

# Formatting (2A)

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This document was produced by using OpenOffice and Octave.

# Formatting and Source Coding

## Formatting

Make the source signal compatible with digital processing

## Transmit Formatting

A transformation from source information to **digital symbols**

## Source Coding

Formatting + Data Compression

## Baseband Signal

From DC up to some finite frequency ( $<$  a few MHz)

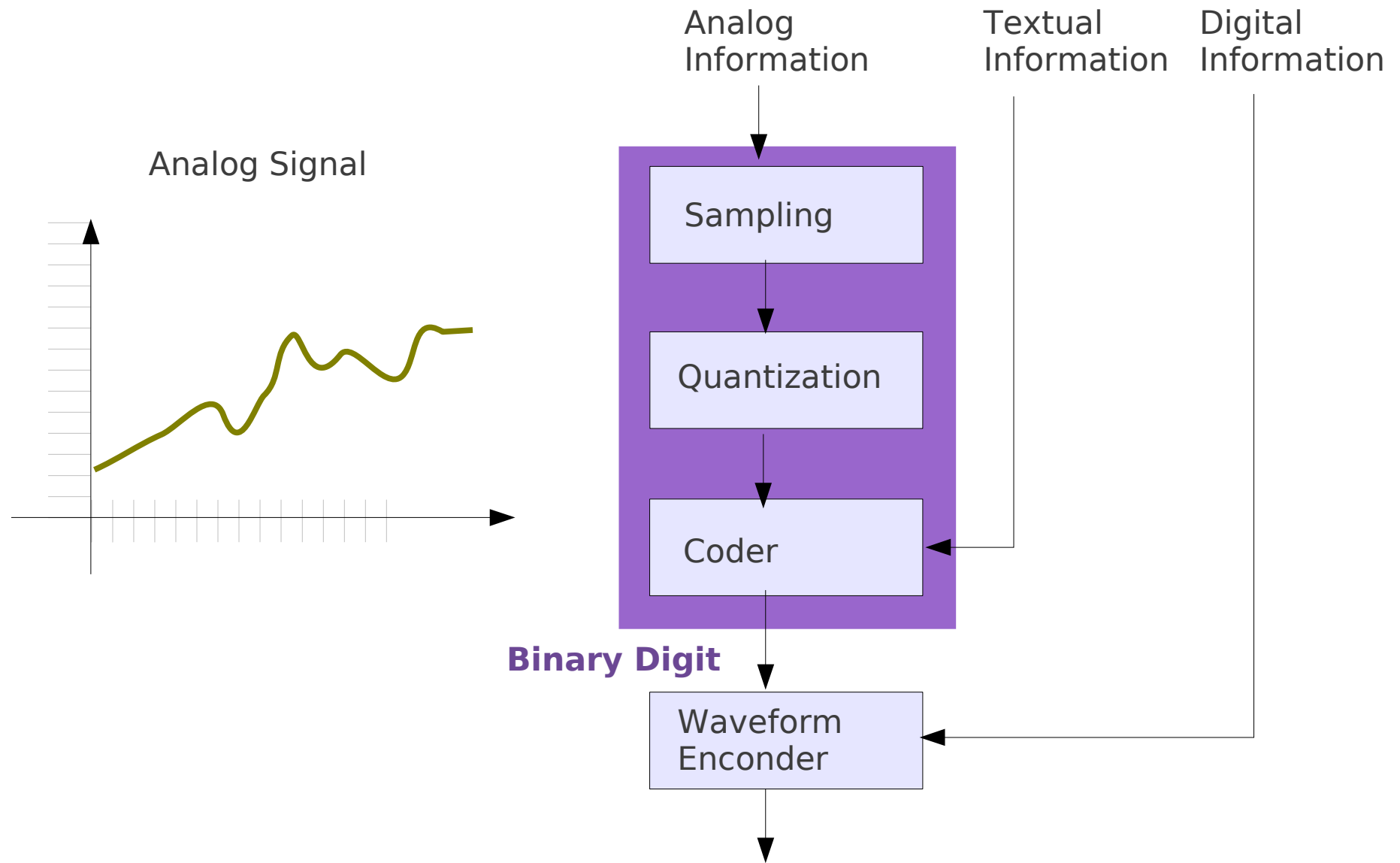
Transmitted over the cable

Not appropriate to transmit over long distance  $\rightarrow$  Bandpass Mod

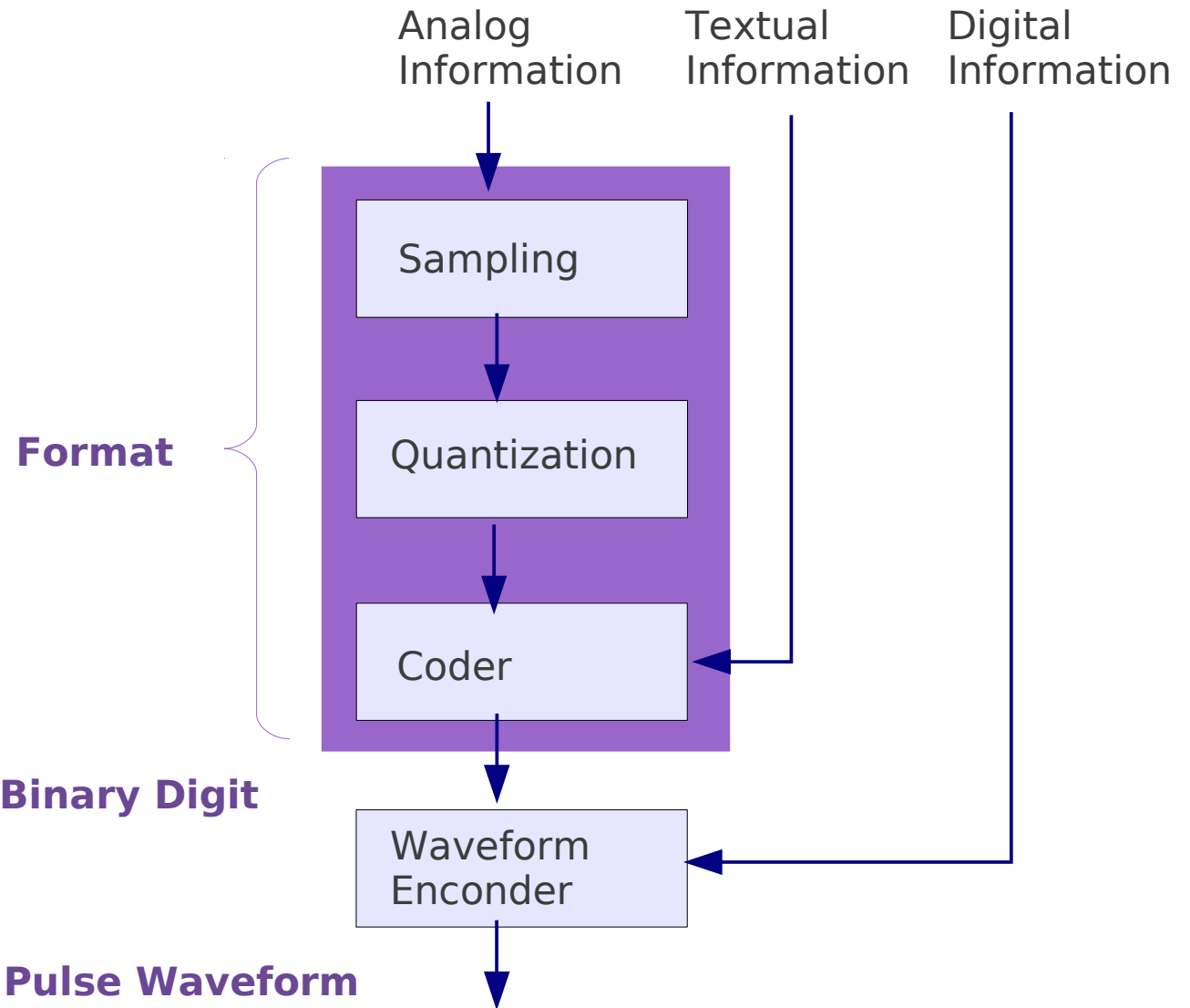
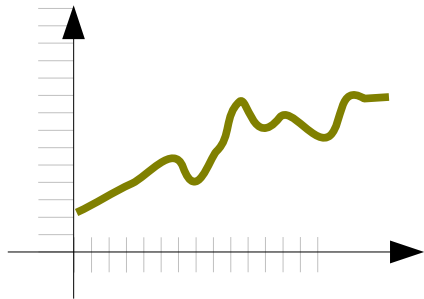
## Pulse (Baseband) Modulation

Pulse waveforms are assigned that represent formatted symbols

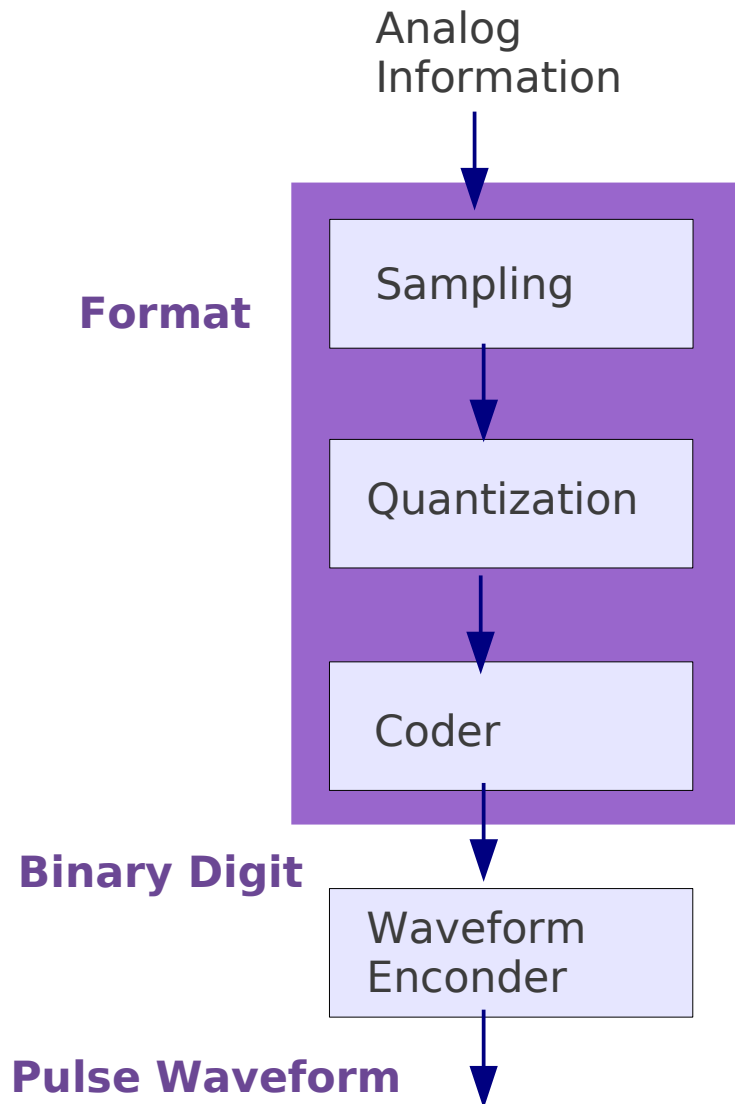
# Baseband Signal



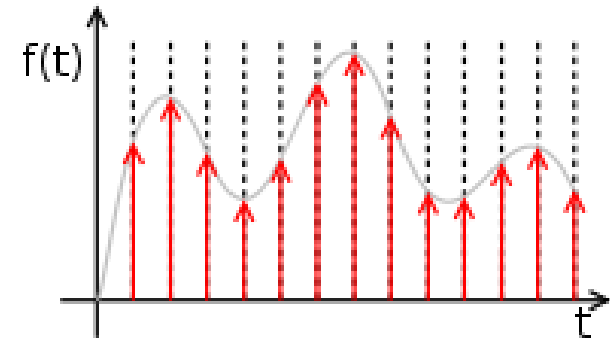
# Baseband Signal Format



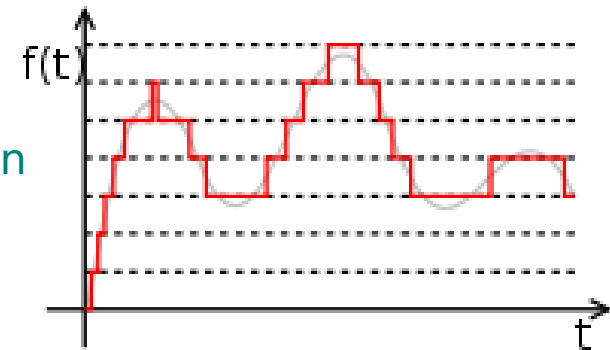
# Sampling and Quantization



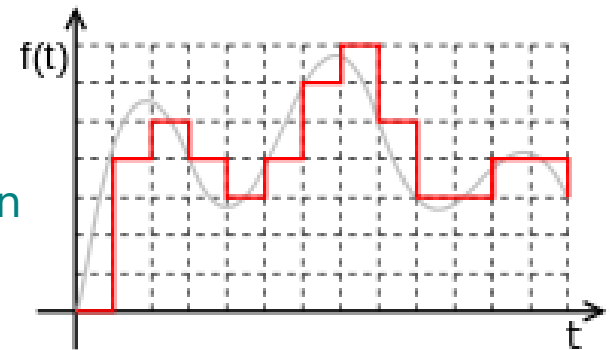
Sampling



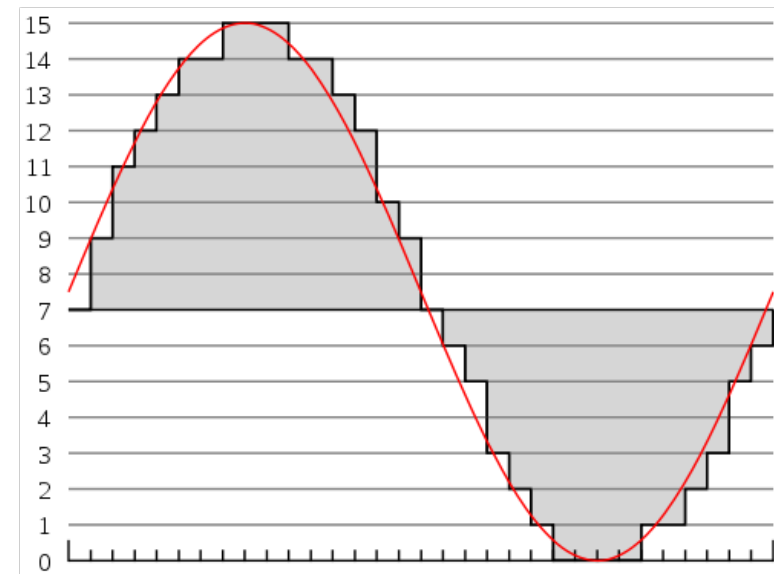
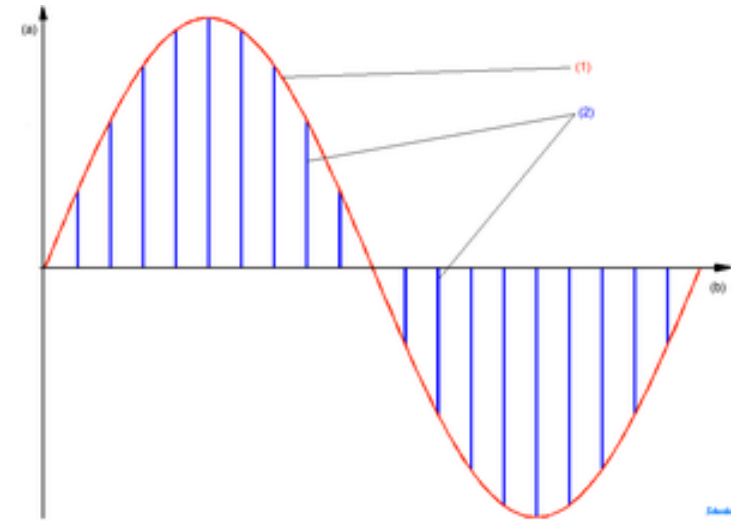
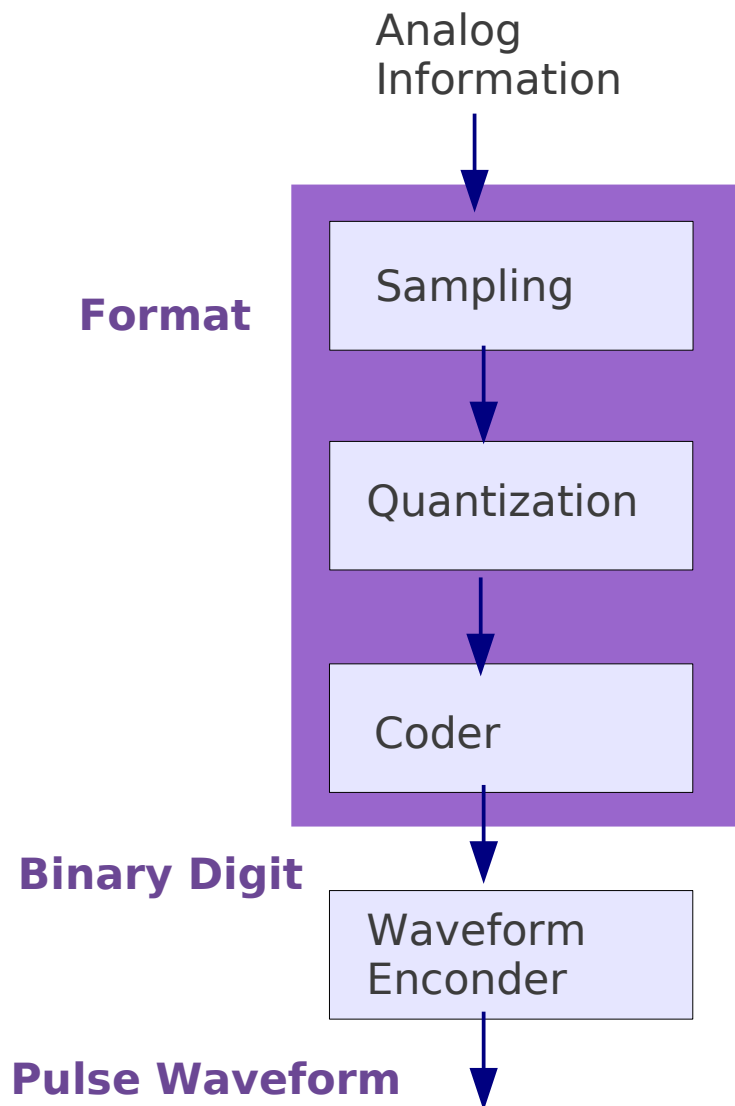
Quantization



Sampling + Quantization



# PAM (Pulse Amplitude Modulation)



# 8-ary Symbol

T H I N K

Message

001010 000100 100100 011100 110100

6-bit ASCII

001 010 000 100 100 100 011 100 110 100

8-ary digits  
(symbols)

1 2 0 4 4 4 3 4 6 4

$s_1(t)$   $s_2(t)$   $s_0(t)$   $s_4(t)$   $s_4(t)$   $s_4(t)$   $s_3(t)$   $s_4(t)$   $s_6(t)$   $s_4(t)$

8-ary(Pulse)  
waveform



# Binary Symbol

T H I N K

Message

001010 000100 100100 011100 110100

6-bit ASCII

0 0 1 0 1 0 0 0 0 1 0 0 1 0 0 0 1 1 1 0 0 1 1 0 1 0 0

0 0 1 0 1 0 0 0 0 1 0 0 1 0 0 0 1 1 1 0 0 1 1

binary digits  
(symbols)

$s_0(t)$   $s_0(t)$   $s_1(t)$   $s_0(t)$   $s_1(t)$   $s_0(t)$   $s_0(t)$   $s_0(t)$   $s_0(t)$   $s_0(t)$   $s_1(t)$   $s_0(t)$   $s_0(t)$

binary (Pulse)  
waveform

# Impulse Sampling

## Impulse train

$$x_{\delta}(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$



$$X_{\delta}(f) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} \delta(f - nf_s)$$

## Shifting property

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

$$x_s(t) = x(t)x_{\delta}(t)$$



$$X_s(f) = X(f) * X_{\delta}(f)$$

$$= \sum_{n=-\infty}^{+\infty} x(t)\delta(t - nT_s)$$

$$= X(f) * \left[ \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} \delta(f - nf_s) \right]$$

$$= \sum_{n=-\infty}^{+\infty} x(nT_s)\delta(t - nT_s)$$

$$= \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X(f - nf_s)$$

# Natural Sampling

## Pulse train

$$x_p(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j2\pi n f_s t}$$



$$c_n = \frac{1}{T_s} \text{sinc}\left(\frac{nT_s}{T_s}\right)$$

$$x_s(t) = x(t) x_p(t)$$



$$X_s(f) = X(f) * X_p(f)$$

$$= x(t) \sum_{n=-\infty}^{+\infty} c_n e^{j2\pi f_s t}$$

$$= \sum_{n=-\infty}^{+\infty} c_n \left[ x(t) e^{j2\pi f_s t} \right]$$

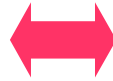


$$= \sum_{n=-\infty}^{+\infty} c_n X(f - n f_s)$$

# Sample and Hold

## Sampled Pulse train

$$x_p(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j2\pi n f_s t}$$



$$c_n = \frac{1}{T_s} \text{sinc}\left(\frac{nT_s}{T_s}\right)$$

$$x_s(t) = p(t) * [x(t) x_\delta(t)]$$



$$X_s(f) = P(f) [X(f) * X_\delta(f)]$$

$$= p(t) * \left[ x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \right]$$



$$= P(f) \left[ X(f) * \left[ \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} \delta(f - n f_s) \right] \right]$$

$$= P(f) \left[ \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X(f - n f_s) \right]$$

# Sampling Theorem

## Uniform Sampling Theorem

A band-limited signal having no spectral components above  $f_m$  Hz can be determined uniquely by values sampled at *uniform intervals* of  $T_s$  seconds

$$T_s \leq \frac{1}{2f_m}$$

*Upper limit of  $T_s$*

$$f_s = \frac{1}{T_s}$$

$$f_s \geq 2f_m$$

*Lower limit of  $f_s$*

*Nyquist Criterion*

*Nyquist Rate  $f_s = 2f_m$*

# Autocorrelation of Energy and Power Signals

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# Ensemble Average

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# WSS (Wide Sense Stationary)

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# Autocorrelation of Random and Power Signals

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# Time Averaging and Ergodicity

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# Autocorrelation of Random and Power Signals

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# Time Averaging and Ergodicity

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## References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] B. Sklar, “Digital Communications: Fundamentals and Applications”