

Convolution (1A)

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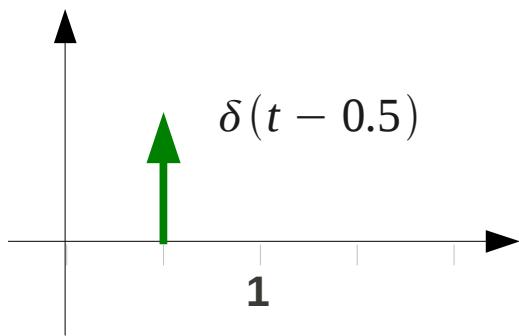
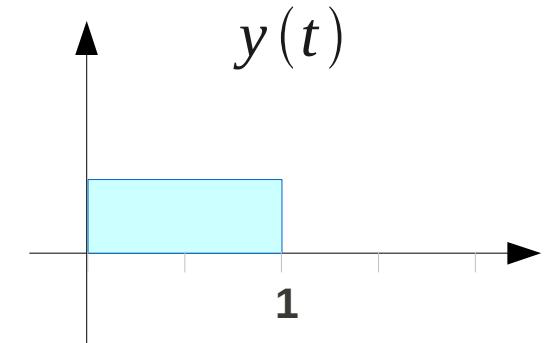
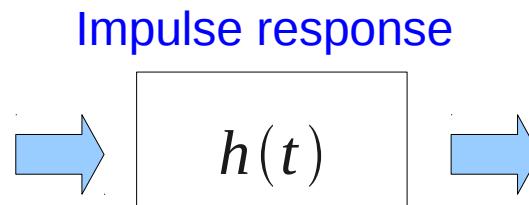
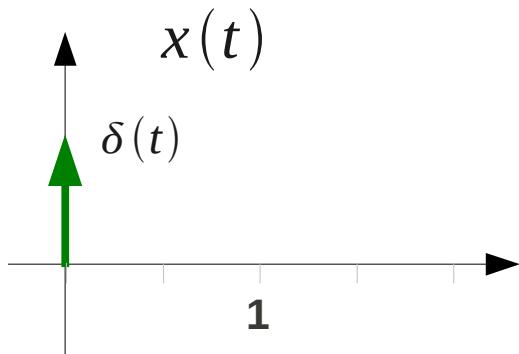
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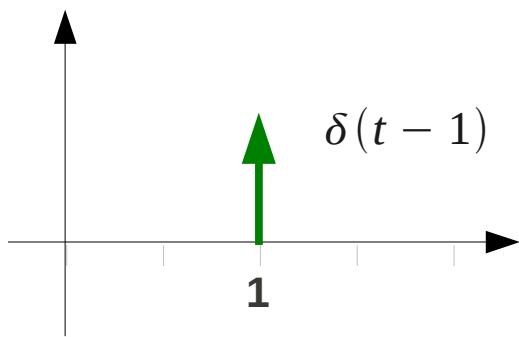
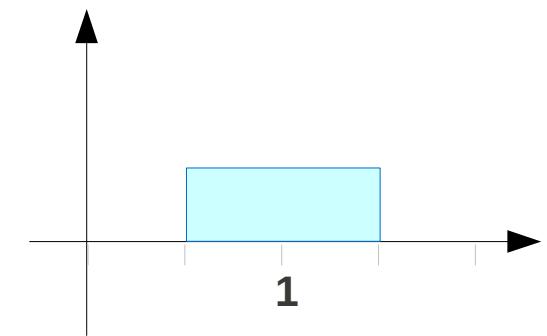
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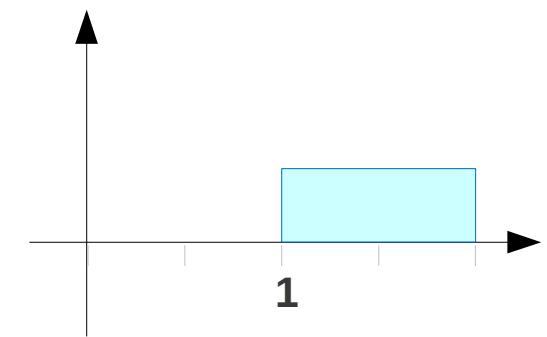
Impulse Response



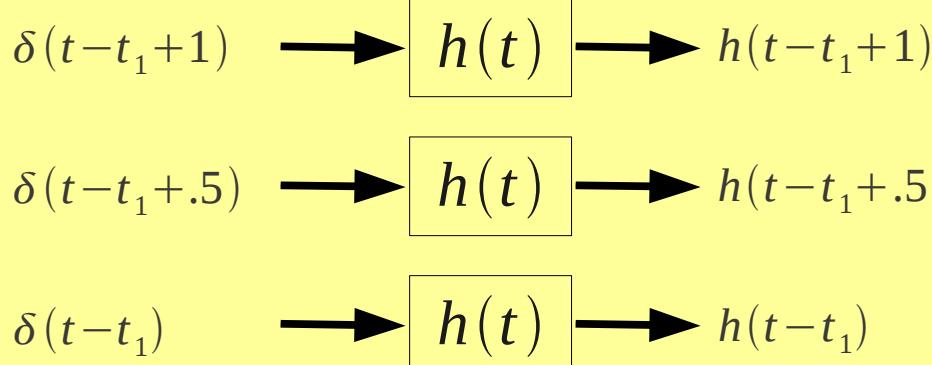
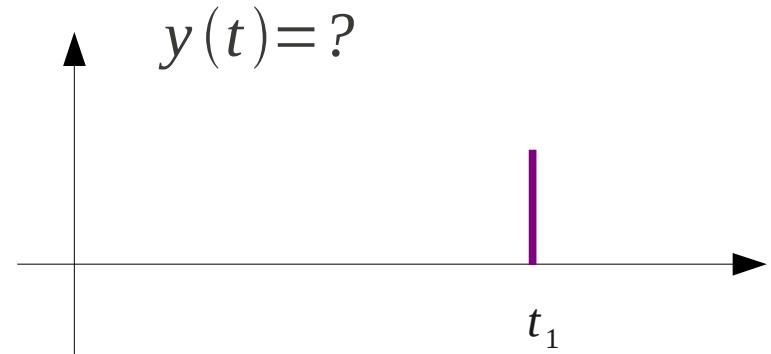
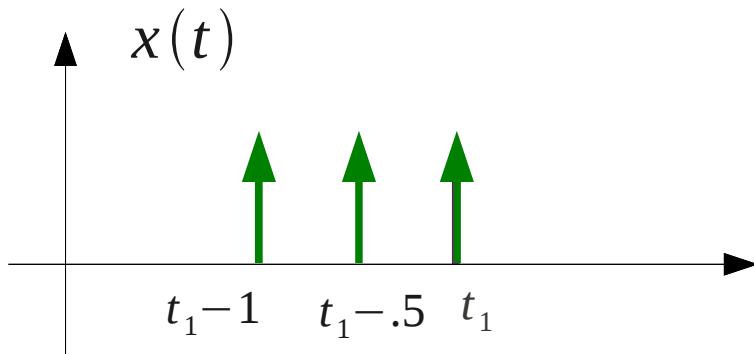
delayed response
by 0.5



delayed response
by 1



LTI System

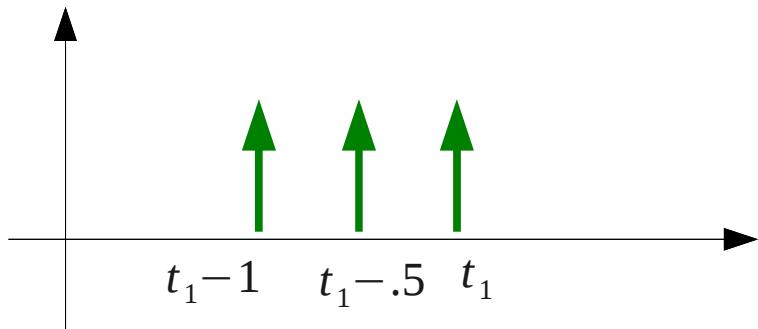


A flowchart showing the addition of the three impulse responses to form the total output $y(t)$:

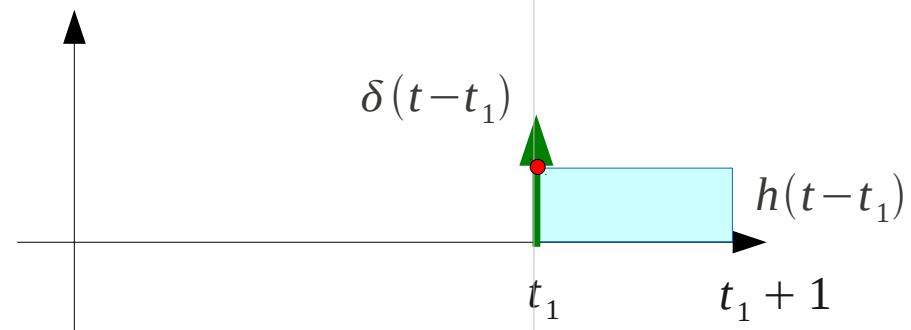
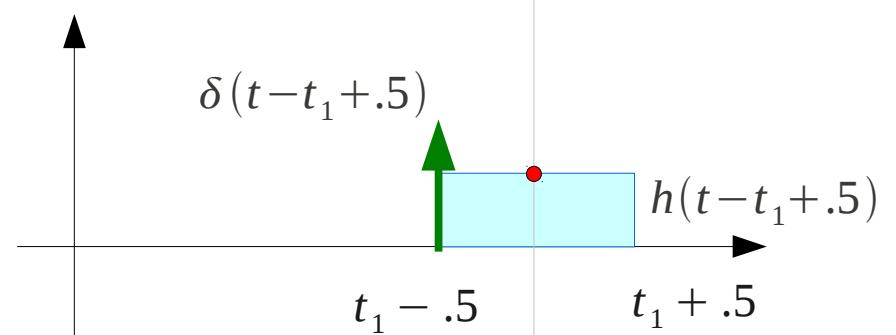
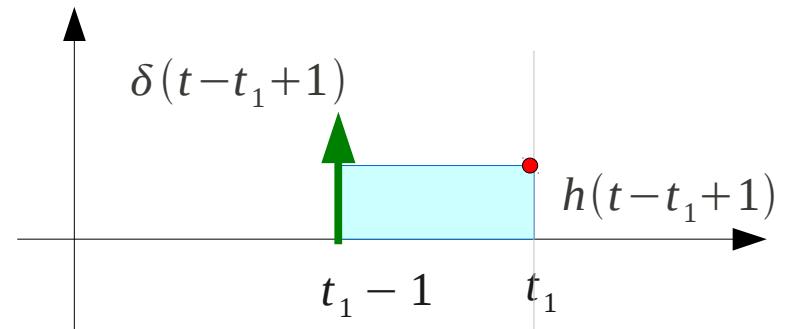
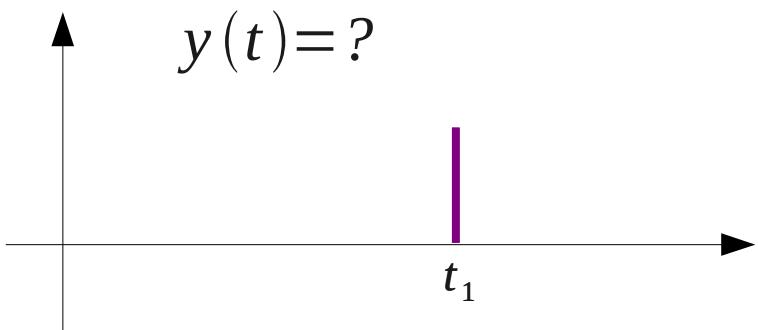
\rightarrow $h(t)$ \rightarrow

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

Output at $t = t_1$



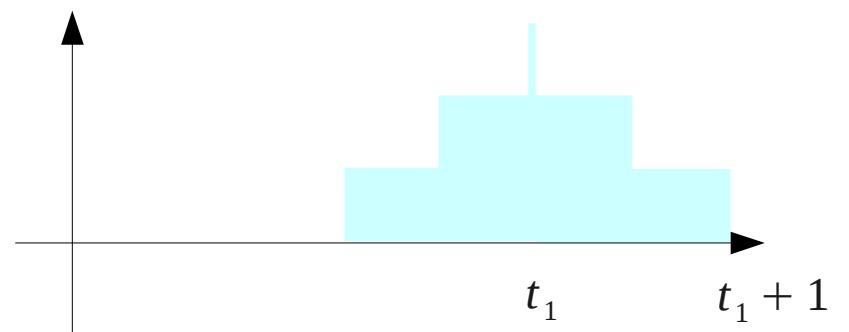
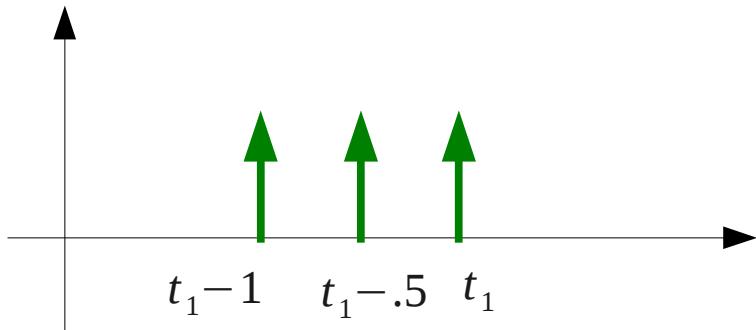
$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$



Using Convolution

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

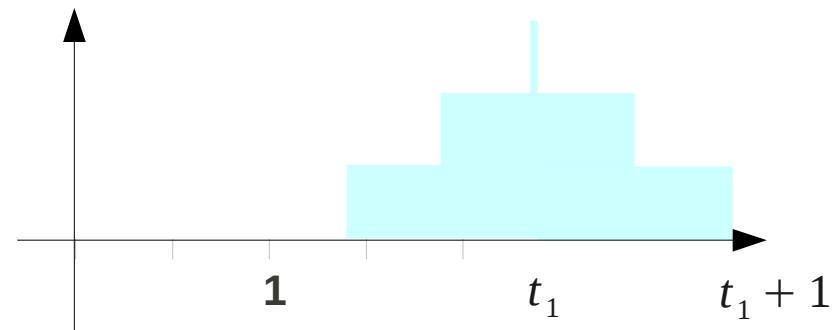
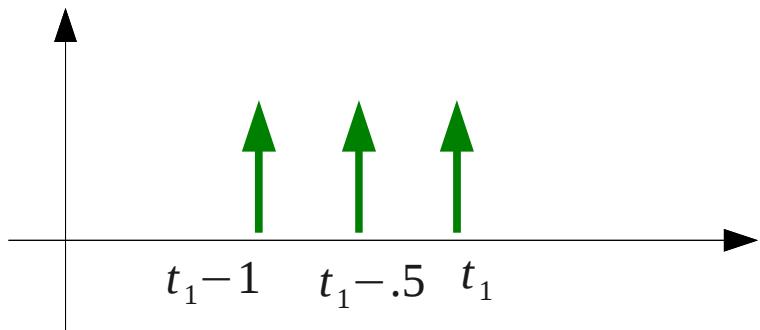
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$



$$\begin{aligned} y(t) &= \int x(v)h(t-v) dv \\ &= \int \delta(v-t_1+1)h(t-v) dv && \rightarrow h(t-t_1+1) \\ &+ \int \delta(v-t_1+.5)h(t-v) dv && \rightarrow h(t-t_1+.5) \\ &+ \int \delta(v-t_1)h(t-v) dv && \rightarrow h(t-t_1) \end{aligned}$$

$$\begin{aligned} y(t_1) &= h(t_1-t_1+1) + h(t_1-t_1+.5) + h(t_1-t_1) \\ &= h(1) + h(.5) + h(0) \end{aligned}$$

N=8 DFT

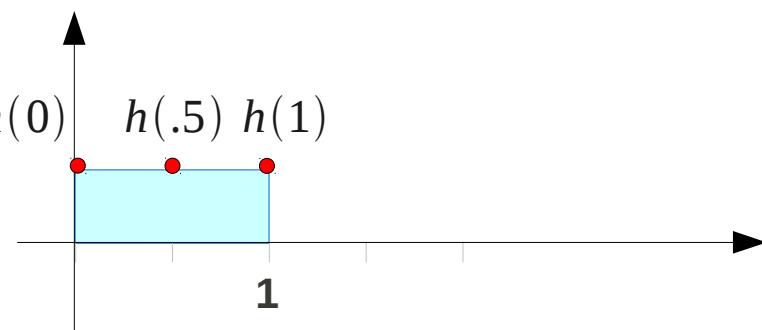
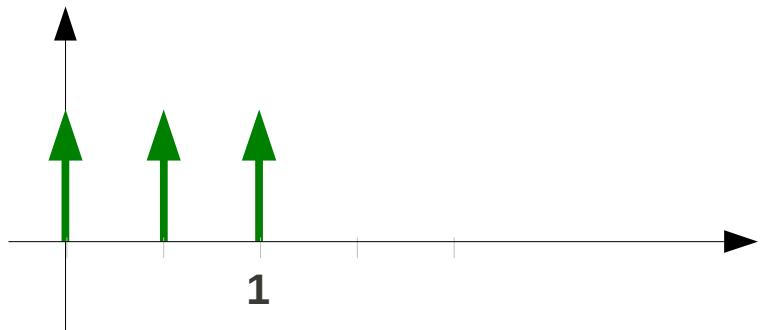


$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$x(t) = \delta(t) + \delta(t-.5) + \delta(t-1)$$

$$y(t_1) = h(1) + h(.5) + h(0)$$



The Meaning of Convolution

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

↓ Change of variables $t \rightarrow v$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

↓ Flip around y axis and shift to the right by t $v \rightarrow t-v$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

$$\begin{aligned} y(t) &= \int x(t-v) h(v) dv \\ &= \underbrace{\int \delta(t-v-t_1+1) h(v) dv}_{\rightarrow h(t-t_1+1)} \\ &\quad + \underbrace{\int \delta(t-v-t_1+.5) h(v) dv}_{\rightarrow h(t-t_1+.5)} \\ &\quad + \underbrace{\int \delta(t-v-t_1) h(v) dv}_{\rightarrow h(t-t_1)} \end{aligned}$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\Rightarrow y(t_1) = h(1) + h(.5) + h(0)$$

The Meaning of Convolution

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

↓ Change of variables $t \rightarrow v$

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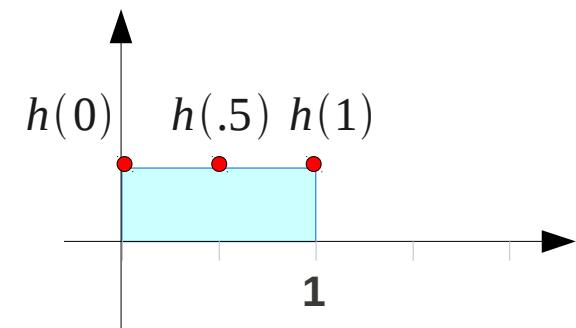
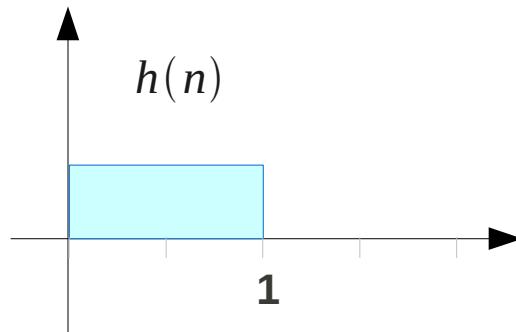
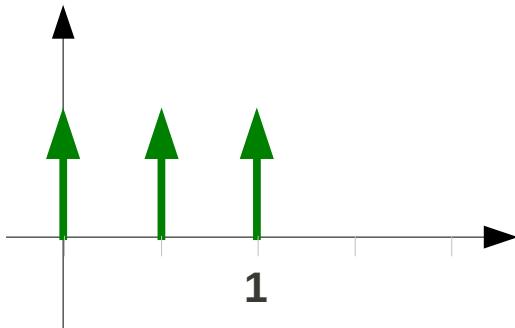
$$\begin{aligned} y(t) &= \int x(t-v) h(v) dv \\ &= \underbrace{\int \delta(t-v-t_1+1) h(v) dv}_{\rightarrow h(t-t_1+1)} \\ &\quad + \underbrace{\int \delta(t-v-t_1+.5) h(v) dv}_{\rightarrow h(t-t_1+.5)} \\ &\quad + \underbrace{\int \delta(t-v-t_1) h(v) dv}_{\rightarrow h(t-t_1)} \end{aligned}$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$\Rightarrow y(t_1) = h(1) + h(.5) + h(0)$$

The Meaning of Convolution

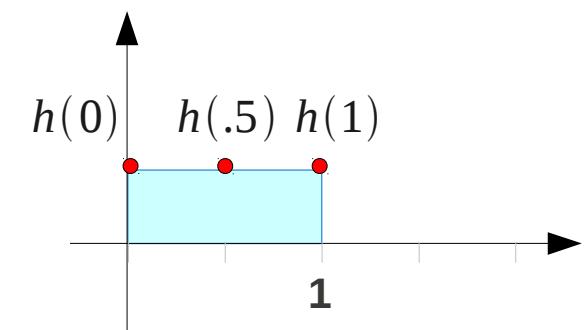
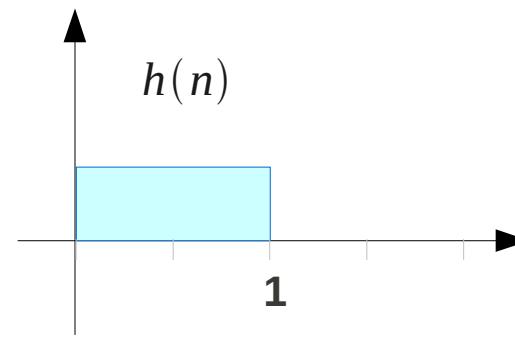
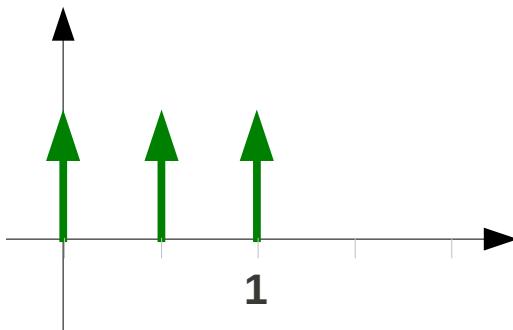
$$\delta(t) + \delta(t-.5) + \delta(t-1)$$



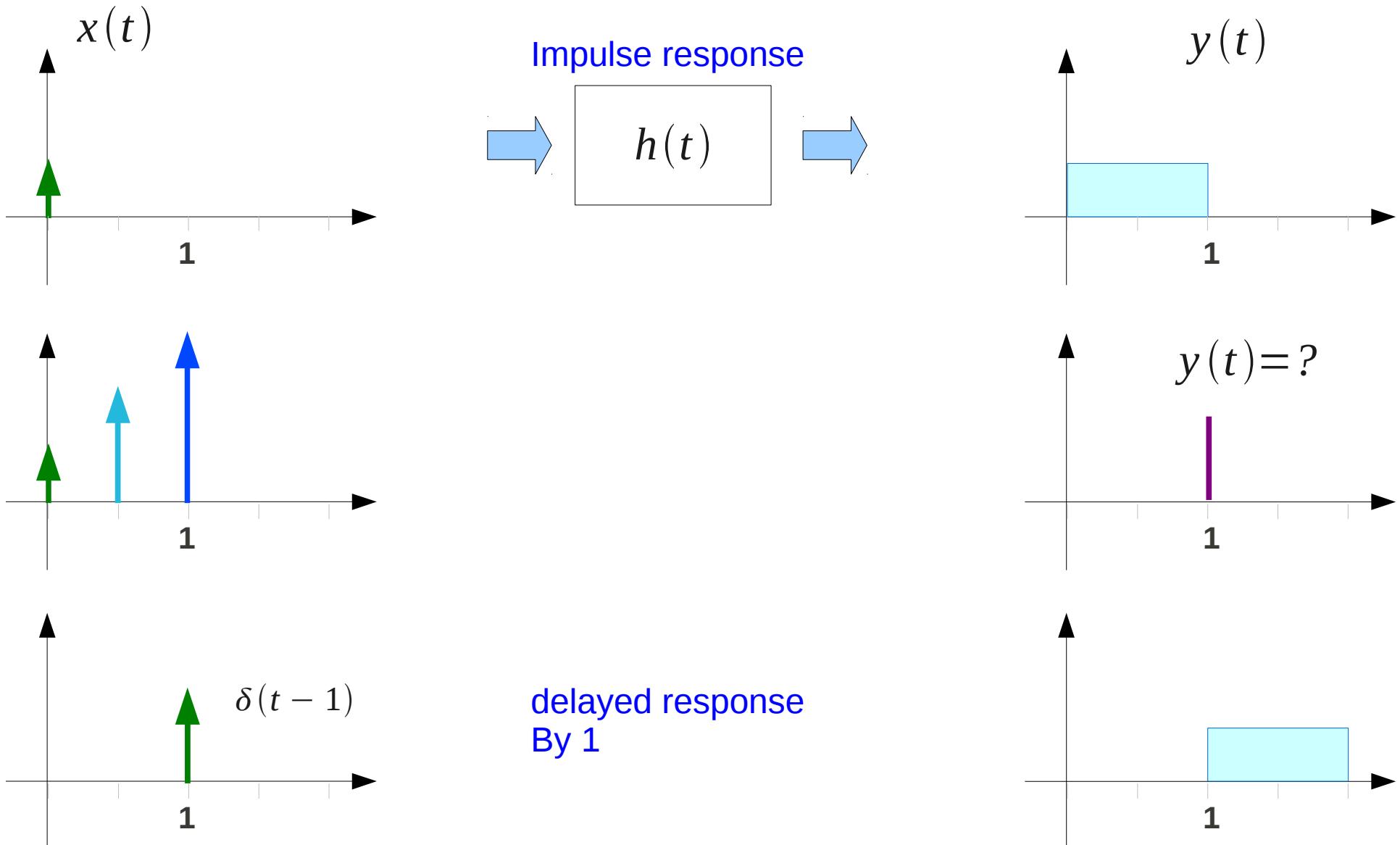
N=8 DFT

$$\begin{aligned}
 y(t) &= \int x(v)h(t-v) dv \\
 &= \int \delta(v-t_1+1)h(t-v) dv && \rightarrow h(t-t_1+1) \\
 &+ \int \delta(v-t_1+.5)h(t-v) dv && \rightarrow h(t-t_1+.5) \\
 &+ \int \delta(v-t_1)h(t-v) dv && \rightarrow h(t-t_1)
 \end{aligned}$$

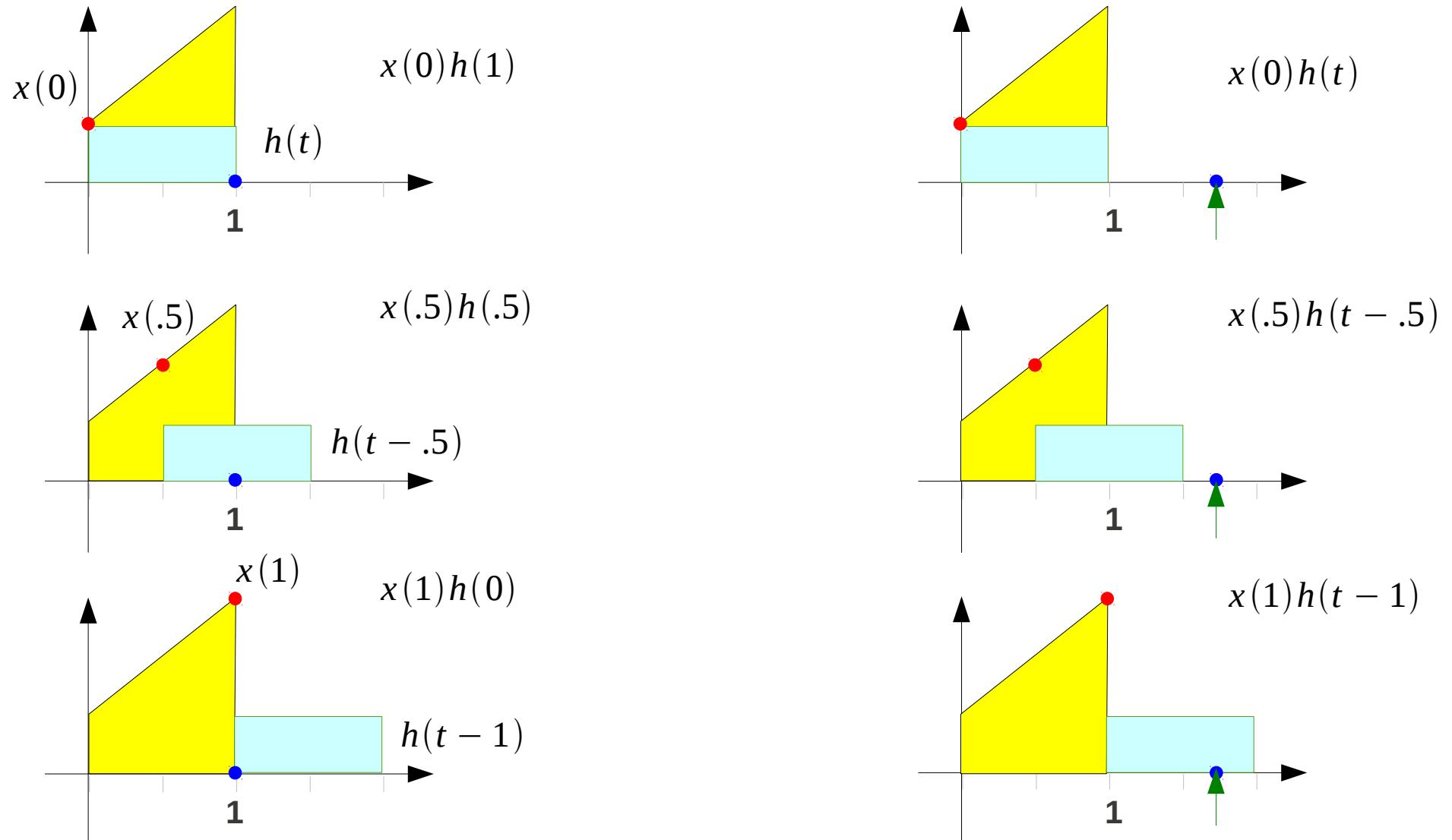
$$\begin{aligned}
 y(t) &= \int x(t-v)h(v) dv \\
 &= \int \delta(t-v-t_1+1)h(v) dv && \rightarrow h(t-t_1+1) \\
 &+ \int \delta(t-v-t_1+.5)h(v) dv && \rightarrow h(t-t_1+.5) \\
 &+ \int \delta(t-v-t_1)h(v) dv && \rightarrow h(t-t_1)
 \end{aligned}$$



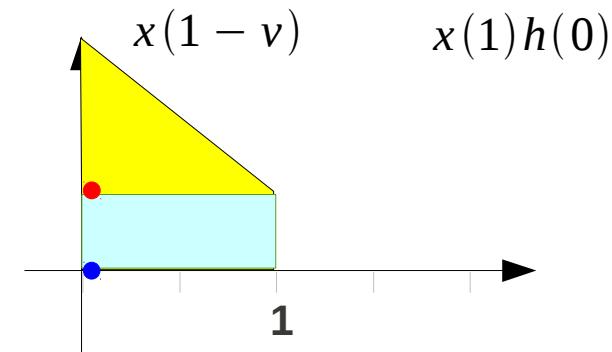
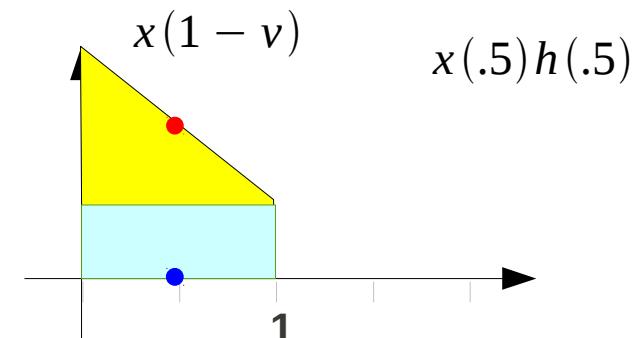
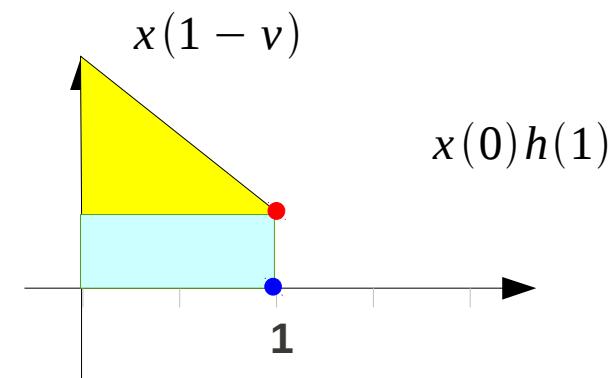
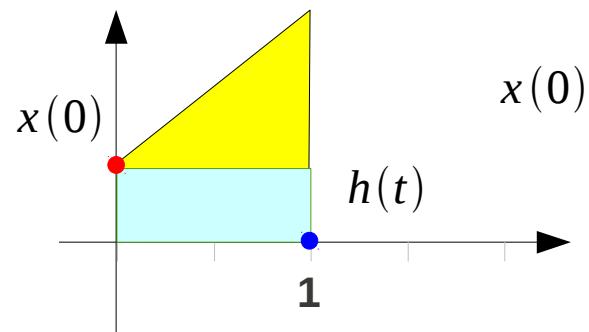
Impulse Response



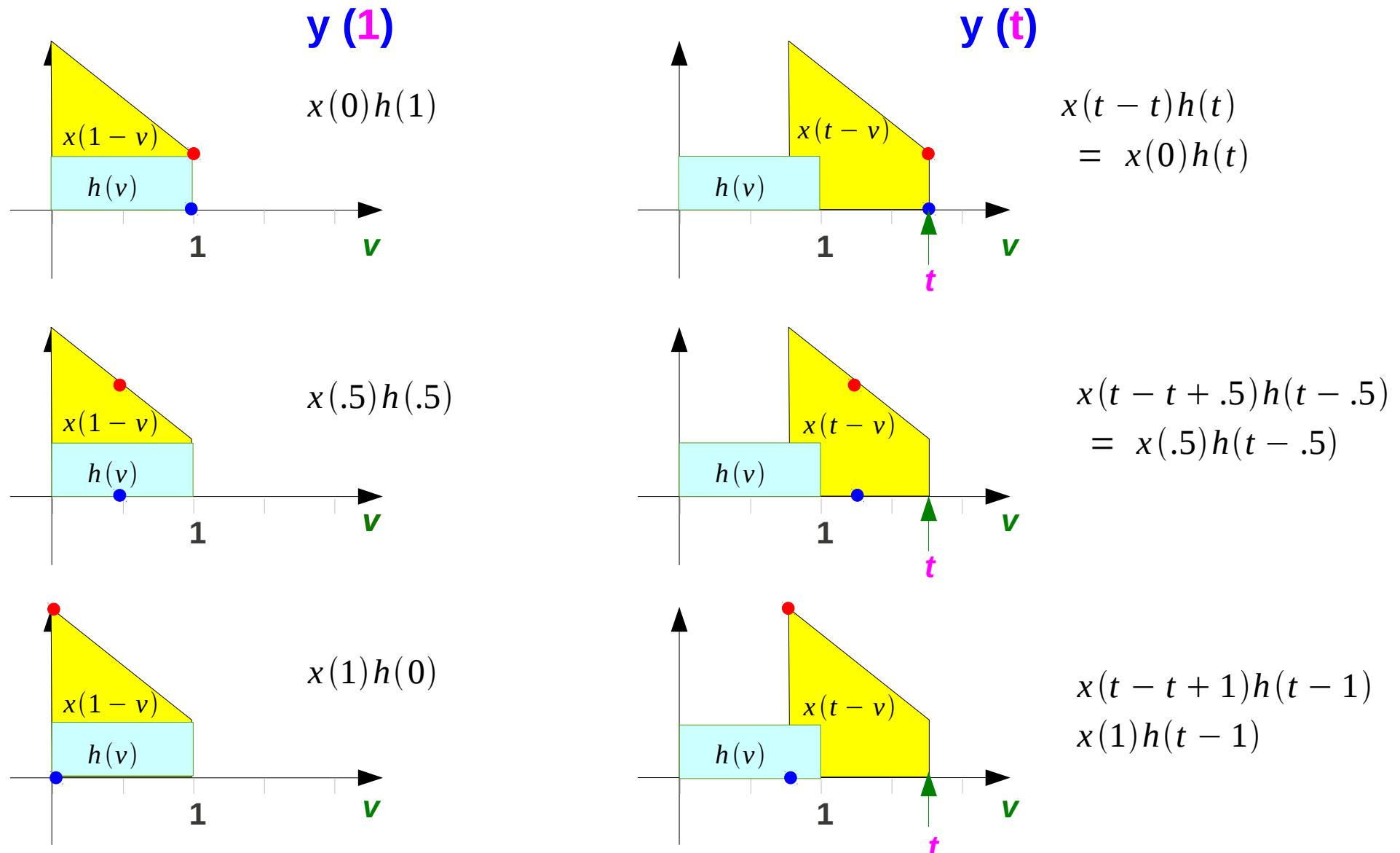
Impulse Response



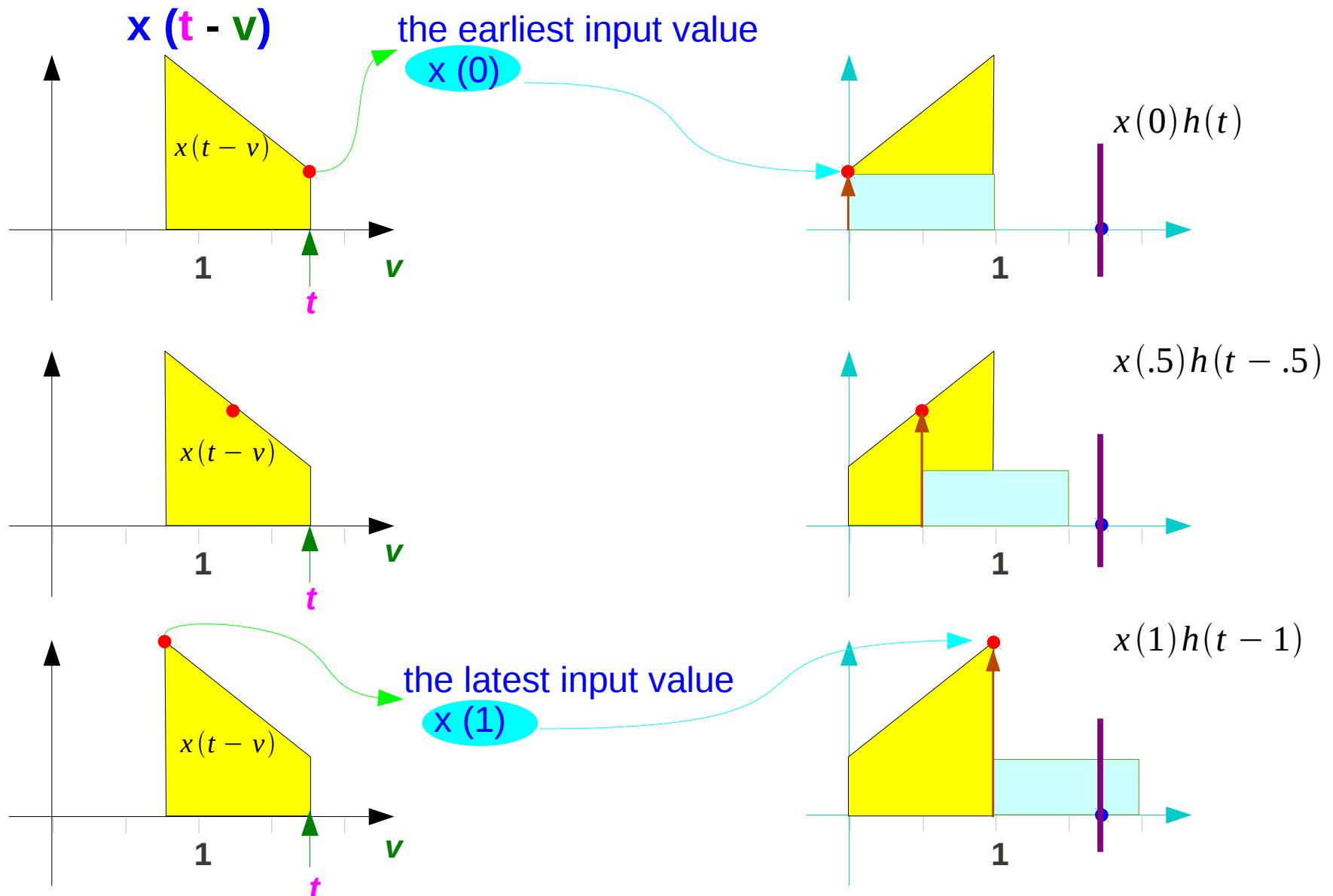
Impulse Response



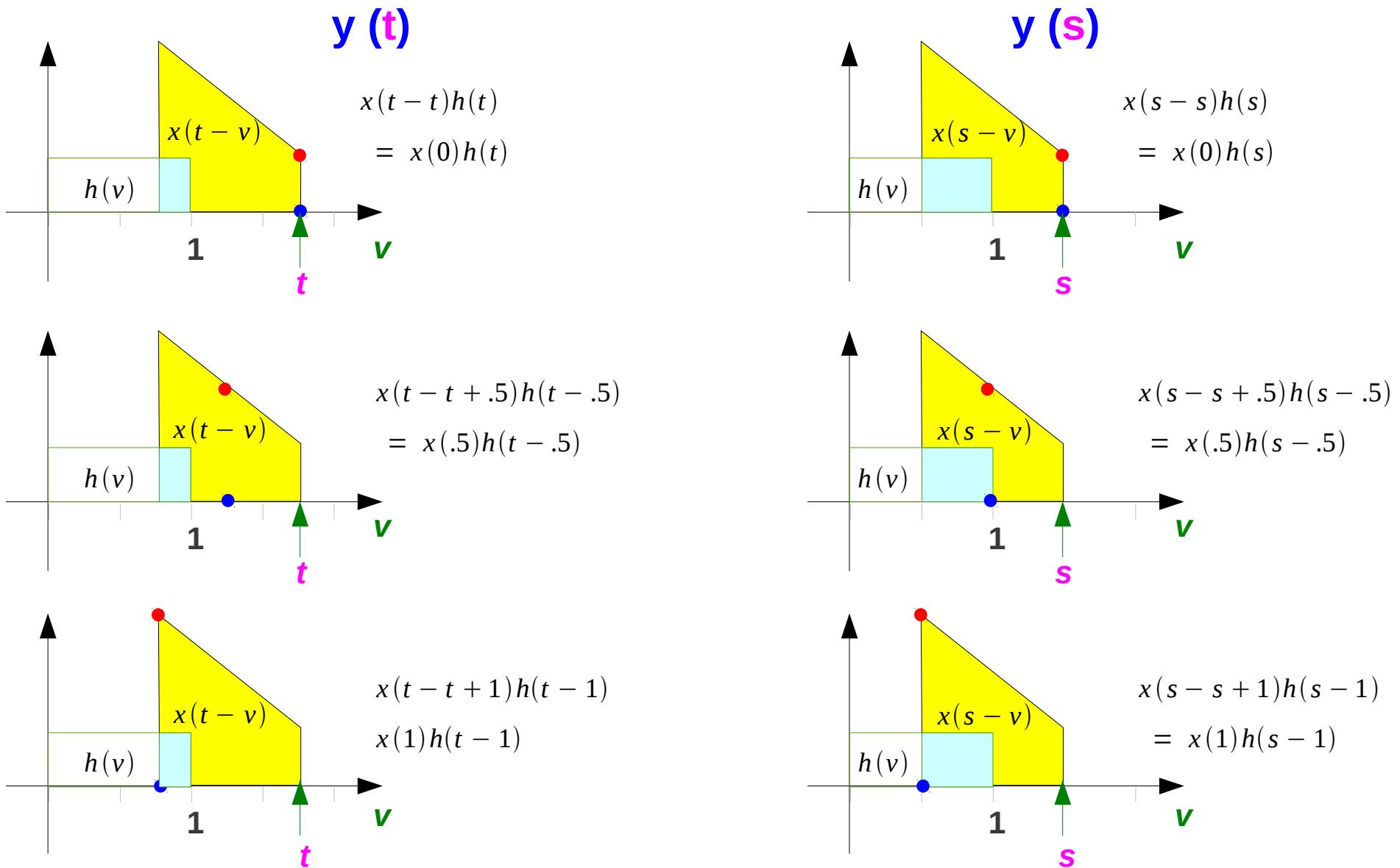
Impulse Response



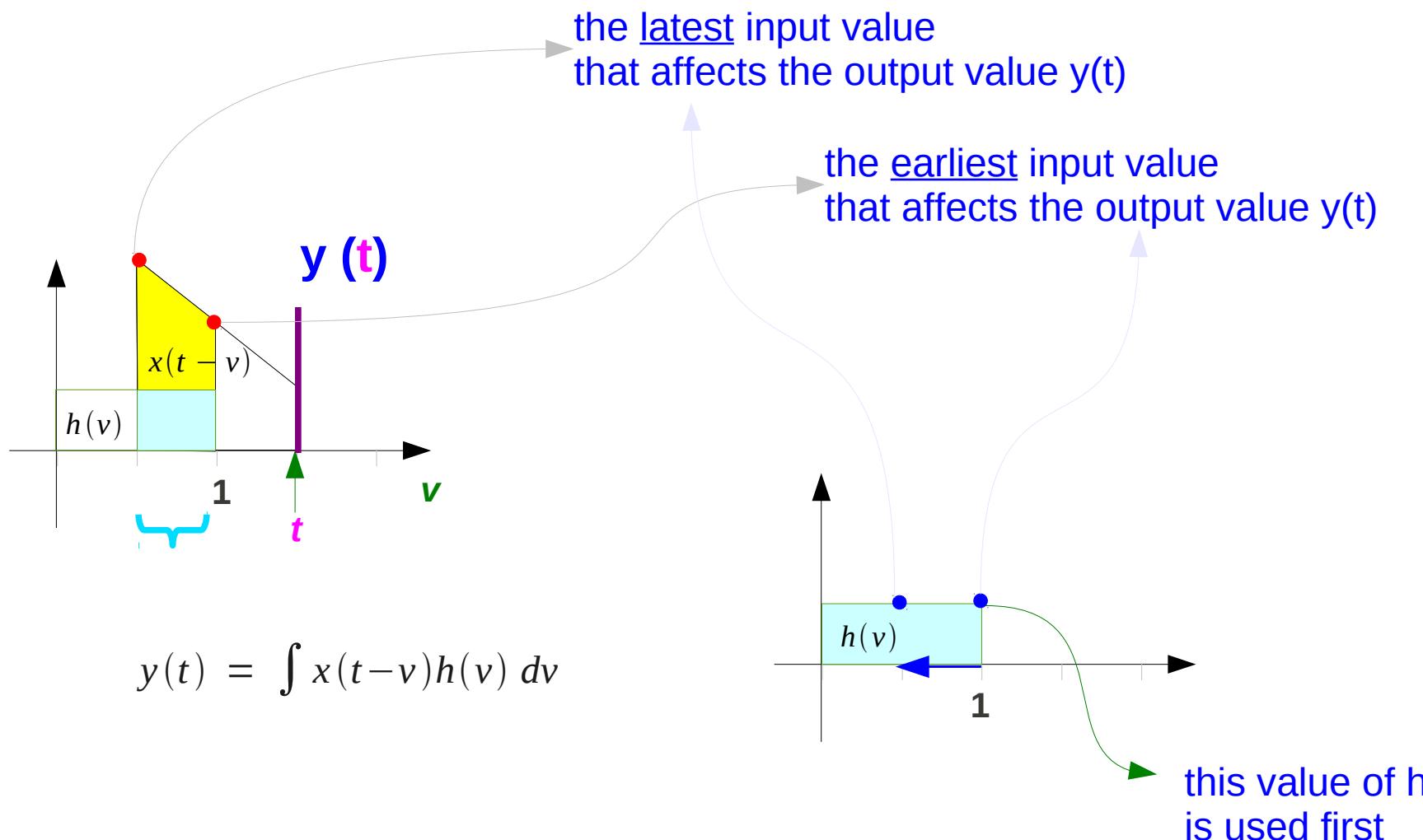
Impulse Response



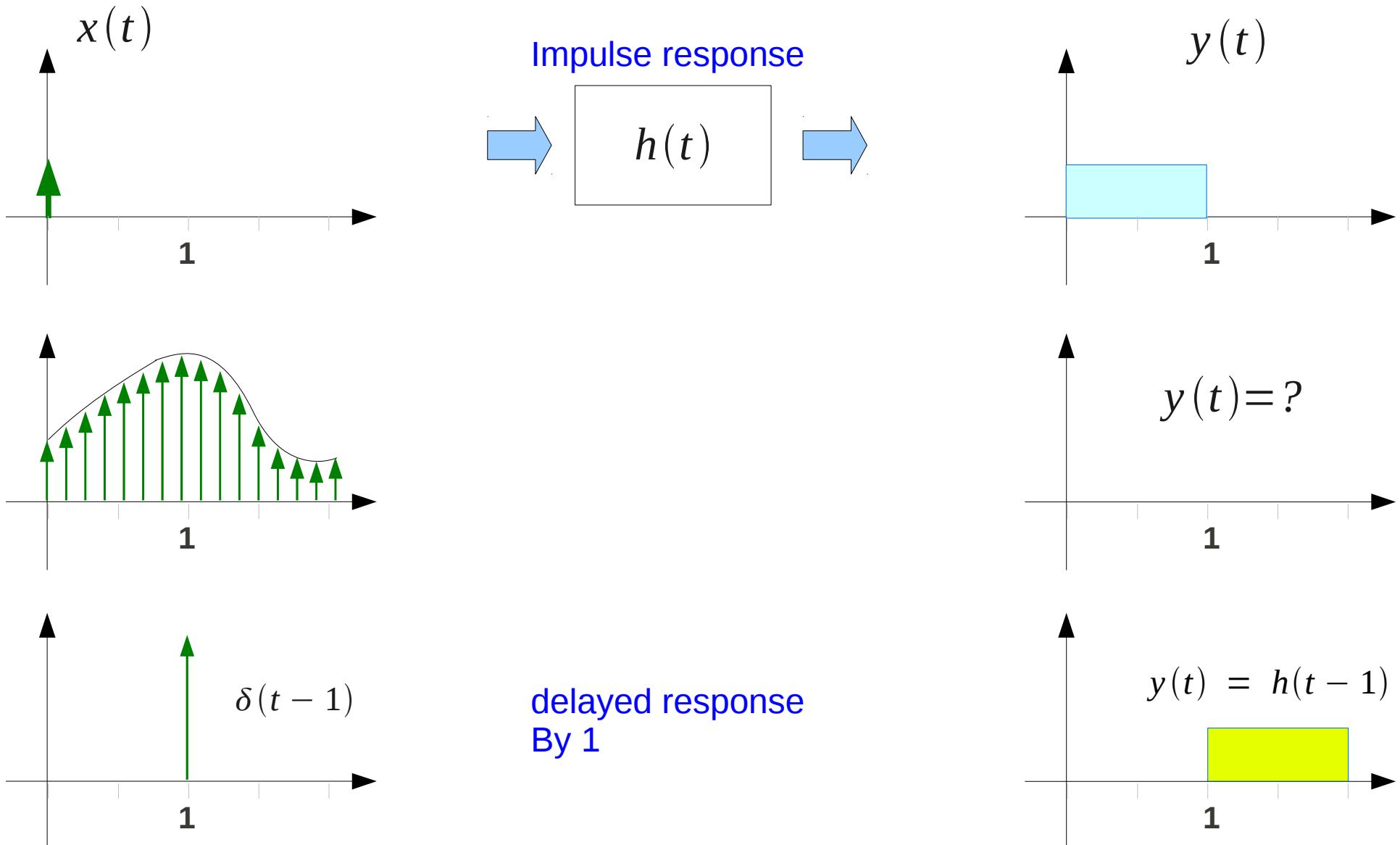
Impulse Response



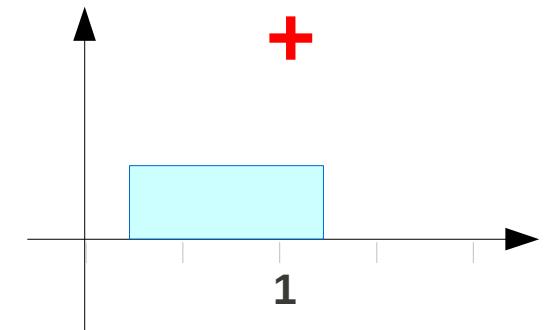
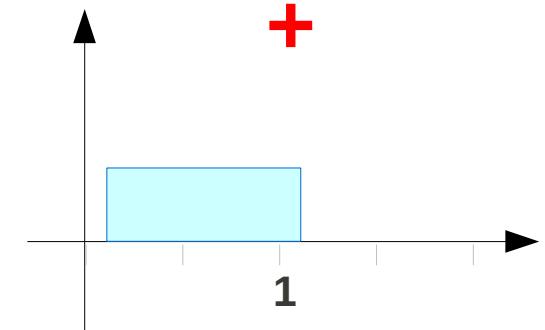
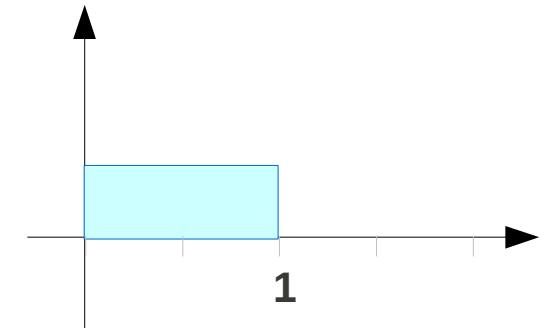
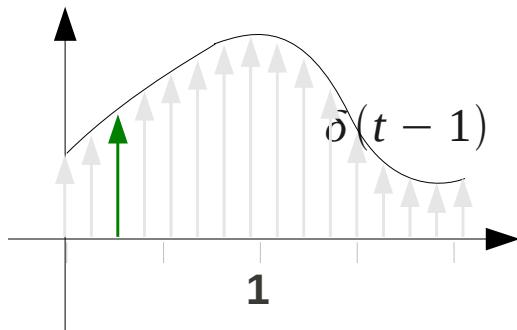
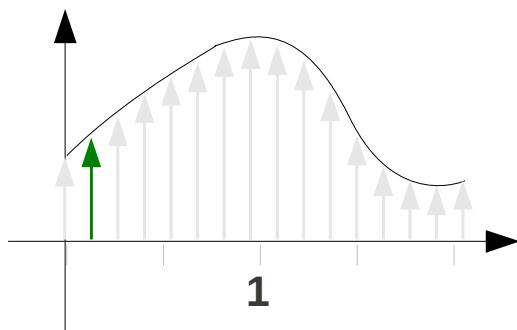
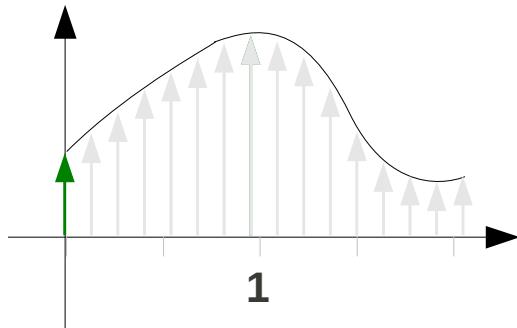
Impulse Response



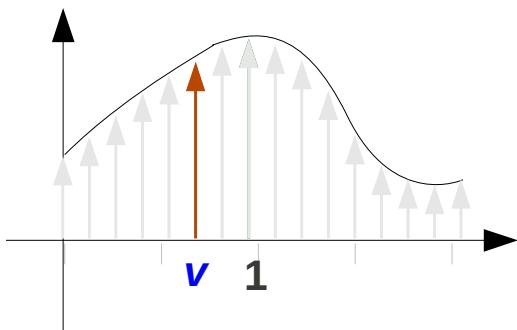
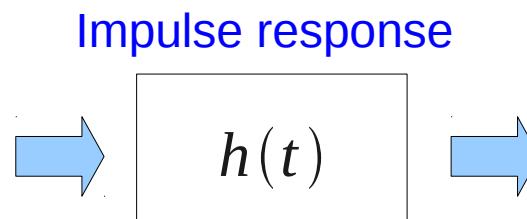
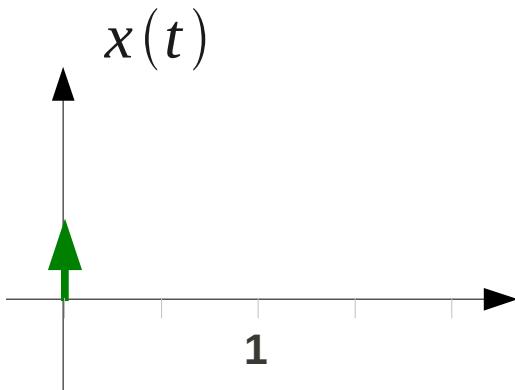
Impulse Response



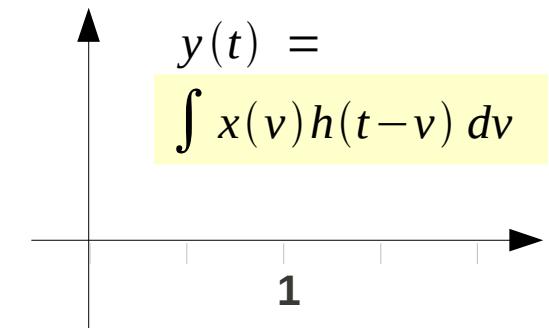
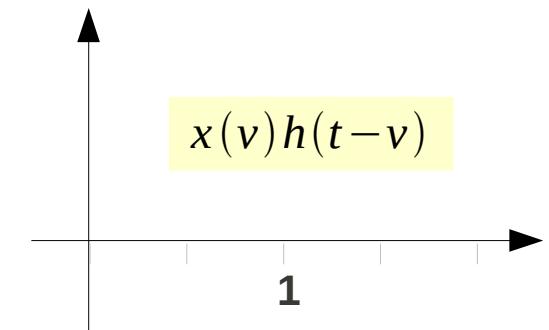
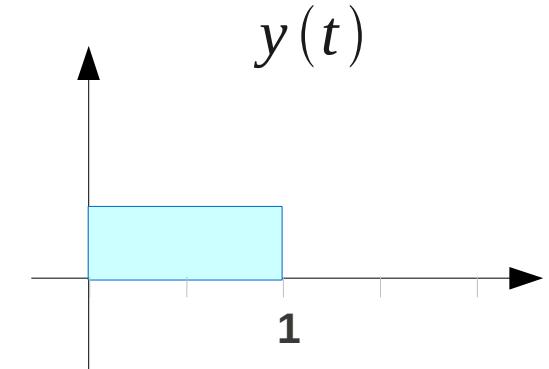
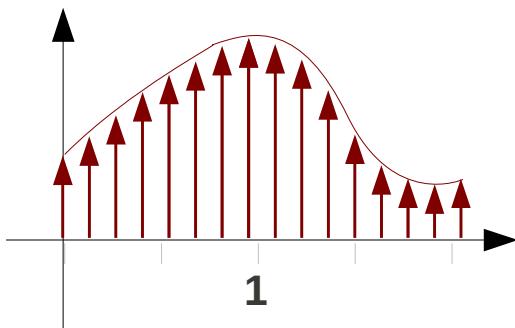
Impulse Response



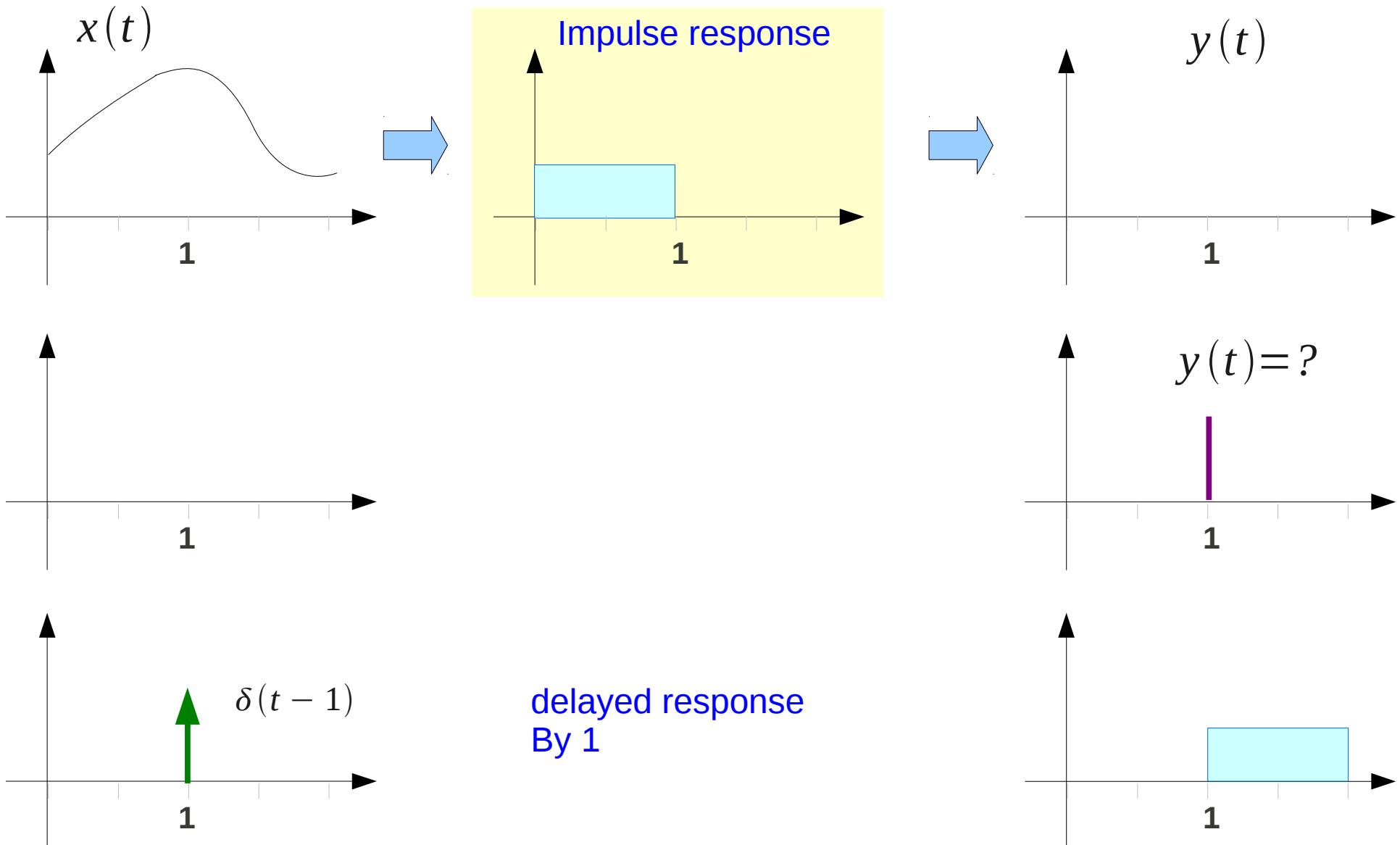
Impulse Response



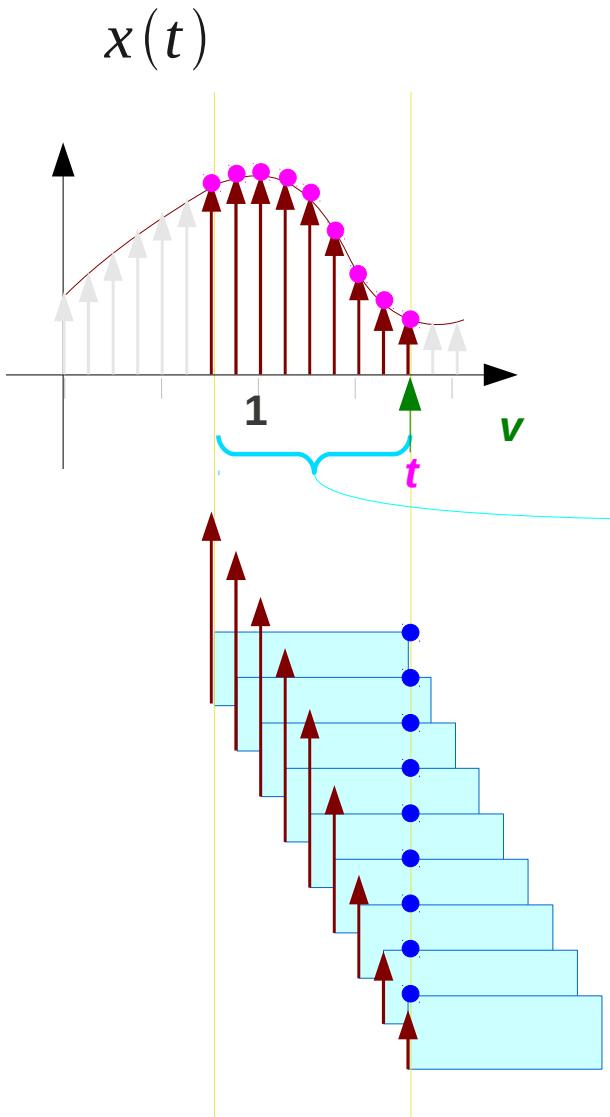
input value at time v
→ $x(v)$
delayed impulse response
→ $h(t-v)$



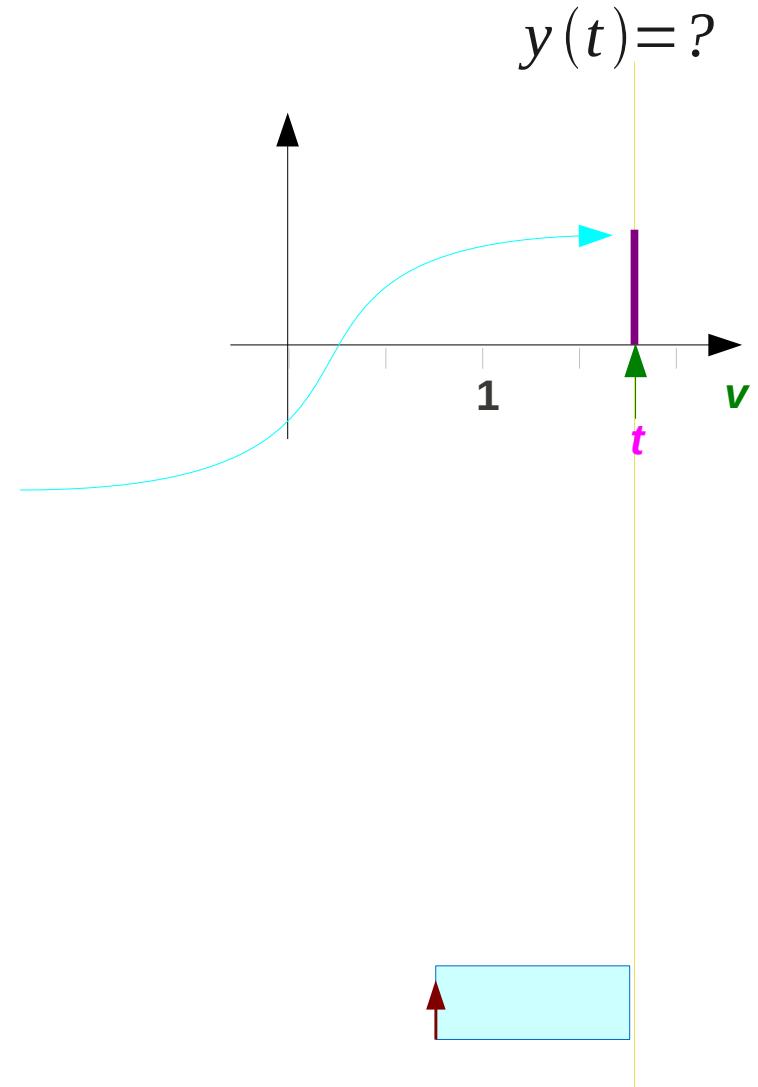
Impulse Response



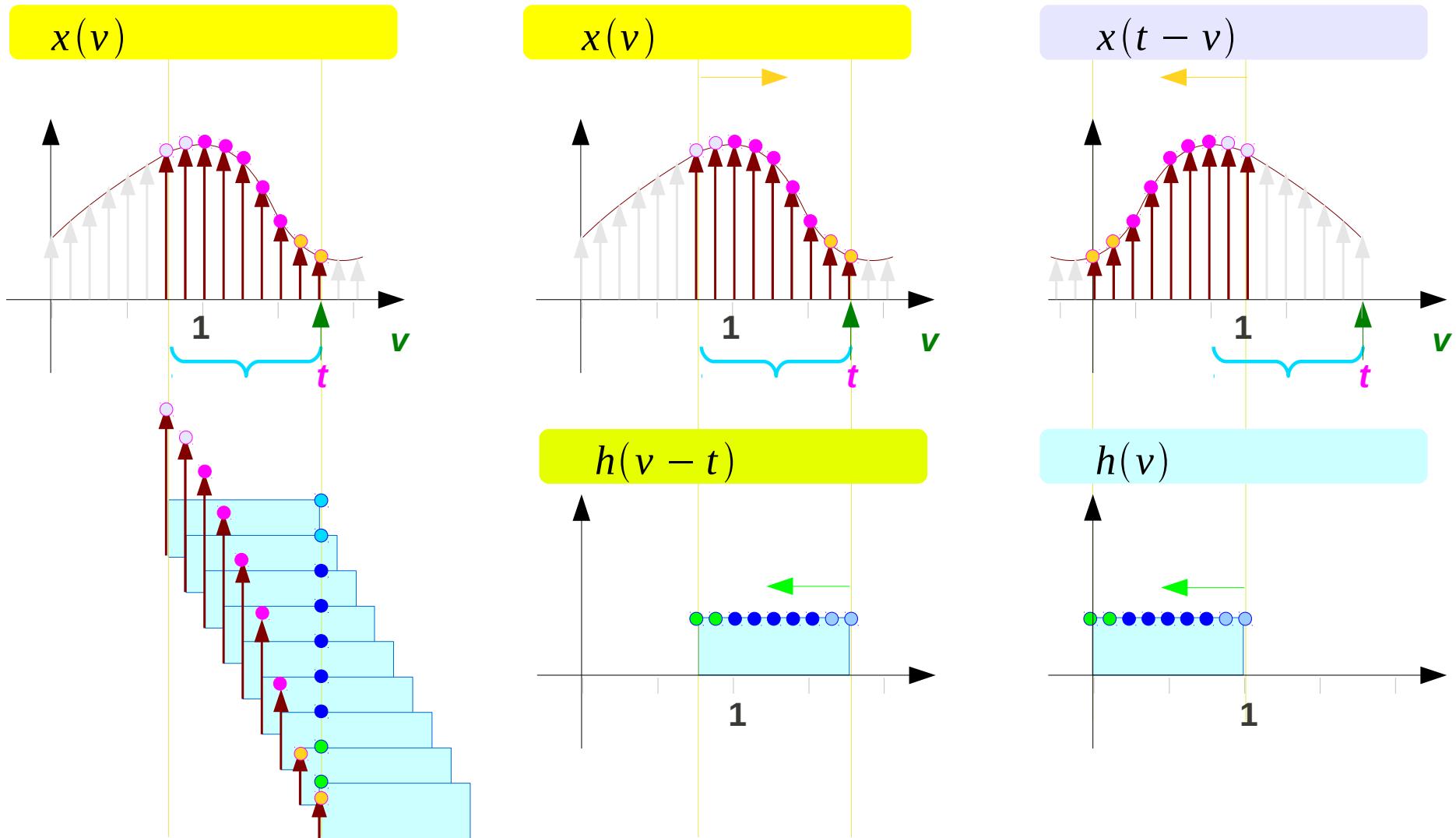
Impulse Response



these inputs affects
the output value at t



Impulse Response



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003