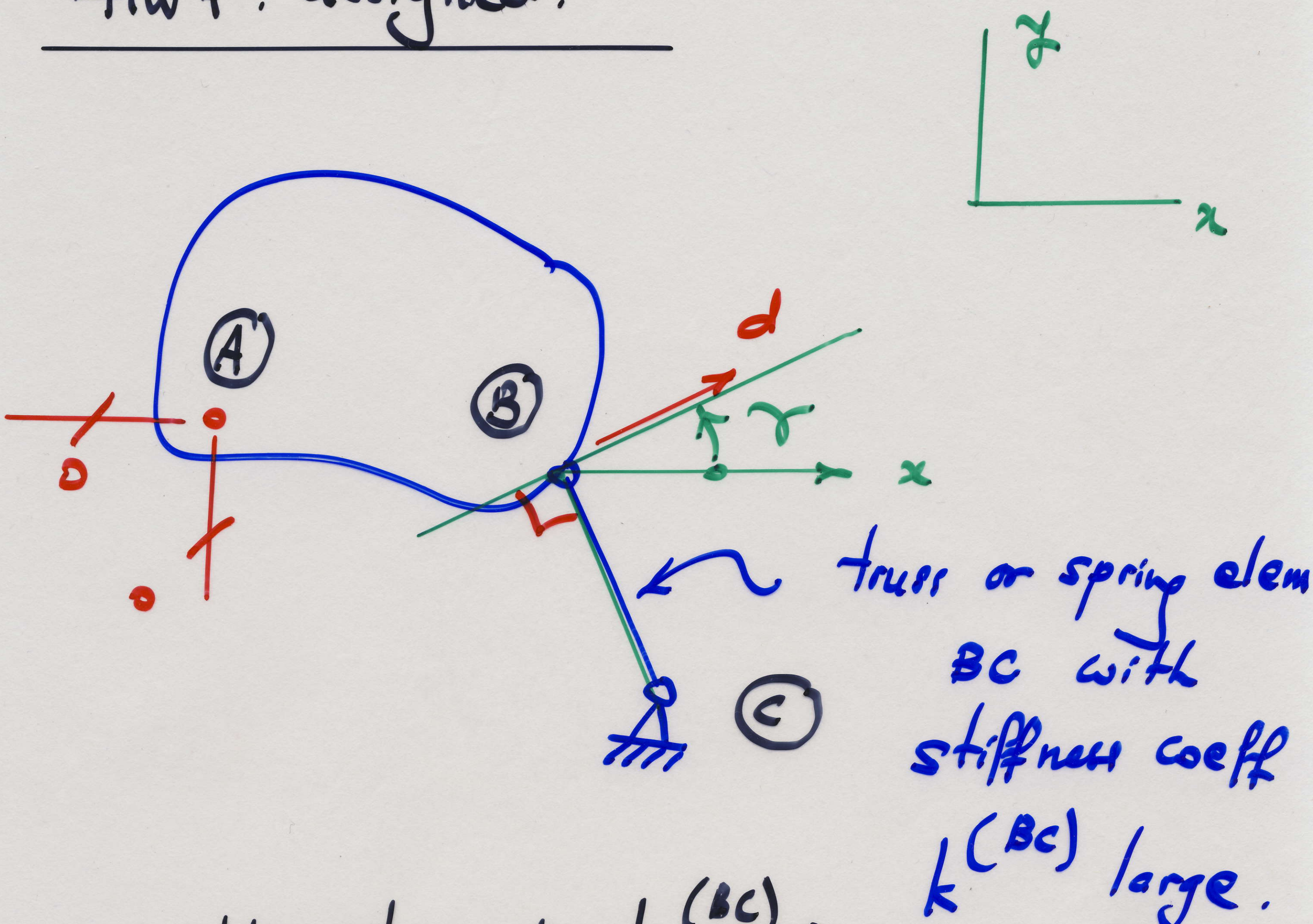
Mtg 19: Mon, 9 Oct 06

HW7: assigned.



Q: How to set $k^{(BC)}$?

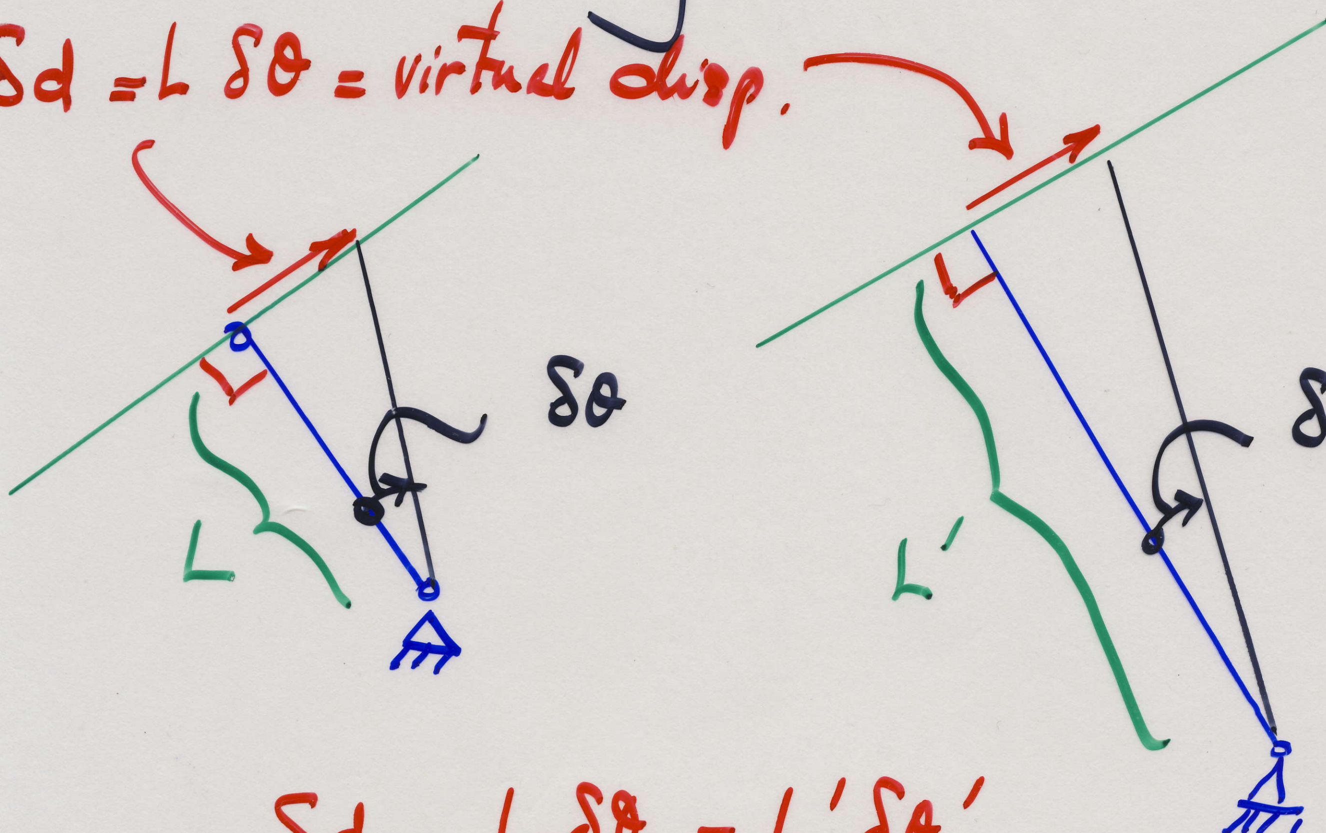
Ans: $k^{(BC)}$ should be large so that the disp. of B along BC dir. is "small" compared to other disp.

But $k^{(BC)}$ should not be ∞, due to numerical ill-conditioning, i.e., the numerical soln would not be accurate (pb. of inverting the stiffness matrix that contains very large coeff).

$k^{(BC)}$ = penalty coeff. $= \mu$

$\underline{K}$ = stiffness matrix before adding elem BC.

$\delta d = L\,\delta\theta$ = virtual disp.



$$\delta d = L\,\delta\theta = L'\,\delta\theta'$$

1-norm of $\underline{K}$:

$$\| \underline{K} \|_1 = \max_{i,j} | K_{ij} |$$

$$k^{(BC)} = 10^n \| \underline{K} \|_1$$

Rule of thumb : $n = 5$

Ref: Book, p. 80, where the penalty coeff is denoted by μ, instead of $k^{(BC)}$.

Mtg 20: Wed, 11 Oct 06

Plan: Principle of virtual work (PVW)

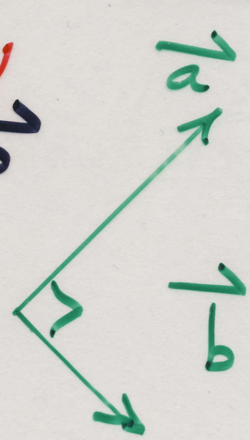
* Transf. of FD rel. from axial dof's to global dof's
* Treatment of inclined roller.

Thm: If $\vec{a} \cdot \vec{b} = 0$ for all possible vector $\vec{a}$ then $\vec{b} = \vec{0}$.

Rem: Let $\vec{a}$, $\vec{b}$ be 2 vectors.
If $\vec{a} \cdot \vec{b} = 0$, is it true that $\vec{b} = 0$? No.

If $\vec{a} = \vec{0} \Rightarrow \vec{a} \cdot \vec{b} = 0$ and $\vec{b}$ need not be $\vec{0}$.

Consider a vector $\vec{a} \neq \vec{0}$

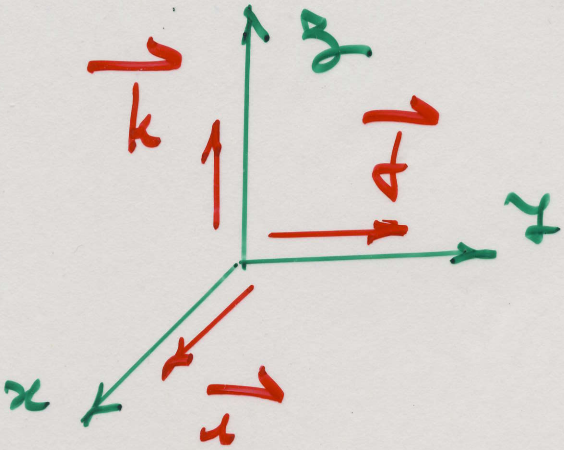


$\vec{b} \neq \vec{0}$, and $\vec{a} \cdot \vec{b} = 0$

There are infinitely many such $\vec{b}$.

Pf of thm:

3-D first:



$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$

$$\vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$= \lfloor a_x \ a_y \ a_z \rfloor \begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix}$$

$= 0$ for all possible $\vec{a}$

Choice 1: Select $\vec{a}$ st

$$a_x = 1, \ a_y = a_z = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 1 \cdot b_x = 0$$

$$\Rightarrow b_x = 0$$

Jarita:

Choice 2: Select $\vec{a}$ st

$$a_x = 0, \ a_y = 1, \ a_z = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 1 \cdot b_y = 0 \Rightarrow b_y = 0$$

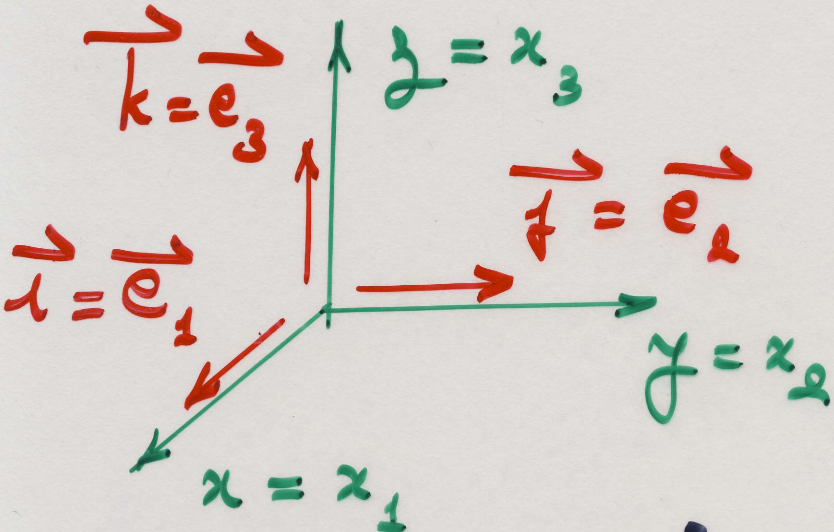
Choice 3: Select $\vec{a}$ st

$$a_x = a_y = 0, \quad a_z = 1$$

$$\Rightarrow \quad b_z = 0 \qquad \text{(HW)}$$

More convenient notation for generalization to n-dim case: Indicial notation.

(index)



$$\vec{a} = \sum_{i=1}^{3} a_i \vec{e}_i$$

$$\vec{b} = \sum_{i=1}^{3} b_i \vec{e}_i$$

$$\vec{a} \cdot \vec{b} = \sum_{i=1}^{3} a_i b_i$$

$$= \lfloor a_1 \;\; a_2 \;\; a_3 \rfloor \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

In n-dim. case (or space):

$$\vec{a} = \sum_{i=1}^{n} a_i \vec{e}_i \quad , \quad \vec{b} = \sum_{i=1}^{n} b_i \vec{e}_i$$

$$\vec{a}\cdot\vec{b} = \sum_{i=1}^{n} a_i b_i$$

$$= \underbrace{\lfloor a_1\ a_2\ \dots\ a_n \rfloor}_{1\times n} \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Bmatrix}_{n\times 1}$$

HW: Make appropriate choices to show that

$$b_i = 0 \quad \text{for } i = 1, \dots, n.$$

Axial FD rel (Equil. eq.)

(end proof)

p. 5-3:

$$\underset{2\times 2}{\underline{\tilde{k}}^{(e)}}\ \underset{2\times 1}{\underline{\delta}^{(e)}} = \underline{P}^{(e)}_{2\times 1}$$

$$\Rightarrow \underbrace{\underline{\tilde{k}}^{(e)}\ \underline{\delta}^{(e)} - \underline{P}^{(e)}}_{\underline{b}} = \underline{0}_{2\times 1}$$

Russell:

Amruta

PVW

$$\underline{a}\cdot\underline{b} = 0_{1\times 1} \text{ for all possible } \underline{a}$$

$$\Rightarrow \underline{b} = \underline{0}_{2\times 1}$$

or : $\underline{a}^{(e)} \cdot [\tilde{\underline{k}}^{(e)} \underline{\delta}^{(e)} - \underline{p}^{(e)}] = 0$

$$\Rightarrow \quad \underline{a}^{(e)} \cdot (\tilde{\underline{k}}^{(e)} \underline{\delta}^{(e)}) = \underline{a}^{(e)} \cdot \underline{p}^{(e)} \qquad (1)$$

The matrix $\underline{a}^{(e)}$ can be interpreted as a virtual disp., then $\underline{a}^{(e)} \cdot \underline{p}^{(e)}$ is a virtual work.

Treat disp and virtual disp in the same manner under transf. of coord (from local axial dof's to global dof's).

$$\underset{2\times1}{\underline{\delta}^{(e)}} = \underset{2\times4}{\underline{T}^{(e)}} \; \underset{4\times1}{\underline{d}^{(e)}} \qquad (2)$$

$$\underset{2\times1}{\underline{a}^{(e)}} = \underset{2\times4}{\underline{T}^{(e)}} \; \underset{4\times1}{\underline{c}^{(e)}} \qquad (3)$$

Use (3) & (2) in (1) :

$$\Rightarrow \left[\underbrace{\underline{T}^{(e)}\,\underline{c}^{(e)}}_{\underline{a}^{(e)}}\right] \cdot \left[\underline{\tilde{k}}^{(e)}\,\underbrace{\underline{T}^{(e)}\,\underline{d}^{(e)}}_{\underline{\delta}^{(e)}}\right] = \left[\underbrace{\underline{T}^{(e)}\,\underline{c}^{(e)}}_{\underline{a}^{(e)}}\right] \cdot \underline{p}^{(e)}$$

$$\Rightarrow \underline{c}^{(e)} \cdot \left[\overbrace{\underline{T}^{(e)T}\,\underline{\tilde{k}}^{(e)}\,\underline{T}^{(e)}}^{\underline{k}^{(e)}}\,\underline{d}^{(e)}\right]$$

$$= \underline{c}^{(e)} \cdot \underbrace{\underline{T}^{(e)T}\,\underline{p}^{(e)}}_{\underline{f}^{(e)}}$$

$$(\underline{A}\,\underline{B}) \cdot \underline{C} = \underline{B} \cdot (\underline{A}^T \underline{C})$$

Rem:

$$\boxed{(\underline{A}\,\underline{b})\cdot\underline{c} \overset{?}{=} \underline{b}\cdot(\underline{A}^T\underline{c})}$$

Case 1: $\quad n\times n \quad n\times 1 \quad n\times 1 \qquad\qquad n\times 1 \quad n\times n \quad n\times 1$

Case 2: $\quad \underbrace{n\times m \quad m\times 1}_{n\times 1} \quad n\times 1 \qquad\qquad m\times 1 \quad \underbrace{m\times n \quad n\times 1}_{m\times 1}$

Even in Case 1, $\underline{A}_{n\times n}$ may not be invertible, i.e., $\underline{A}^{-1}$ does not exist.

In Case 2, $\underline{A}^{-1}$ does not exist, since $\underline{A}$ is rectangular.

Define $\underline{d} = \underline{A}\,\underline{b}$

$\underline{d}$	$\underline{A}$	$\underline{b}$	
$n\times 1$	$n\times n$	$n\times 1$	Case 1
$n\times 1$	$n\times m$	$m\times 1$	Case 2

$$\underbrace{(\underline{A}\,\underline{b})}_{\underline{d}}\cdot\underline{c} = \underline{d}\cdot\underline{c} = \underbrace{\underline{d}^T_{1\times n}\,\underline{c}_{n\times 1}}_{1\times 1}$$

$$\underline{d} = \begin{Bmatrix} d_1 \\ \vdots \\ d_n \end{Bmatrix}_{n\times 1} \qquad \underline{c} = \begin{Bmatrix} c_1 \\ \vdots \\ c_n \end{Bmatrix}_{n\times 1}$$

$$\alpha = (\underline{A}\,\underline{b})^T \underline{c} = (\underline{b}^T \underline{A}^T)\,\underline{c}$$

Recall: (Chris)

$$\underbrace{\big(\underbrace{\underset{n\times m}{\underline{E}}\ \underset{m\times q}{\underline{F}}}_{n\times q}\big)^T}_{q\times n} = \underbrace{\underset{q\times m}{\underline{F}^T}\ \underset{m\times n}{\underline{E}^T}}_{q\times n}$$

$$\underline{E}\,\underline{F}\,\underline{G} = (\underline{E}\,\underline{F})\,\underline{G} = \underline{E}(\underline{F}\,\underline{G})$$

$$\alpha = \underline{b}^T(\underline{A}^T\underline{c}) = \underline{b}\cdot(\underline{A}^T\underline{c})$$

p. 20-6: The PVW in global dof's now reads as follows:

$$\underline{c}^{(e)}\cdot\left[\underline{k}^{(e)}\underline{d}^{(e)}\right] = \underline{c}^{(e)}\cdot\underline{f}^{(e)}$$

for all possible (virt. disp) $\underline{c}^{(e)}$

p. 20-1 : By thm on this page

$$\Rightarrow \boxed{\begin{aligned} &\underset{4\times 4}{\underline{k}^{(e)}}\ \underset{4\times 1}{\underline{d}^{(e)}} = \underset{4\times 1}{\underline{f}^{(e)}} \\ &\underline{k}^{(e)} = \underline{T}^{(e)T}\ \underline{\tilde{k}}^{(e)}\ \underline{T}^{(e)} \end{aligned}}$$

Case of inclined roller : (p. 18-2)

$$\underset{n\times n}{\underline{K}}\ \underset{n\times 1}{\underline{d}} = \underset{n\times 1}{\underline{F}}$$

FD rel without ess. b.c.'s.

1) (Chris) $\underline{K}\,\underline{d} - \underline{F} = \underset{n\times 1}{\underline{0}}$

2) (Chris) $\underset{n\times 1}{\underline{a}} \cdot \left[\underline{K}\,\underline{d} - \underline{F}\right] = \underset{1\times 1}{0}$ (Sida)

for all possible virt. disp $\underline{a}$

$$\underline{d} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_n \end{Bmatrix} , \quad \underline{d}' = \left\{ \begin{array}{c} d_1' \\ d_2' \\ \hline d_3 \\ \vdots \\ d_n \end{array} \right\}$$

$$\underline{d} = \underline{T} \, \underline{d}'$$

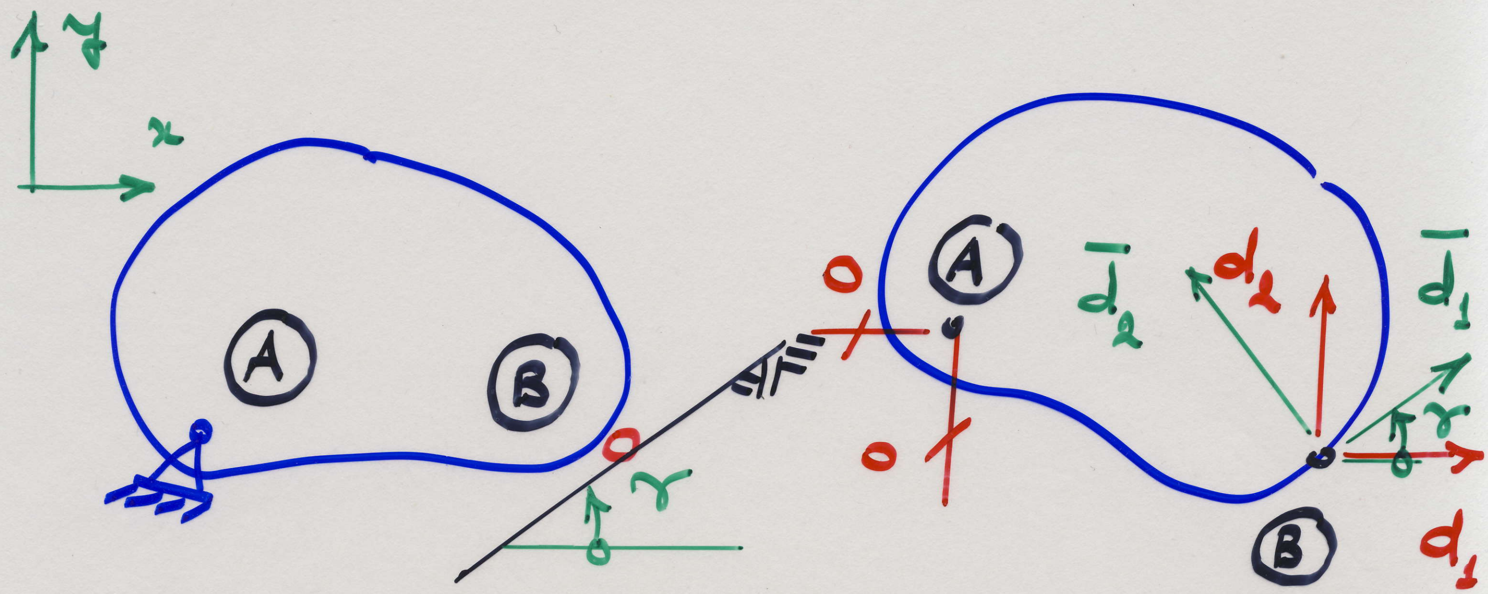
$$d_2' = \begin{bmatrix} \sin\gamma & -\cos\gamma \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$$

$$d_1 = d_1' \cos\gamma + d_2' \sin\gamma$$

$$d_2 = d_1' (\qquad) + d_2' (\qquad)$$

Mtg 22: Mon, 16 Oct 06

Thanksgiving make-up lecture: + 5 min each lect.



$$\underline{d} = \begin{Bmatrix} d_1 \\ d_2 \\ \hline d_3 \\ \vdots \\ d_n \end{Bmatrix}_{n \times 1} , \quad \underline{\bar{d}} = \begin{Bmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \hline d_3 \\ \vdots \\ d_n \end{Bmatrix}_{n \times 1}$$

$$\underline{K}\,\underline{d} = F$$

Find $\underline{T}$ st $\underset{n\times 1}{\underline{d}} = \underset{n\times n}{\underline{T}}\ \underline{\bar{d}}_{n\times 1}$

$$\underset{n\times 1}{\underline{a}} = \underset{n\times n}{\underline{T}}\ \underset{n\times 1}{\underline{c}}$$

$$\underline{a} \cdot [\underline{K}\,\underline{d} - \underline{F}] = 0$$ for all poss. virt. disp. $\underline{a}$

Equivalently:

$$(\underline{T}\,\underline{c}) \cdot [\underline{K}(\underline{T}\,\bar{\underline{d}}) - \underline{F}] = 0$$ for all poss. virt. disp. $\underline{c}$

$\downarrow$ p. 21-1

$$\Rightarrow \underline{c} \cdot [\underline{T}^T \underline{K}\,\underline{T}\,\bar{\underline{d}} - \underline{T}^T\underline{F}] = 0$$ for all poss. $\underline{c}$

$\downarrow$ p. 20-2

$$\Rightarrow (\underset{n\times n}{\underline{T}^T}\ \underset{n\times n}{\underline{K}}\ \underset{n\times n}{\underline{T}})\ \underset{n\times 1}{\bar{\underline{d}}} = \underset{n\times n}{\underline{T}^T}\ \underset{n\times 1}{\underline{F}}$$

(Trm)

Rem: A short cut approach is to eliminate a-priori the known zero dof $\overline{d_2}$, by using the following transf.

$$\underset{n\times 1}{\underline{d}} = \underset{(n-1)\times n}{\underline{\tilde{T}}} \underbrace{\begin{Bmatrix} \bar{d}_1 \\ d_3 \\ d_4 \\ \vdots \\ d_n \end{Bmatrix}}_{(n-1)\times 1}$$

$$\Rightarrow \Big(\underset{(n\times 1)\times n}{\underline{\tilde{T}}^T} \; \underset{n\times n}{\underline{K}} \; \underset{n\times(n-1)}{\underline{\tilde{T}}}\Big) \underset{\cancel{n\times}(n-1)\times 1}{\underline{\tilde{d}}} = \underset{(n-1)\times n}{\underline{\tilde{T}}^T} \, \underset{n\times 1}{\underline{F}}$$

Method valid only when $\bar{d}_2 = 0$.
If $\bar{d}_2 \neq 0$, use matrix partitioning method.

///

Mtg 23: Wed, 18 oct 06

p. 17-1: Two fund. questions.

1) why $k^{(e)} = \dfrac{E^{(e)} A^{(e)}}{L^{(e)}}$?

2) why + in assembly op. A, i.e.,

$$\underline{K} = \underset{e=1}{\overset{nel}{A}} \underline{k}^{(e)}$$

nel = n° of elems

why not −, *, ÷ ?

Ans: 1) (Chris)

p. 3-3: $\sigma = E \varepsilon$ Hooke's law (Peter)

$$\sigma = \frac{f_2^{(e)}}{A^{(e)}}$$ (Peter)

$$\varepsilon = \frac{d_2 - d_1}{L^{(e)}}$$ (Peter, Shawn)

$$\Rightarrow \quad f_2^{(e)} = \frac{E^{(e)} A^{(e)}}{L^{(e)}} (d_2 - d_1)$$

p. 3-3.

Equiv.

$$\boxed{P_2^{(e)} = \frac{E^{(e)} A^{(e)}}{L^{(e)}} \left(\delta_2^{(e)} - \delta_1^{(e)} \right)}$$

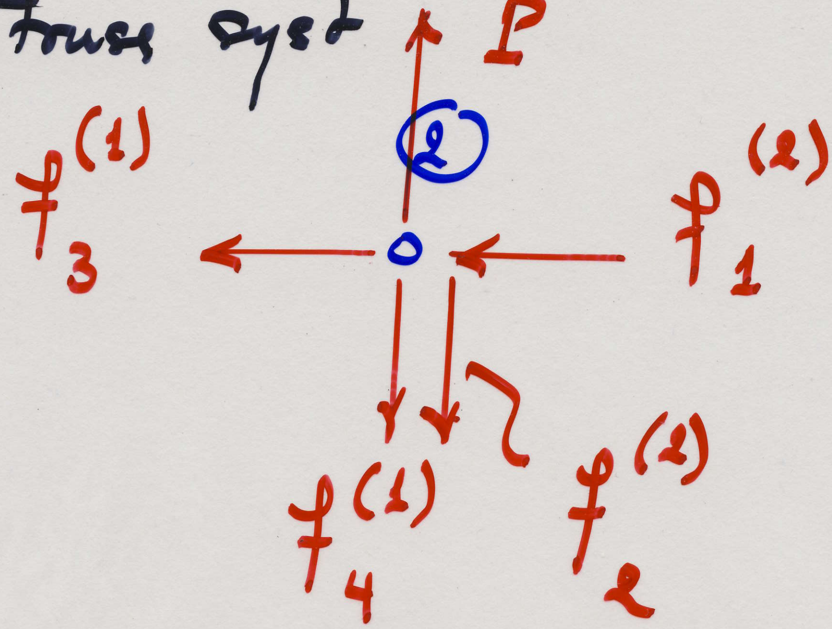
2) Assembly op. : Refer to HW7, equil. of node 2 ; or to 2-bar syst on p. 10-1, also look at equil. of node 2.

Step 1 : Isolate node 2, and write equil. eqs for node 2.

$$\sum F_x = 0$$

$$\sum F_y = 0$$

2-bar truss syst



$$\Sigma F_x = 0 = -f_3^{(1)} - f_1^{(2)}$$

$$\Rightarrow f_3^{(1)} + f_1^{(2)} = 0$$

$$\Sigma F_y = 0 = P - f_4^{(1)} - f_2^{(2)}$$

$$\Rightarrow f_4^{(1)} + f_2^{(2)} = P$$

Step 2: Relate $f_i^{(e)}$ to $\underline{d}^{(e)}$ and then to $\underline{d}$:

$$\underline{k}^{(e)}\,\underline{d}^{(e)} = \underline{f}^{(e)}$$

$\underline{d}^{(e)}$ is related to $\underline{d}$ by

lmm array (Master location matrix)

$$d_3^{(1)} = d_1^{(2)} = d_3$$
$$d_4^{(1)} = d_2^{(2)} = d_4$$

} Chris

Step 3: Put everything together to obtain $\underline{K}$ as on p. 9-5.

Space truss: Philip, Jarita, Mike

$$\underset{n \times n}{\underline{K}} \; \underset{n \times 1}{\underline{d}} = \underset{n \times 1}{\underline{F}}$$

nnp = nº of $\underbrace{\text{nodal points}}_{\text{nodes}}$

$$n = 3 * nnp$$

p. 13-1: $\underset{\sim}{T}^{(e)}$ ← (p. 5-3)

$\underline{k}_{2\times 2}$ remains same (axial def's)

Larry

Brian

23-5

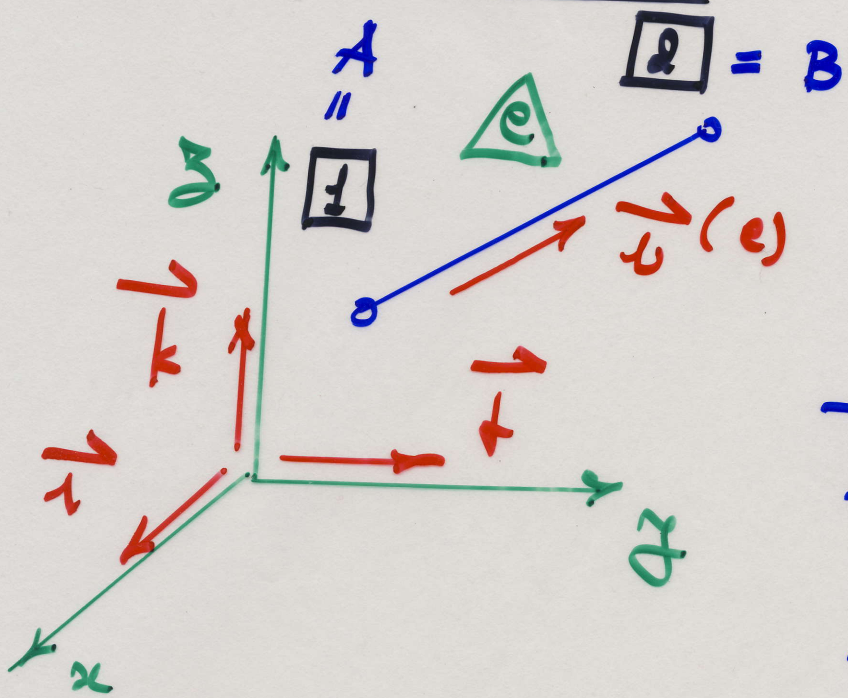
(Mike) $$\underline{T}^{(e)}_{2\times 6} = \begin{bmatrix} l^{(e)} & m^{(e)} & n^{(e)} & 0 & 0 & 0 \\ 0 & 0 & 0 & l^{(e)} & m^{(e)} & n^{(e)} \end{bmatrix}$$

(7.13-1)

$$\vec{i}^{(e)} = l^{(e)}\vec{i} + m^{(e)}\vec{j} + n^{(e)}\vec{k}$$

Mtg 24: Fri, 20 Oct 06 + 5 min (Lect 1)

Plan:
- space truss (cont'd)
- HW: Exam 2 preparation
- Beams & Frames



Q: How to find $l^{(e)}, m^{(e)}, n^{(e)}$

Tim: $\alpha^{(e)}, \beta^{(e)}, \gamma^{(e)}$ be 3 angles betw $\vec{i}^{(e)}$ and $\vec{i}, \vec{j}, \vec{k}$, resp.

$$\begin{cases} l^{(e)} = \cos\alpha^{(e)} \\ m^{(e)} = \cos \cancel{\alpha}\, \beta^{(e)} \\ n^{(e)} = \cos\gamma^{(e)} \end{cases}$$

How to find $\alpha^{(e)}, \beta^{(e)}, \gamma^{(e)}$?

Hugh:
$$l^{(e)} = \vec{i}^{(e)} \cdot \vec{i}$$
$$m^{(e)} = \vec{i}^{(e)} \cdot \vec{j}$$

$$n^{(e)} = \vec{i}^{(e)} \cdot \vec{k}$$

Cesar: $\vec{a} \cdot \vec{b} = \|\vec{a}\| \, \|\vec{b}\| \cos(\vec{a}, \vec{b})$

$(\vec{a}, \vec{b})$ = angle betw. $\vec{a}$ and $\vec{b}$

$$\vec{i}^{(e)} \cdot \vec{i} = \underbrace{\|\vec{i}^{(e)}\|}_{=1} \, \underbrace{\|\vec{i}\|}_{=1} \cos(\vec{i}^{(e)}, \vec{i})$$

$$= \cos \alpha^{(e)} = l^{(e)}$$

Since $\alpha^{(e)} = (\vec{i}^{(e)}, \vec{i}) =$ angle betw $\vec{i}^{(e)}$ and $\vec{i}$.

Another expl.:

$$\vec{i}^{(e)} \cdot \vec{i} = \left(l^{(e)} \vec{i} + m^{(e)} \vec{j} + n^{(e)} \vec{k} \right) \cdot \vec{i}$$

$$= l^{(e)} \underbrace{(\vec{i} \cdot \vec{i})}_{\|\vec{i}\|^2 = 1^2 = 1} + m^{(e)} \underbrace{(\vec{j} \cdot \vec{i})}_{0} + n^{(e)} \underbrace{(\vec{k} \cdot \vec{i})}_{0}$$

Similarly ~~for~~ $m^{(e)}$ and $n^{(e)}$.

Def: $\vec{\imath}^{(e)} = \dfrac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|}$

$$\overrightarrow{AB} = (x_B - x_A)\vec{\imath} + (y_B - y_A)\vec{\jmath} + (z_B - z_A)\vec{k}$$

$$\|\overrightarrow{AB}\| = \left[(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2 \right]^{1/2}$$

$$\ell^{(e)} = \frac{(AB)_x}{\|\overrightarrow{AB}\|} = \frac{x_B - x_A}{\|\overrightarrow{AB}\|}$$

$$\overrightarrow{AB} = (AB)_x \vec{\imath} + (AB)_y \vec{\jmath} + (AB)_z \vec{k}$$

Similarly for $m^{(e)}$ and $n^{(e)}$.

Set up $\underline{T}^{(e)}$ (p. 23-5)

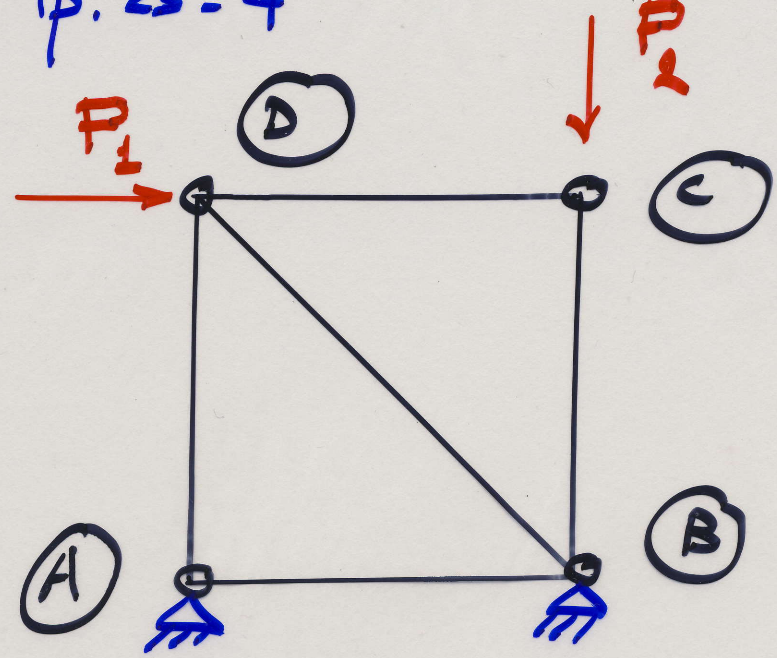
p. 13-1:

$$\underset{6\times 6}{\underline{k}^{(e)}} = \underset{6\times 2}{\underline{T}^{(e)T}} \; \underset{2\times 2}{\underline{\tilde{k}}^{(e)}} \; \underset{2\times 6}{\underline{T}^{(e)}}$$

$$\underline{K} = \underset{e=1}{\overset{nel}{\mathrm{A}}} \underline{k}^{(e)}$$

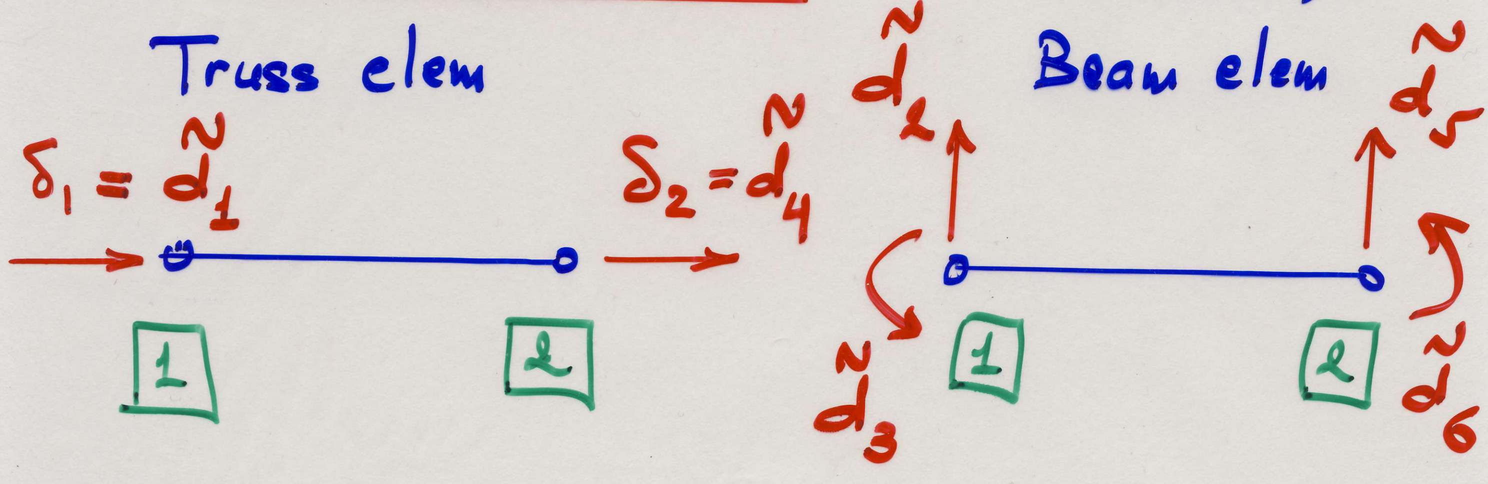
p. 23-4

HW:



1) Find $\underline{K}$
2) Find $\underline{F}$
3) Consider prescribed disp. at A

Beams and Frames: (Peter)



Beam dof's ($\tilde{d}_2, \tilde{d}_3, \tilde{d}_5, \tilde{d}_6$) only.

$$\underset{4\times 4}{\tilde{\underline{k}}^{(e)}_B} \; \underset{4\times 1}{\tilde{\underline{d}}^{(e)}_B} = \underset{4\times 1}{\tilde{\underline{f}}^{(e)}_B}$$

$$\tilde{\underline{d}}^{(e)}_B = \begin{Bmatrix} \tilde{d}_2 \\ \tilde{d}_3 \\ \tilde{d}_5 \\ \tilde{d}_6 \end{Bmatrix} \qquad \tilde{\underline{f}}^{(e)}_B = \begin{Bmatrix} \tilde{f}^{(e)}_2 \\ \tilde{f}^{(e)}_3 \\ \vdots \\ \tilde{f}^{(e)}_6 \end{Bmatrix}$$

mom.

$\tilde{\underline{k}}^{(e)}_B$ p. 247 Book.

Mtg 25: Mon, 23 Oct 06 + 5 min (Lect 2)

Plan: - PVW revisited, HW7
- Beams, PVW.

- HW10 assigned (Design proj.)

p. 24-5: Book, p. 247

$$\underset{\sim}{\underline{k}}^{(e)}_{B} = \frac{2EI}{L^3} \begin{array}{c} \begin{bmatrix} 6 & 3L & -6 & 3L \\ & 2L^2 & -3L & L^2 \\ & \text{Sym} & 6 & -3L \\ & & & 2L^2 \end{bmatrix} \\ \begin{array}{cccc} \delta\tilde{d}_2 & \tilde{d}_3 & \tilde{d}_5 & \tilde{d}_6 \end{array} \end{array}$$

Transv. disp.

rot.

Frame = Truss ⊕ Beam.

uncoupled in local dof's. $\underline{\tilde{d}}$

$$\underset{6\times 1}{\underline{\tilde{d}}} = \left\{ \begin{array}{c} \underline{\tilde{d}}_T \\ \hline \underline{\tilde{d}}_B \end{array} \right\}_{6\times 1} \qquad \underset{6\times 1}{\underline{\tilde{f}}} = \left\{ \begin{array}{c} \underline{\tilde{f}}_T \\ \hline \underline{\tilde{f}}_B \end{array} \right\}_{6\times 1}$$

$$\underset{6\times 6}{\underline{\tilde{k}}^{(e)}} = \begin{bmatrix} \underset{2\times 2}{\underline{\tilde{k}}^{(e)}_T} & \underline{0}_{2\times 4} \\ \underset{4\times 2}{\underline{0}} & \underset{4\times 4}{\underline{\tilde{k}}^{(e)}_B} \end{bmatrix} \qquad \overset{6\times 1}{\left\{ \begin{array}{c} \tilde{d}_1 \\ \tilde{d}_4 \\ \hline \tilde{d}_2 \\ \tilde{d}_3 \\ \tilde{d}_5 \\ \tilde{d}_6 \end{array} \right\}} \begin{array}{c} \underline{\tilde{d}}_T \\ \\ \underline{\tilde{d}}_B \end{array}$$

$$\underline{\tilde{d}}^T = \lfloor \tilde{d}_1 \ \tilde{d}_2 \ \tilde{d}_3 \ \tilde{d}_4 \ \tilde{d}_5 \ \tilde{d}_6 \rfloor$$

$\underset{6\times 6}{\underline{\tilde{k}}^{(e)}}$: Book, p. 266.

$$
\underset{6\times 6}{\underline{\tilde{k}}^{(e)}} =
\begin{array}{cc}
\begin{array}{cccccc} \tilde{d}_1 & \tilde{d}_2 & \tilde{d}_3 & \tilde{d}_4 & \tilde{d}_5 & \tilde{d}_6 \end{array} & \\
\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
 & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
 & & & \frac{EA}{L} & 0 & 0 \\
 & \text{Sym.} & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
 & & & & & \frac{4EI}{L}
\end{bmatrix}_{6\times 6} &
\begin{array}{c} \tilde{d}_1 \\ \tilde{d}_2 \\ \tilde{d}_3 \\ \tilde{d}_4 \\ \tilde{d}_5 \\ \tilde{d}_6 \end{array}
\end{array}
$$

Sam : Check dimen!!

Pam : $[E] = \frac{F}{L^2}$

$$\sigma = E\,\varepsilon$$

$$\Rightarrow [\sigma] = [E]\underbrace{[\varepsilon]}_{\substack{\shortparallel \\ 1}}$$

(dimen of.)

$$\varepsilon = \frac{\Delta L}{L} \Rightarrow [\varepsilon] = \frac{L}{L} = 1$$

$$[E] = [\sigma] = \frac{F}{L^2}$$

$$\left[\frac{EA}{L}\right] = \frac{[E]\,[A]}{L} = \frac{(F/L^2)(L^2)}{L}$$

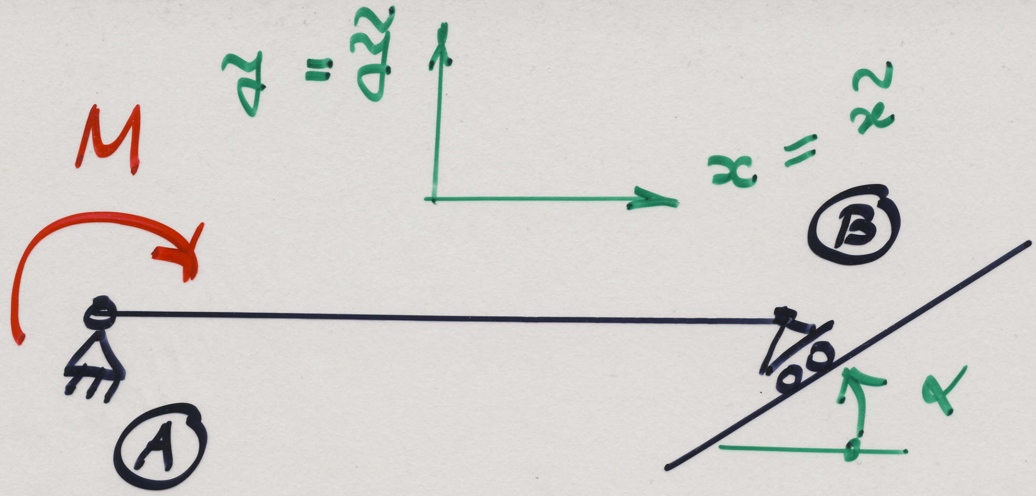
$$= \frac{F}{L}$$

FD rel: $\tilde{f}_1 = \frac{EA}{L}(\tilde{d}_1 - \tilde{d}_4)$

$$[\tilde{f}_1] = F$$

$$\left[\frac{EA}{L}(\tilde{d}_1 - \tilde{d}_4)\right] = \frac{F}{L}\,L = F$$

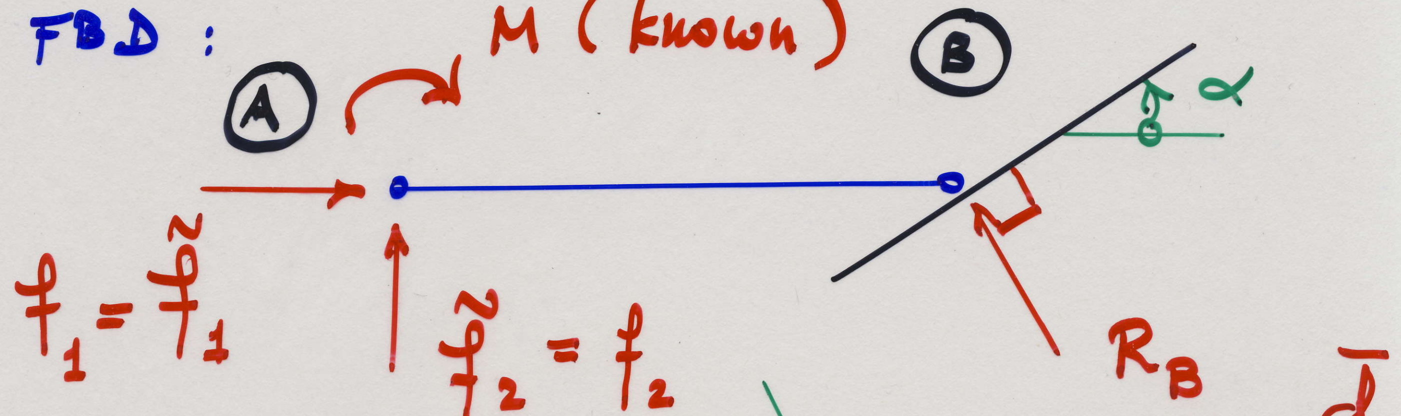
25-4



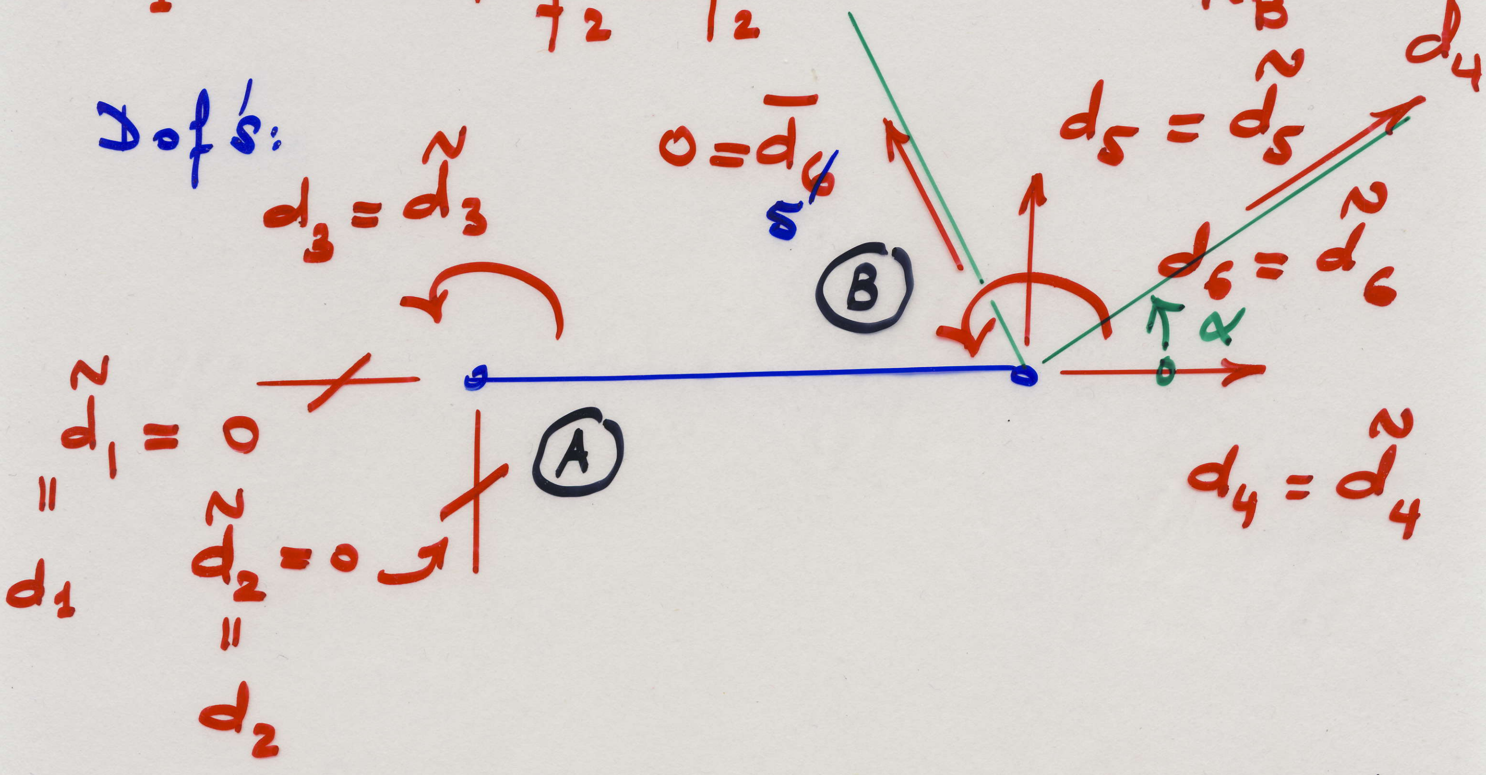
Mtg 26: Wed, 25 Oct 06 + 5 min (Lect 3)

p. 25-4 (Cont'd)

FBD :



Dof's:



$$\underset{6\times 6}{\underline{K}}\ \underset{6\times 1}{\underline{d}} = \underset{6\times 1}{\underline{F}} \quad \text{(before ess. b.c.'s)}$$

with $\underset{6\times 6}{\underline{K}} = \underset{6\times 6}{\underline{k}^{(1)}} = \underset{6\times 6}{\underline{\tilde{k}}^{(1)}}$

$\left(\begin{array}{l} x = \tilde{x} \\ y = \tilde{y} \end{array} \right)$

$$\underline{d}^b = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \underline{0}$$

$$\underline{d}^i = \begin{Bmatrix} d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix}$$

$$\underline{k}^{(1)} = \begin{bmatrix} \underline{K}^{bb} & \underline{K}^{bi} \\ \underline{K}^{ib} & \underline{K}^{ii} \end{bmatrix}$$

$$\underline{F} = \begin{Bmatrix} \underline{F}^b \\ \underline{F}^i \end{Bmatrix}, \quad \underline{F}^b = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}, \quad \underline{F}^i = \begin{Bmatrix} f_3 \\ f_4 \\ f_5 \\ f_6 \end{Bmatrix}$$

2nd matrix eq:

$$\underline{K}^{ib} \underbrace{\underline{d}^b}_{=\underline{0}} + \underline{K}^{ii} \underline{d}^i = \underline{F}^i$$

$$\underline{F}^i = \begin{Bmatrix} -M \\ \neq 0 \\ \neq 0 \\ 0 \end{Bmatrix} \begin{matrix} f_4 \\ f_5 \end{matrix}$$

Chris

David

$$\underset{4\times1}{\underline{d}^i} = \underset{4\times3}{\underline{T}} \; \underset{3\times1}{\underline{\bar{d}}_i}, \quad \underset{3\times1}{\underline{\bar{d}}_i} = \begin{Bmatrix} d_3 \\ \bar{d}_4 \\ d_6 \end{Bmatrix}_{3\times1}$$

$$d_4 = (\cos\alpha)\, \bar{d}_4$$

$$d_5 = (\sin\alpha)\, \bar{d}_4$$

$$\underset{4\times3}{\underline{T}} = \begin{array}{c} \begin{array}{ccc} d_3 & \bar{d}_4 & d_6 \end{array} \\ \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & \cos\alpha & 0 \\ 0 & \sin\alpha & 0 \\ 0 & 0 & 1 \end{array}\right]_{4\times3} \end{array} \begin{array}{l} \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{array}$$

David, Siola

Tim

Brad, Shawn:

Recipe: $$\underbrace{\underset{3\times4}{\underline{T}^T}\ \underset{4\times4}{\underline{K}^{ii}}\ \underset{4\times3}{\underline{T}}}_{3\times3}\ \underset{3\times1}{\underline{\bar{d}}^i} = \underbrace{\underset{3\times4}{\underline{T}^T}\ \underset{4\times1}{\underline{F}^i}}_{3\times1}$$

Larry: If $\underline{d}^b \neq 0$:

$$\underline{T}^T \underline{K}^{ii} \underline{T} \underline{\bar{d}}^i = \underline{T}^T \left(\underline{F}^i - \underline{K}^{ib} \underline{d}^b \right)$$

Review PVW.

Mtg 27: Fri, 27 Oct 06

Exam ~~1~~ 2