2.16.1

Int $\Rightarrow I = \int_{-5}^{5} \frac{1}{1+x^2}\, dx$

find Int. using Newton Coats
with $n = 1, 2 \ldots 15$

Newton Coats Polynomials

$$P_n(x_j) = \sum_{i=0}^{n} l_i(x_j)\, f(x_i) = f(x_j)$$

nodal points

$$l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$$I_n = \int_a^b P_n(x) = \sum_{i=0}^{n} \underbrace{\int_a^b l_i(x)\,dx}_{w_i}\, f(x_i)$$

One can do easly integration
for $n=1$ (Trap)
$n=2$ (Simpson)

However for $n > 2$ would be nice
to have a method to compute w_i

2.16.2

Based on Methods of Numerical Int Davis/Rabinowitz
P 75

$$\int_a^b K(x) f(x) dx = \sum_{j=0}^{n} w_j f(x_j)$$

$$\underbrace{\qquad}_{\sum f_n} \rightarrow \mathcal{L}(f;x) \equiv L_n(f;x)$$

$$E(f) = \int_a^b K(x) f(x) - \sum_{j=1}^{n} w_j f(x_j)$$

$$= \int_a^b K(x)(f(x) - \mathcal{L}(f;x))dx$$

take $K(X) = 1$ for Newton Cotes

Since $E(1) \cdots E(x^n) = 0$
demand Error 0 @ nodes

Leads to system of eq

$$w_0 + w_1 + \ldots w_n = \int_a^b K(x) dx = m_0$$

$$w_0 x_0 + w_1 x_1 + \ldots w_n x_n = \int_a^b K(x) x\, dx = m_1$$

$$\vdots$$

$$w_0 x_0^n + w_1 x_1^n + \ldots w_n x_n^n = \int_a^b K(x) x^n dx = m_n$$

This means the $w(i)$ is dependent on $x(i)$
location of nodal points and start end points but not function

A matrix

to solve use
$$\begin{bmatrix} 1 & 1 \ldots 1 \\ x_0 & x_1 \ldots x_n \\ x_0^n & x_1^n \ldots x_n^n \end{bmatrix} \begin{Bmatrix} w_0 \\ w_1 \\ \vdots \\ w_n \end{Bmatrix} = \begin{Bmatrix} m_0 \\ m_1 \\ \vdots \\ m_n \end{Bmatrix}$$

$$w = A^{-1} m$$

(1 based on $w(i)$ computation

$$I_n = \sum_{i=0}^{n} w_i \times f(x_i)$$

Table of Int. results is shown Int_Table.png

(2 plot f, f_n, $n = 1, 2, 3, 8, 12$

$$f_n(x_j) = \sum_{i=0}^{n} l_i(x_j)\, f(x_i)$$

this is done by first computing l_i at the nodal points & computing f_n for each location of $f(x_j)$

MatLab Code HW_2_16b.m

output plots are shown in fn.jpg

(3 plot I_n Vs n show no convergance from (1 → Matlab Code HW_2_16a.m
Plot is shown Int.jpg

(4 from (1 → Matlab Code HW_2_16a.m
$W(i)$ are tabulated w_Table.png
for $n \geq 8$ some $W(i) < 0$
Lagrange functions l_i $i = 1..8$ are computed in Matlab Code HW_2_16d.m & ploted in li8.pgn