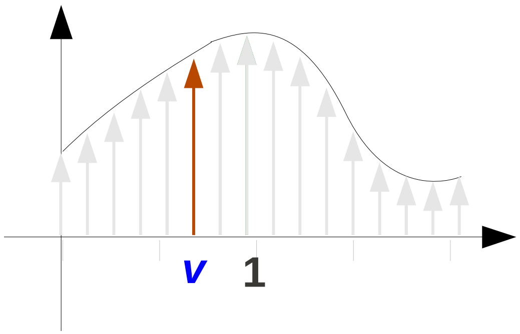


Impulse Response

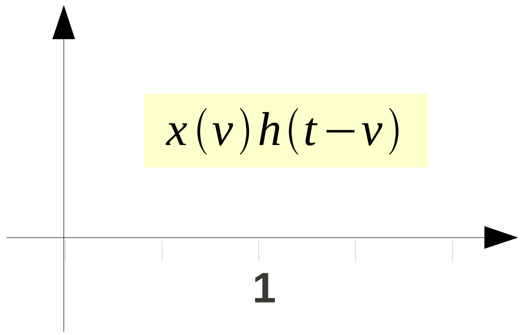


Impulse Response

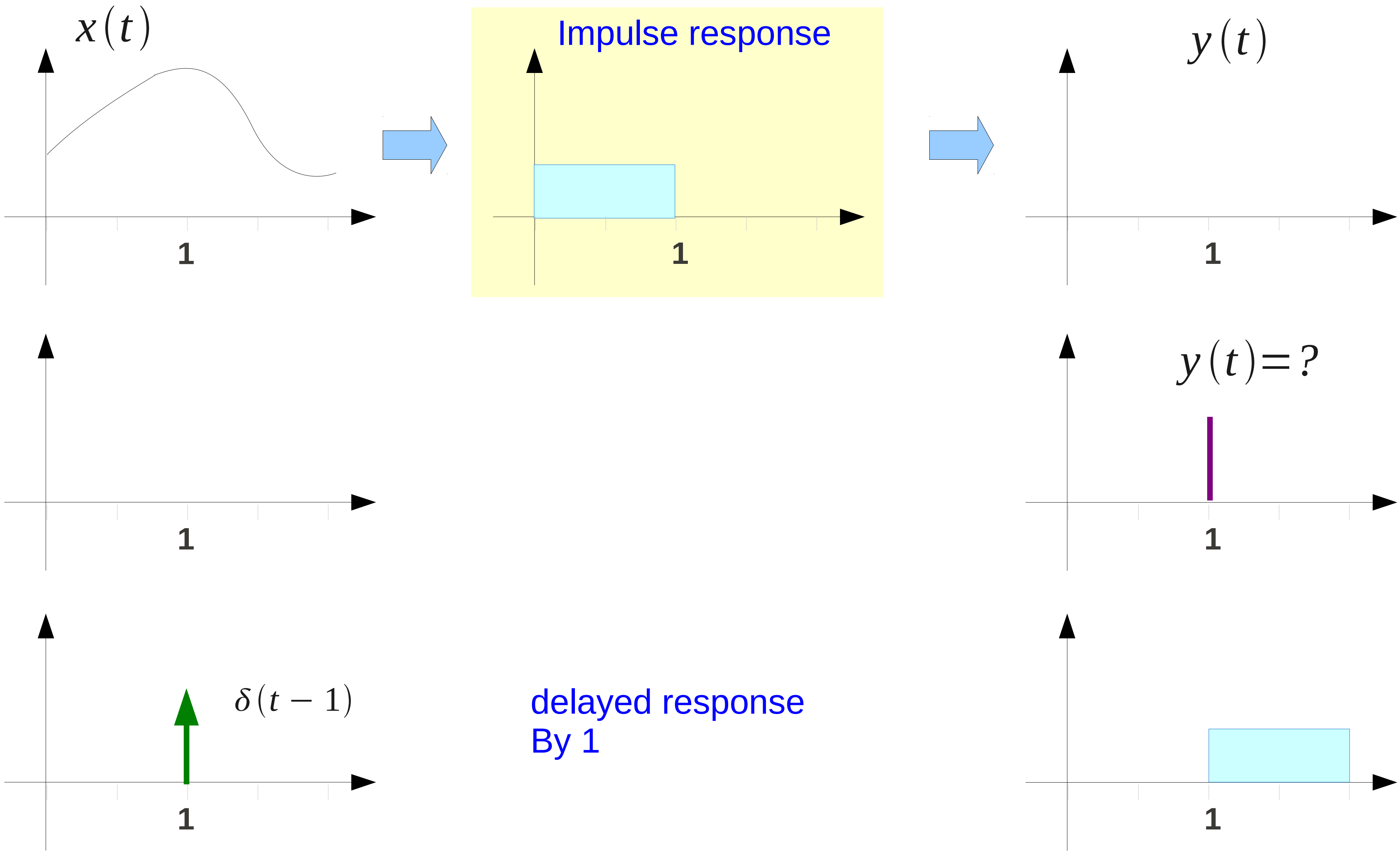




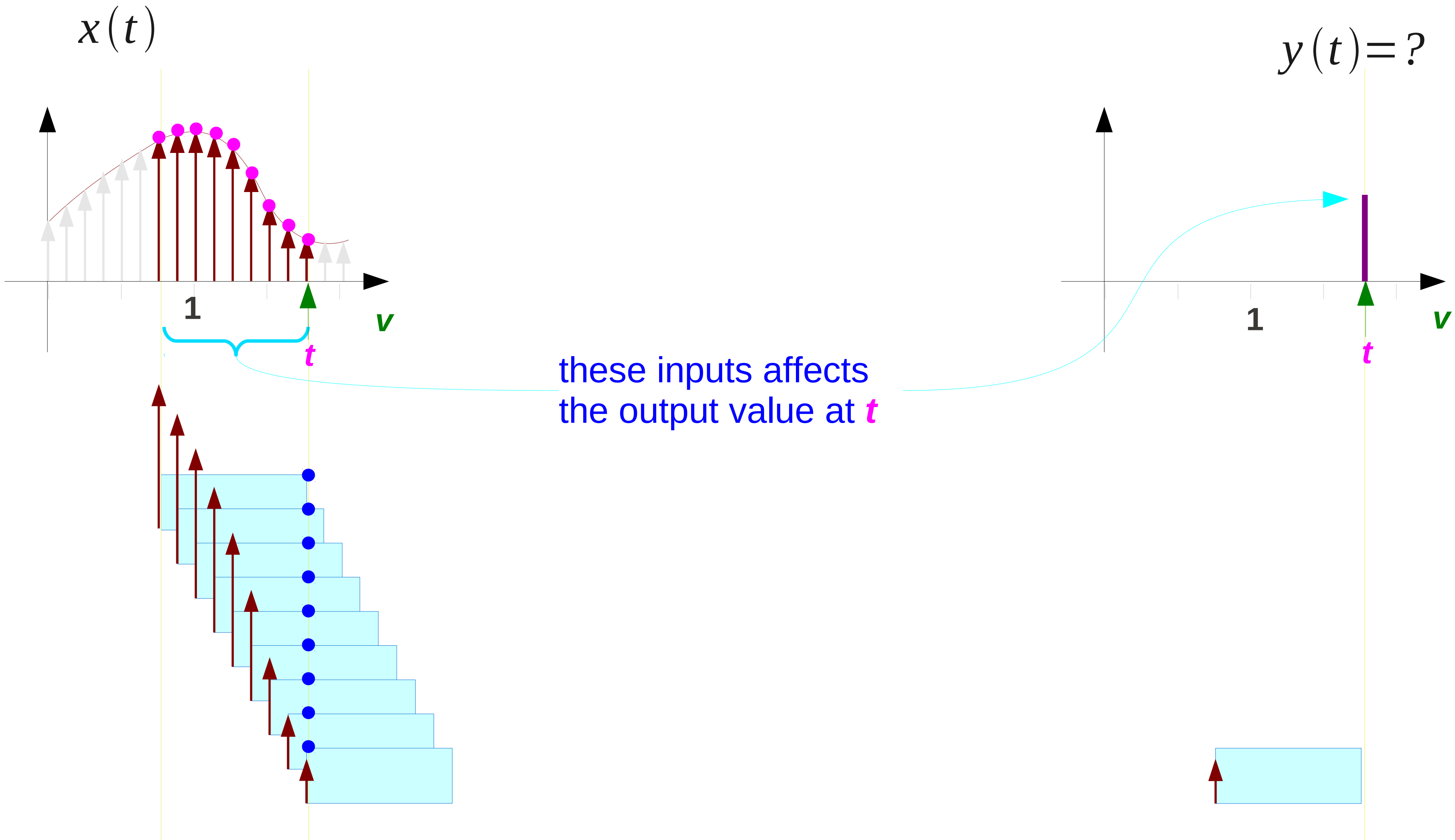
input value at time v
→ x (v)
delayed impulse response
→ h (t - v)



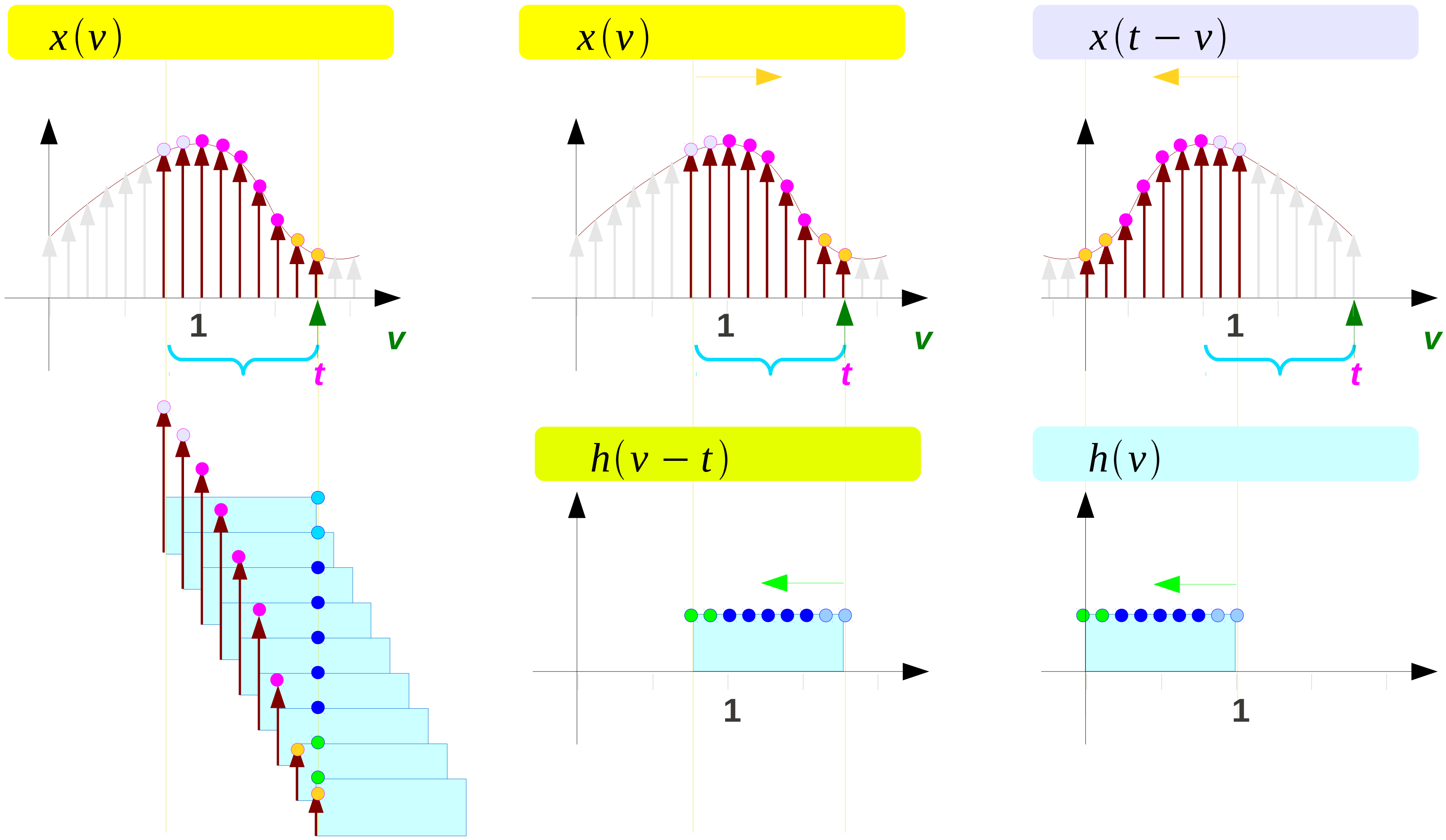
Impulse Response



Impulse Response



Impulse Response



References

[1] http://en.wikipedia.org/

[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003