Group Delay and Phase Delay (1A)

Young Won Lim
3/24/12

Please send corrections (or suggestions) to youngwlim@hotmail.com.

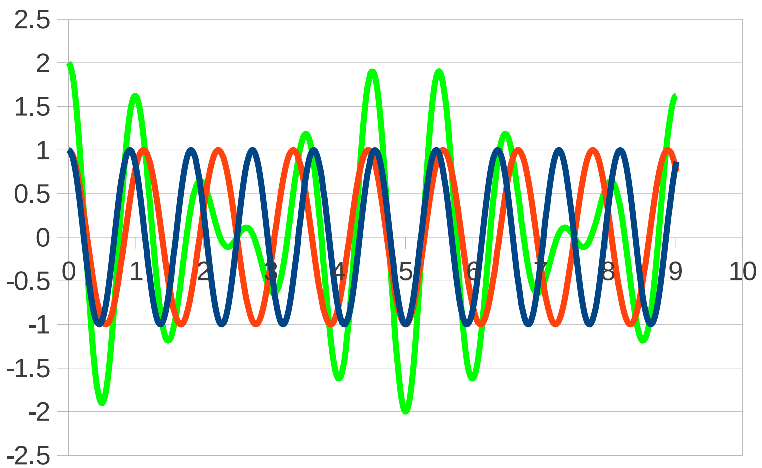
This document was produced by using OpenOffice and Octave.

Beat Signal

Very similar frequency signals

1.1 Hz $\cos(2\pi*1.1*t)$

0.9 Hz $\cos(2\pi*0.9*t)$



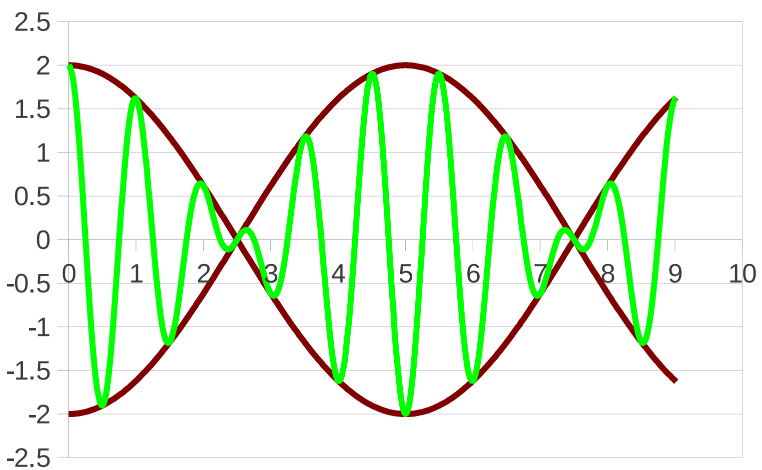
$$\cos(2\pi*1.1*t) + \cos(2\pi*0.9*t)$$

$$= \cos\left(2\pi*\frac{(1.1-0.9)}{2}*t\right)\cdot\cos\left(2\pi*\frac{(1.1+0.9)}{2}*t\right)$$

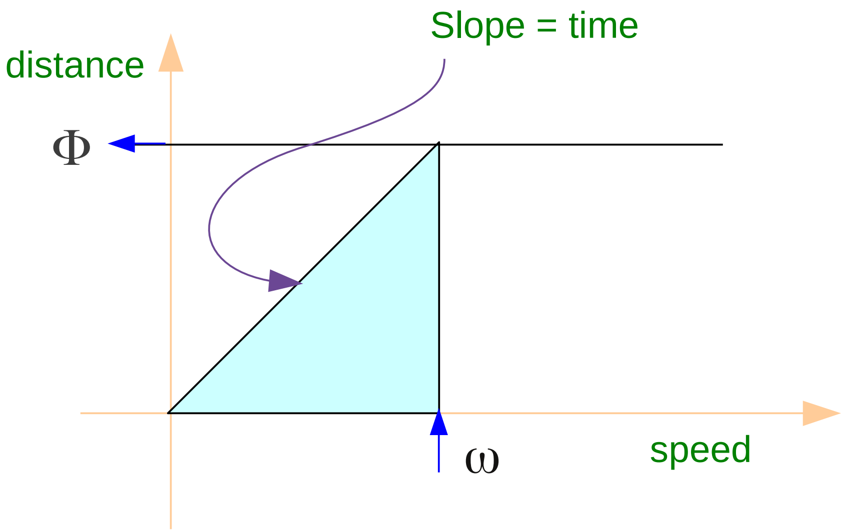
$$= \cos(2\pi*0.1*t)\cdot\cos(2\pi*1.0*t)$$

Slow moving envelop

Fast moving carrier

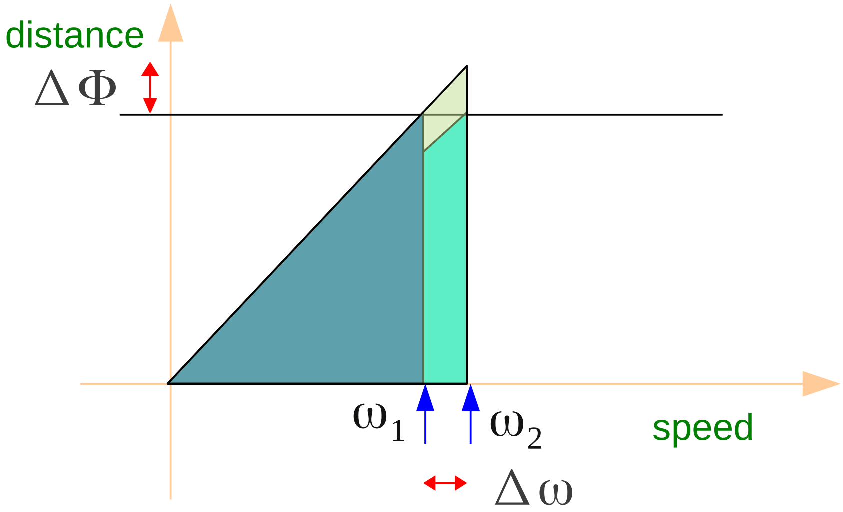


Angle and Angular Speed



$$\Phi = \omega \cdot t$$

$$t = \frac{\Phi}{\omega}$$



$$\Delta\Phi = \Delta\omega \cdot \Delta t$$

$$\Delta t = \frac{\Delta\Phi}{\Delta\omega}$$

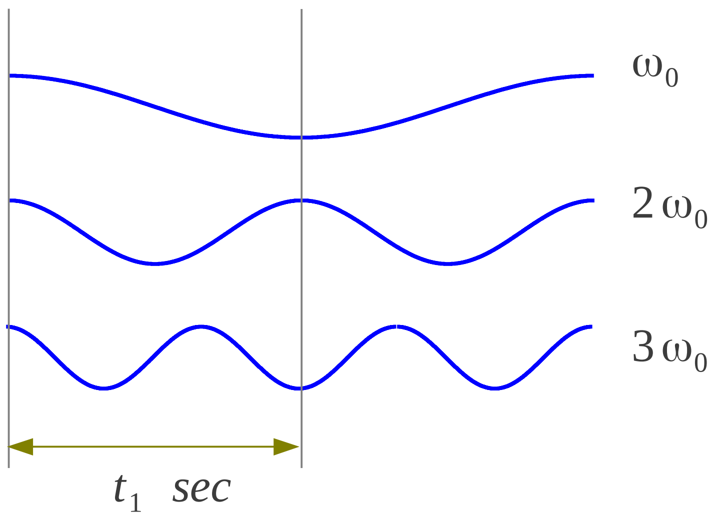
Phase Shift and Time Shift

measure phase shift not in second
*But **in portions** of a cosine wave cycle*

within phase change in one cycle

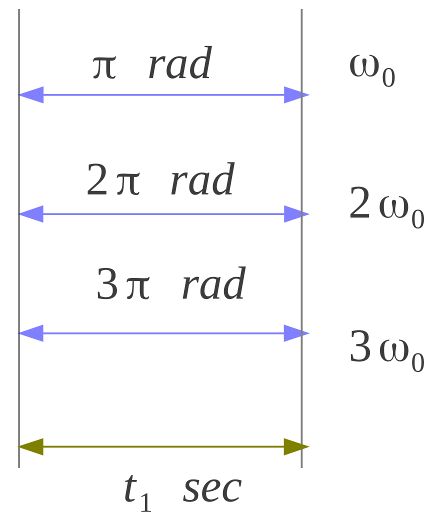
Phase Shift → in radians, degrees
Delay → in seconds (time)

Given time shift (delay)



The actual phase shift is different

According to the frequency

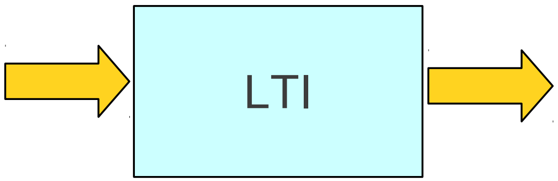


The same <u>delay</u>
applied to all frequencies

The different <u>phase shift</u>
to the different frequency

Frequency Response

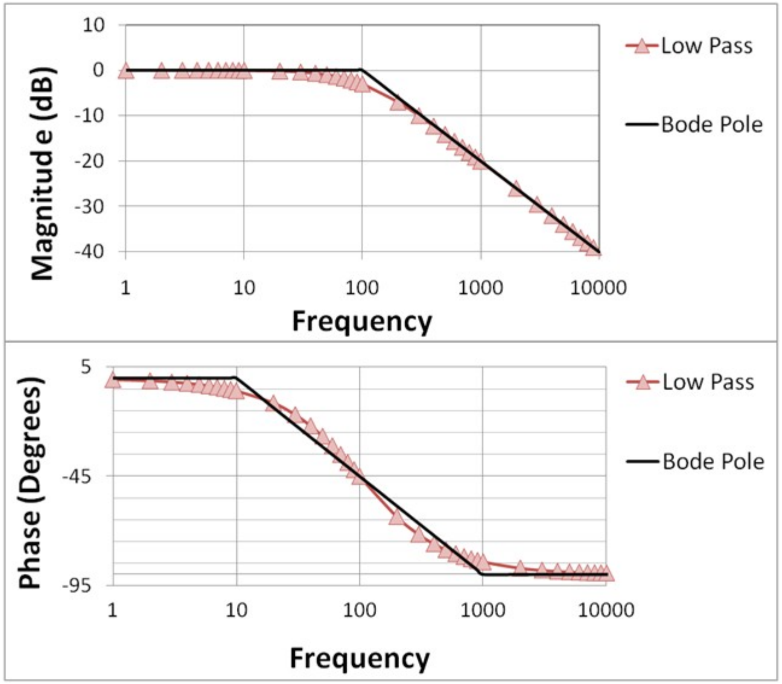
Frequency Response $H(e^{j\omega})$

$e^{j\omega t}$  $H(e^{j\omega})\, e^{j\omega t}$

LPF example

$|H(e^{j\omega})|$ Magnitude Response

$\angle H(e^{j\omega})$ Phase Response



Young Won Lim
3/24/12

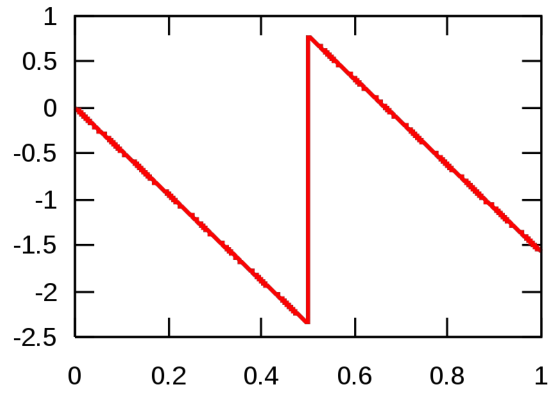
Linear Phase System

Linear Phase System

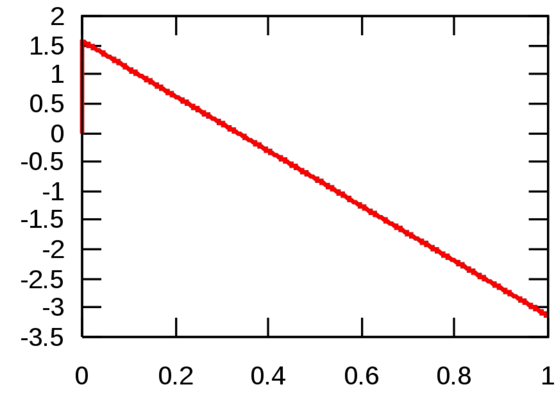
Phase Shift $\propto$ Frequency

$$\angle H\left(e^{j\omega}\right) \propto \omega$$

a) FIR Filter (Type II) having Linear Phase

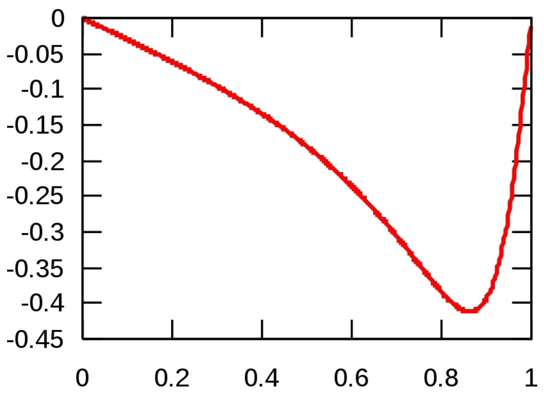


b) FIR Filter (Type IV) having Linear Phase

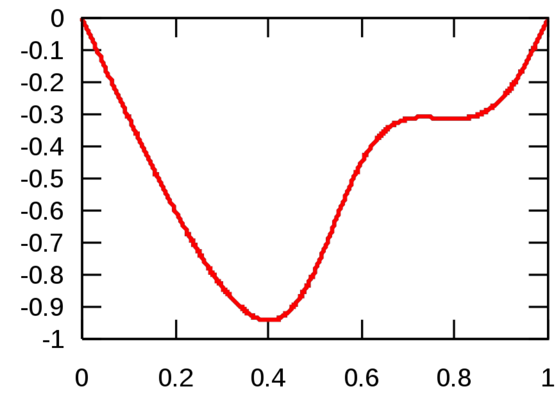


Non-Linear Phase System

c) IIR Filter having Non-Linear Phase

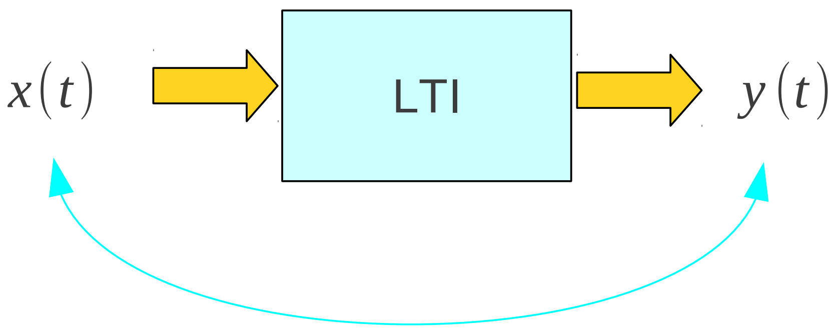


d) FIR Filter having Non-Linear Phase

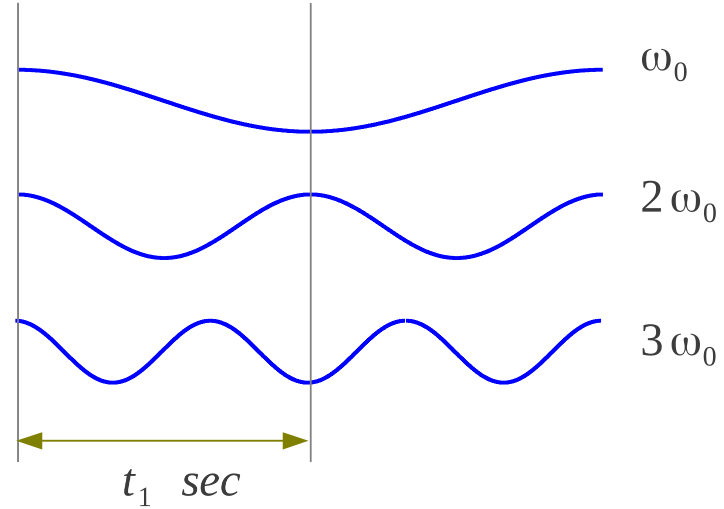


Uniform Time Delay (1)

Frequency Response $H(e^{j\omega})$



The waveform shape can be preserved.

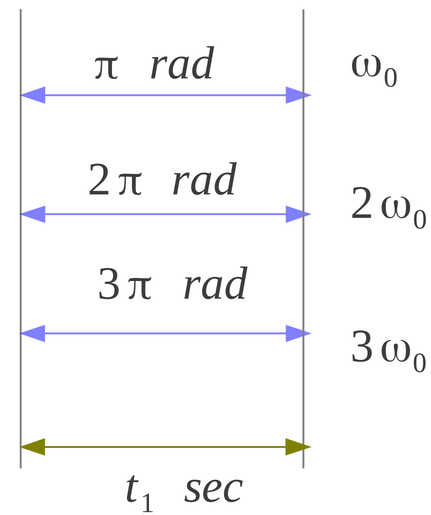


{ uniform magnitude $|H(e^{j\omega})| = c$

{ uniform time delay

⇕

linear phase $\angle H(e^{j\omega}) = k\omega$

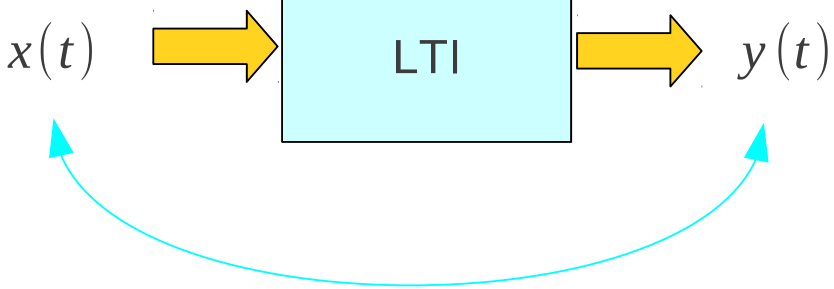


Uniform Time Delay

Could remove delay from the phase response

Uniform Time Delay (2)

Frequency Response $H(e^{j\omega})$



The waveform shape can be preserved.

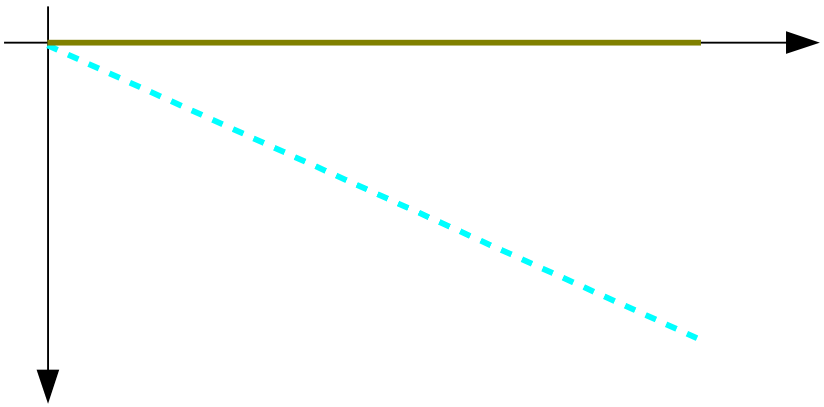
- uniform magnitude $|H(e^{j\omega})| = c$
- uniform time delay

⇕

linear phase $\angle H(e^{j\omega}) = k\,\omega$

Uniform Time Delay

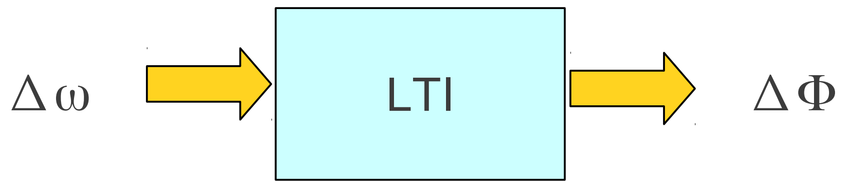
Could remove delay from the phase response to achieve a horizontal line at zero degree (No delay)



Group Delay

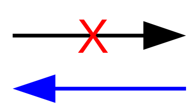
Consider the cosine components at *closely spaced frequencies* and *their phase shifts* in relation to each other

Group Delay:
The phase shift changes for small changes in frequency



A uniform, waveform preserving phase response → linear

Constant Group Delay



Uniform Time Delay (linear phase)

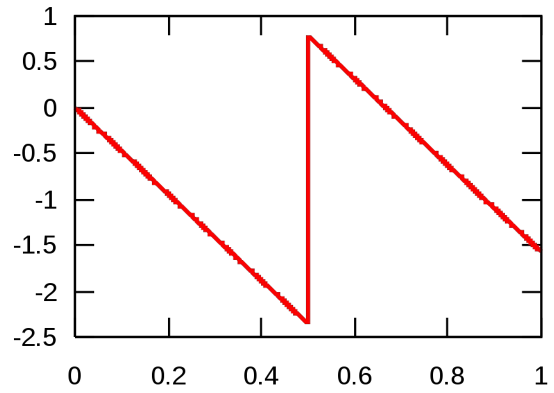
Group Delay (2)

Linear Phase System

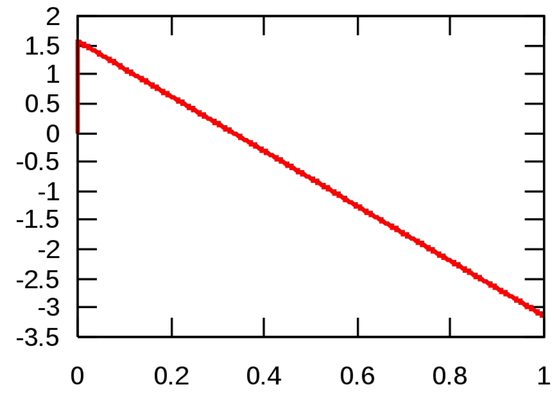
Phase Shift $\propto$ Frequency

$$\angle H\left(e^{j\omega}\right) \propto \omega$$

a) FIR Filter (Type II) having Linear Phase

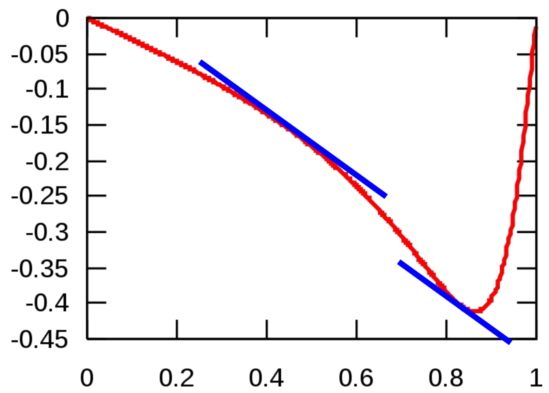


b) FIR Filter (Type IV) having Linear Phase

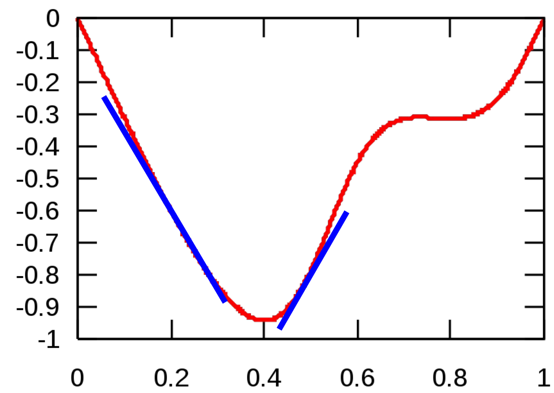


Non-Linear Phase System

c) IIR Filter having Non-Linear Phase



d) FIR Filter having Non-Linear Phase



Group Delay

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] http://www.libinst.com/tpfd.htm