Convolution (1A)

-

Young Won Lim
11/18/10

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

$x(t)$

$\delta(t)$

1

$h(t)$

$y(t)$

1

$\delta(t - 0.5)$

1

1

$\delta(t - 1)$

1

1

$$y(t) = ?$$

t_1

$y(t)$

$t_1 - 1$ t_1

$t_1 - .5$ $t_1 + .5$
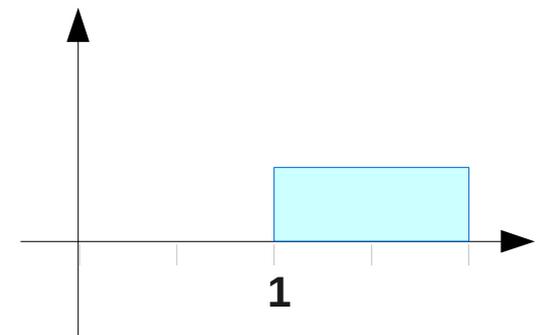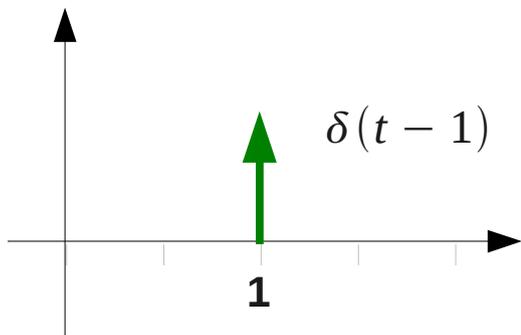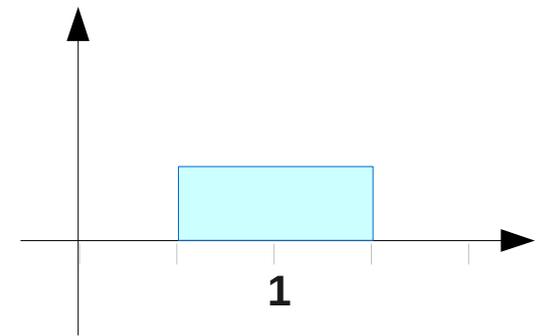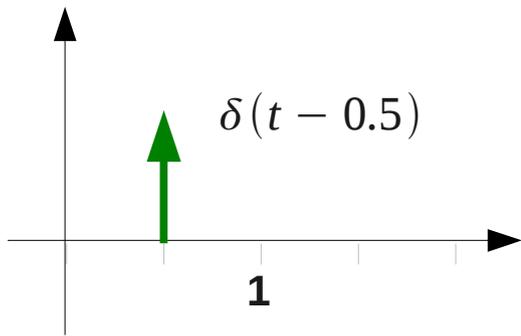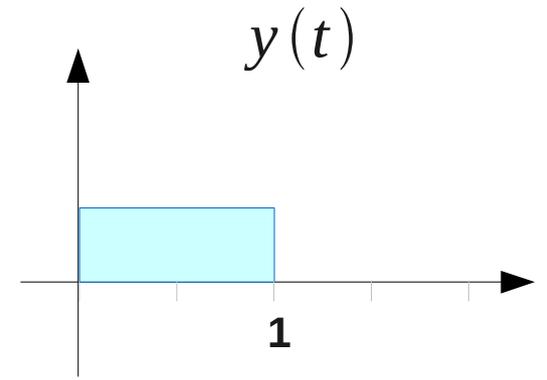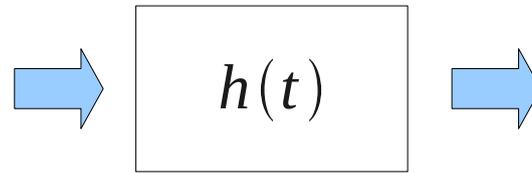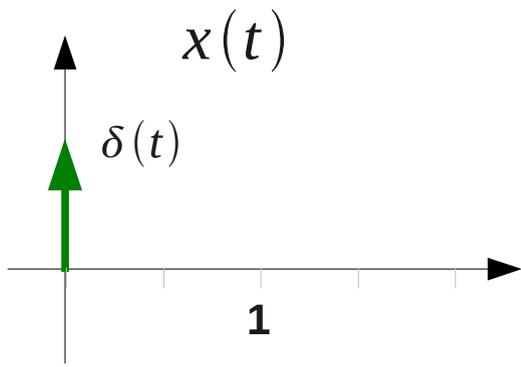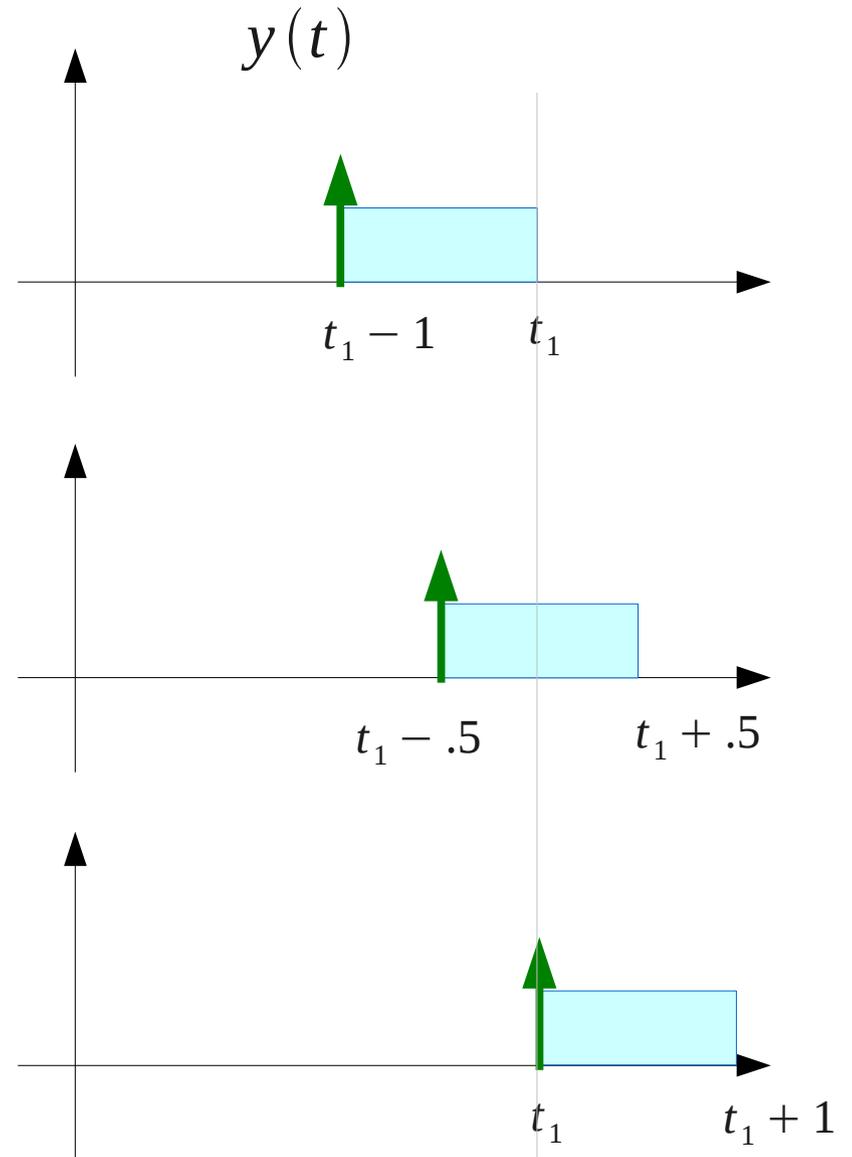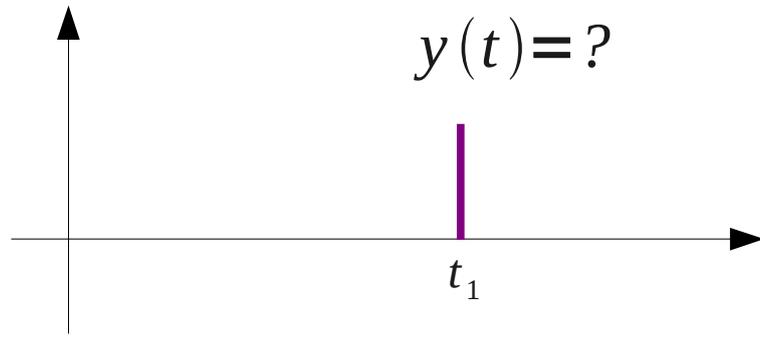
t_1 $t_1 + 1$

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$
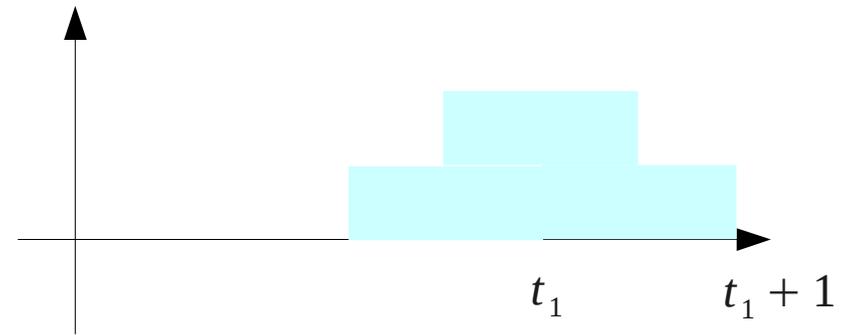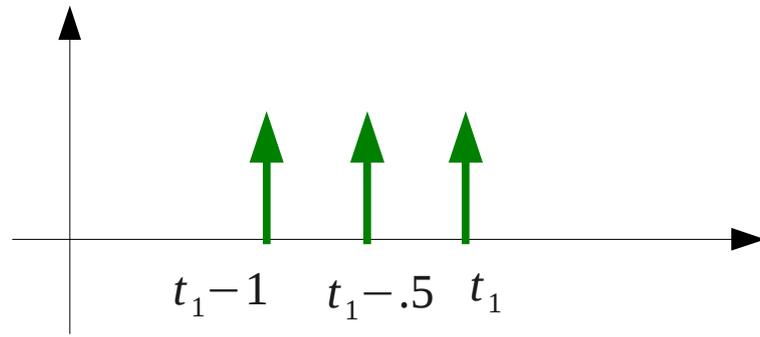
$t_1-1 \quad t_1-.5 \quad t_1$

$t_1 \qquad t_1+1$

$$y(t) = \int x(v)h(t-v)\,dv$$

$$= \int \delta(v-t_1+1)h(t-v)\,dv \qquad \rightarrow \quad h(t-t_1+1)$$

$$+ \int \delta(v-t_1+.5)h(t-v)\,dv \qquad \rightarrow \quad h(t-t_1+.5)$$

$$+ \int \delta(v-t_1)h(t-v)\,dv \qquad \rightarrow \quad h(t-t_1)$$

$$y(t_1) = h(t_1-t_1+1) + h(t_1-t_1+.5) + h(t_1-t_1)$$

$$= h(1) + h(.5) + h(0)$$

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$y(t)=?$$

1
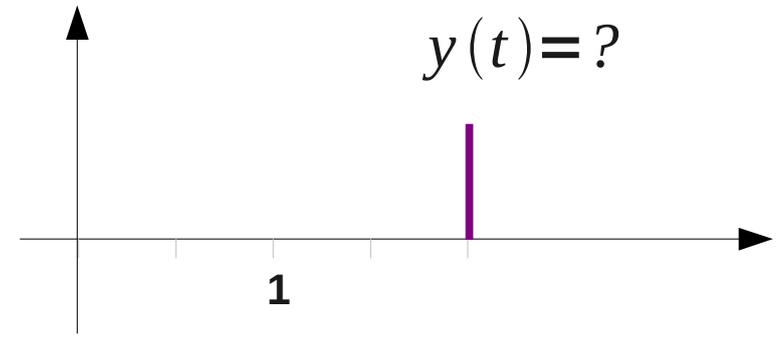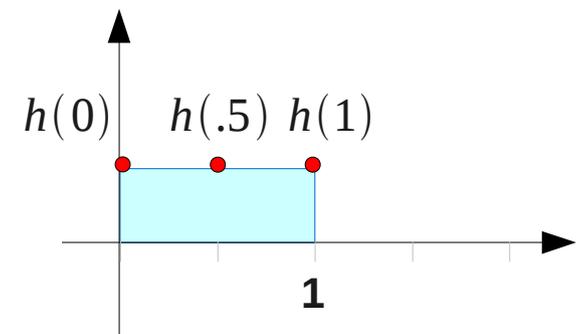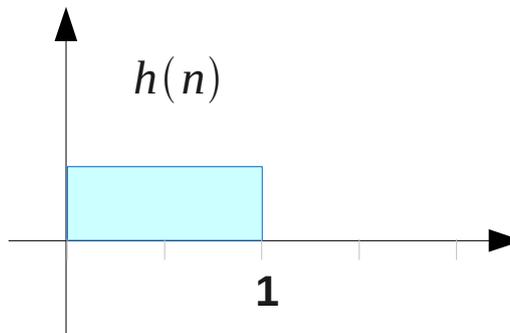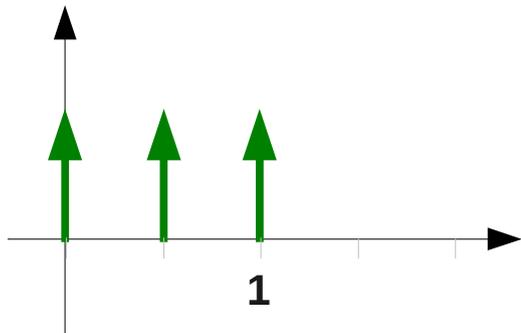
$$y(t) = h(t-t_1+1) + h(t-t_1+.5) + h(t-t_1)$$

$$y(t_1) = h(1) + h(.5) + h(0)$$

$$\delta(t) + \delta(t-.5) + \delta(t-1)$$

1

$h(n)$

1

$h(0)$ $h(.5)$ $h(1)$

1

$$x(t) = \delta(t-t_1+1) + \delta(t-t_1+.5) + \delta(t-t_1)$$

$$x(v) = \delta(v-t_1+1) + \delta(v-t_1+.5) + \delta(v-t_1)$$

$$x(t-v) = \delta(t-v-t_1+1) + \delta(t-v-t_1+.5) + \delta(t-v-t_1)$$

$$y(t) = \int x(t-v)h(v)\,dv$$

$$= \int \delta(t-v-t_1+1)h(v)\,dv \qquad \rightarrow \quad h(t-t_1+1)$$

$$+ \int \delta(t-v-t_1+.5)h(v)\,dv \qquad \rightarrow \quad h(t-t_1+.5)$$

$$\delta(t) + \delta(t-.5) + \delta(t-1) \qquad + \int \delta(t-v-t_1)h(v)\,dv \qquad \rightarrow \quad h(t-t_1)$$

$$y(t) = \int x(v)h(t-v)\,dv$$

$$= \int \delta(v-t_1+1)h(t-v)\,dv \qquad \rightarrow \quad h(t-t_1+1)$$

$$+ \int \delta(v-t_1+.5)h(t-v)\,dv \qquad \rightarrow \quad h(t-t_1+.5)$$

$$+ \int \delta(v-t_1)h(t-v)\,dv \qquad \rightarrow \quad h(t-t_1)$$

$$y(t) = \int x(t-v)h(v)\,dv$$

$$= \int \delta(t-v-t_1+1)h(v)\,dv \qquad \rightarrow \quad h(t-t_1+1)$$

$$+ \int \delta(t-v-t_1+.5)h(v)\,dv \qquad \rightarrow \quad h(t-t_1+.5)$$

$$+ \int \delta(t-v-t_1)h(v)\,dv \qquad \rightarrow \quad h(t-t_1)$$

References
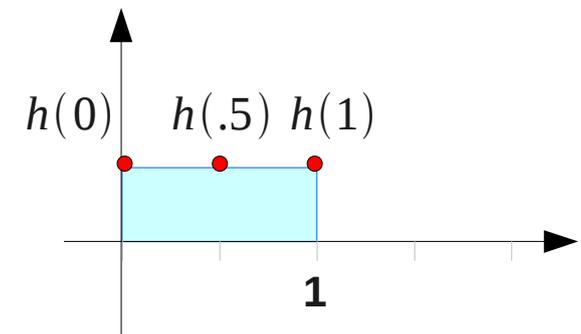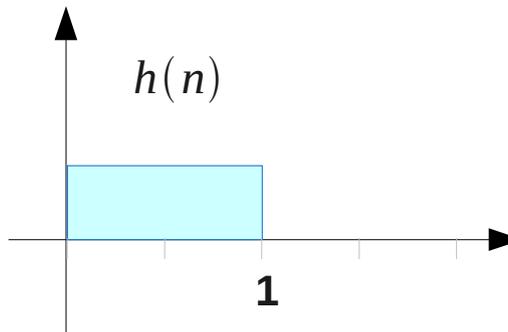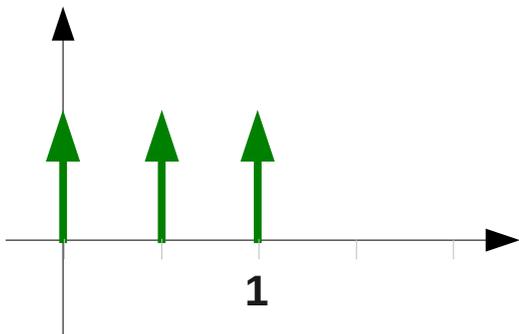
[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003