

# Anti-Image Postfilter (7B)

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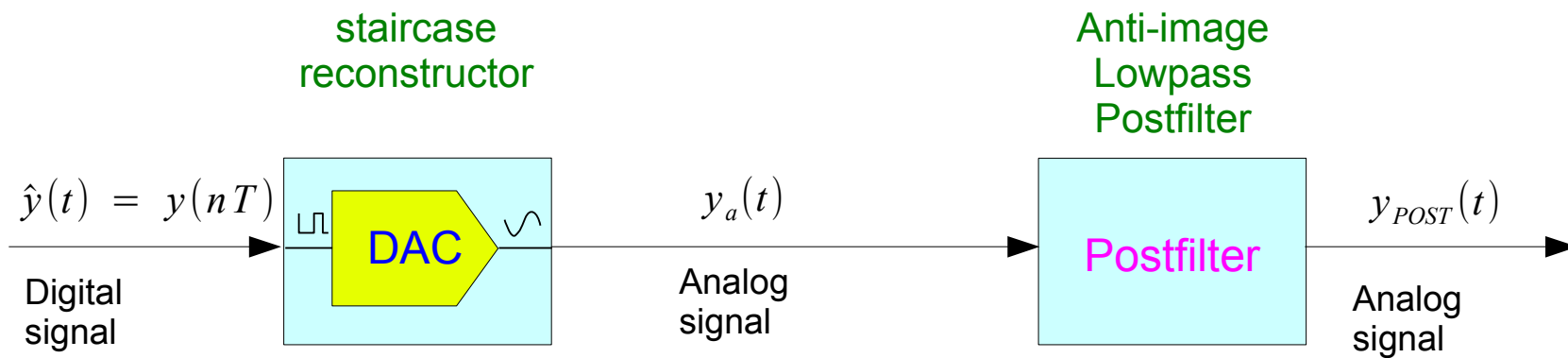
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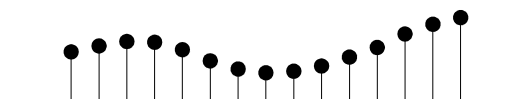
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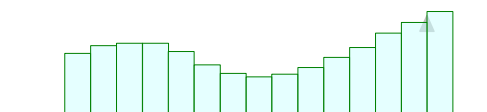
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$$\hat{y}(t) = y(nT)$$

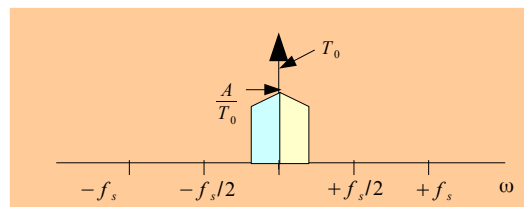
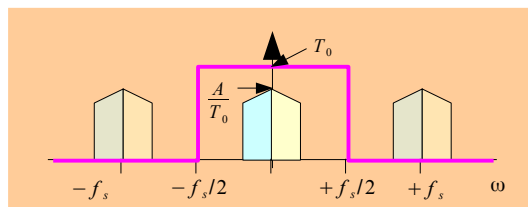
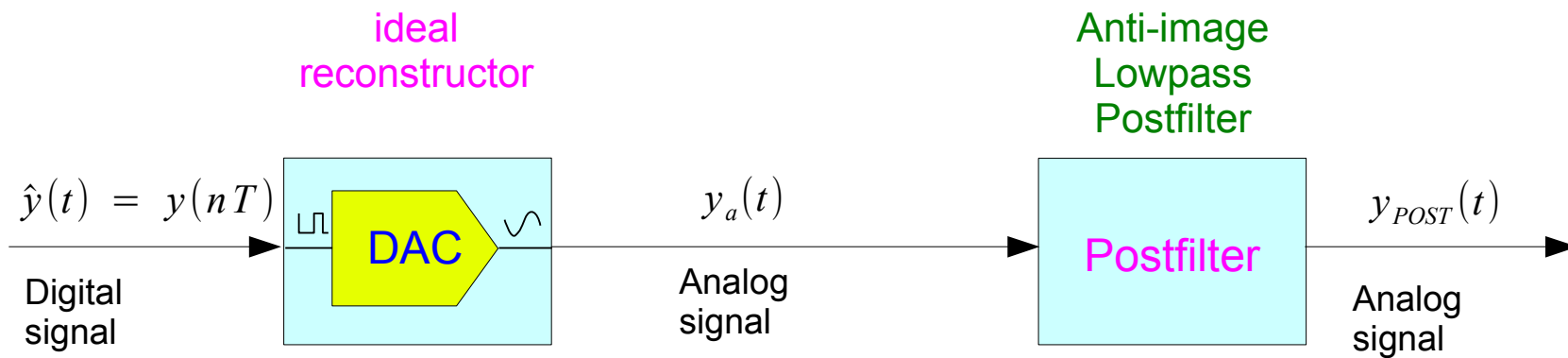


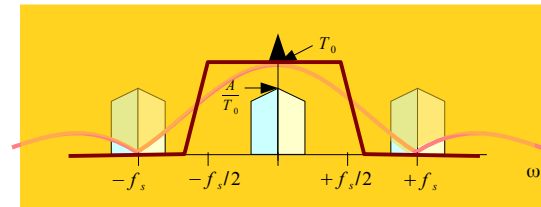
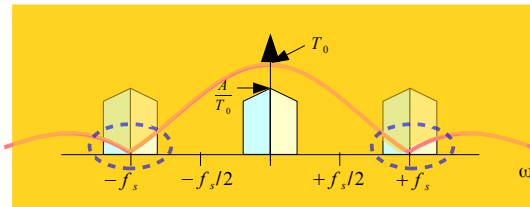
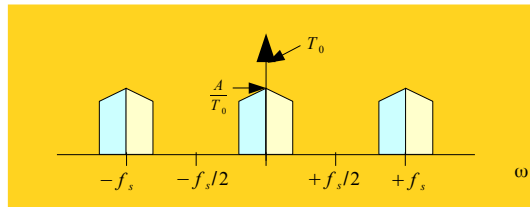
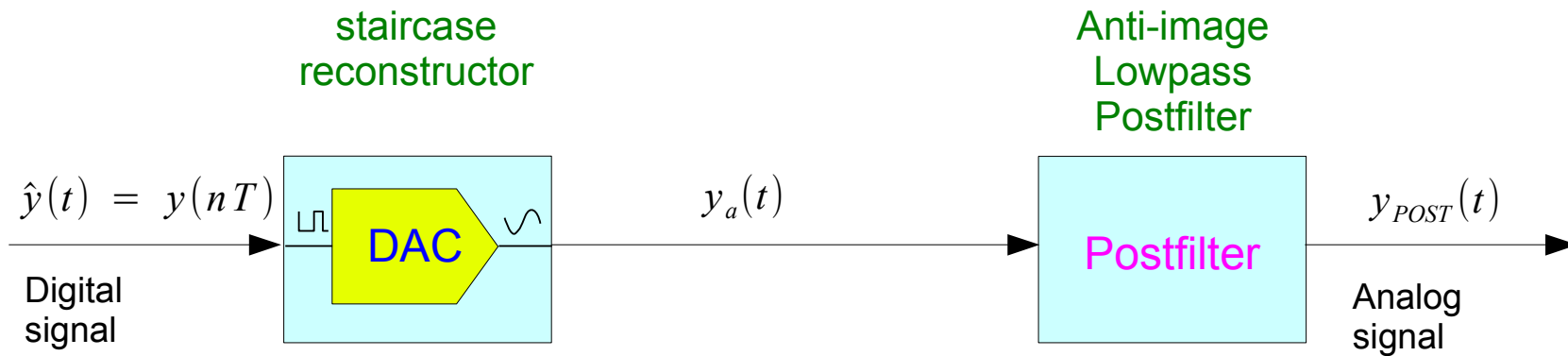
$$y_a(t)$$



$$y_{POST}(t)$$







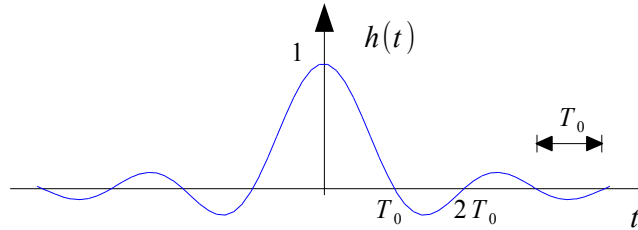
↑ surviving replica

↑ Non-flat: Partially attenuated

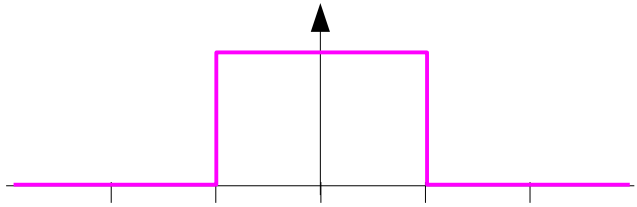
↑ surviving replica

# CTFT of Reconstructors (1)

$t = \pm T_0, \pm 2T_0, \pm 3T_0, \dots \rightarrow h(t) = 0$

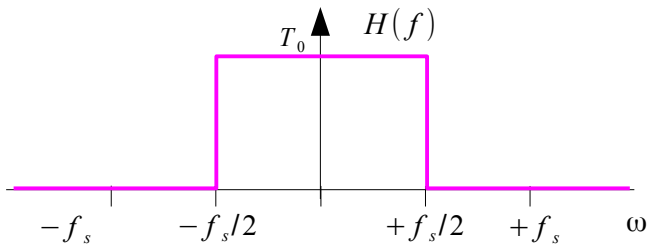


$$\frac{1}{T_0} \equiv f_s$$



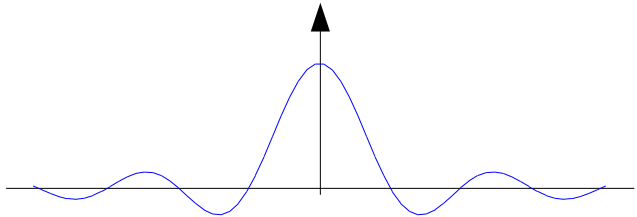
$$h(t) = \frac{\sin(\pi t/T_0)}{\pi t/T_0} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

**CTFT**



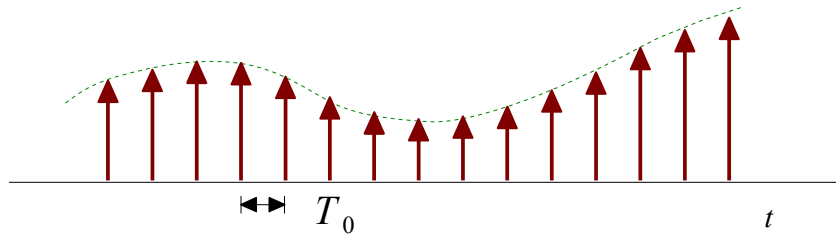
$$H(f) = \begin{cases} T_0, & |f| \leq f_s/2 \\ 0, & \text{otherwise} \end{cases}$$

**CTFT**

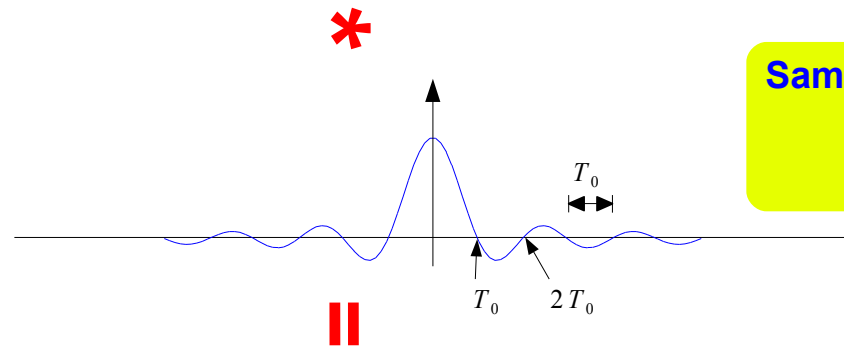
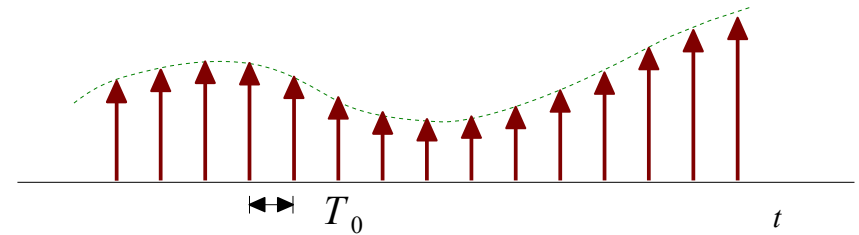


# Reconstruct via Convolution

## Ideal Reconstructor

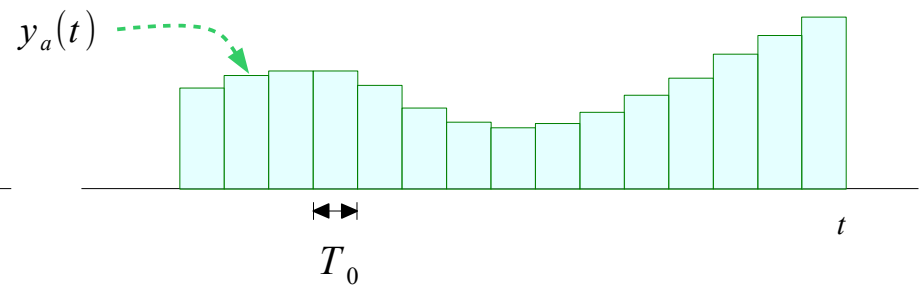
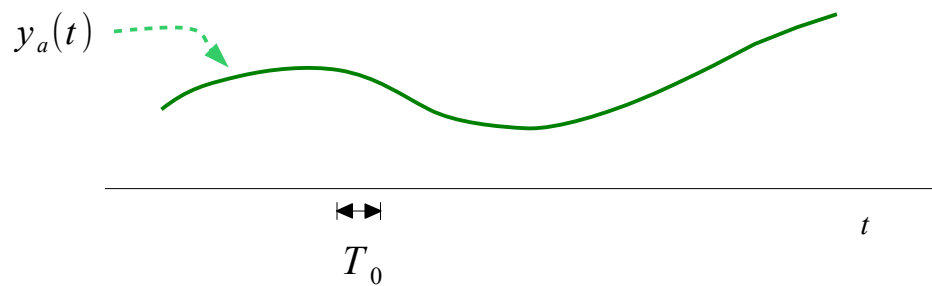
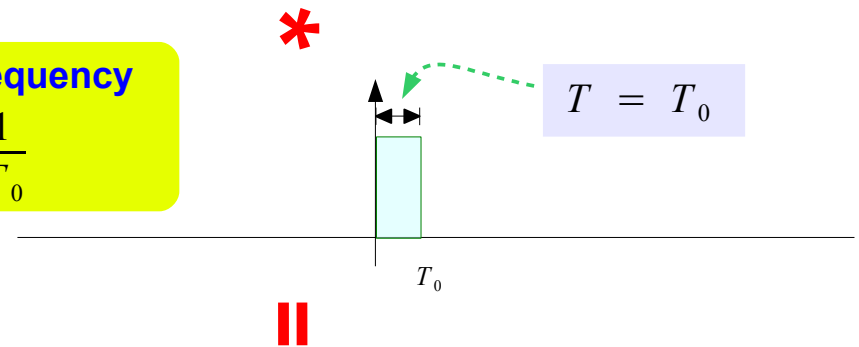


## Practical Reconstructor



Sampling frequency

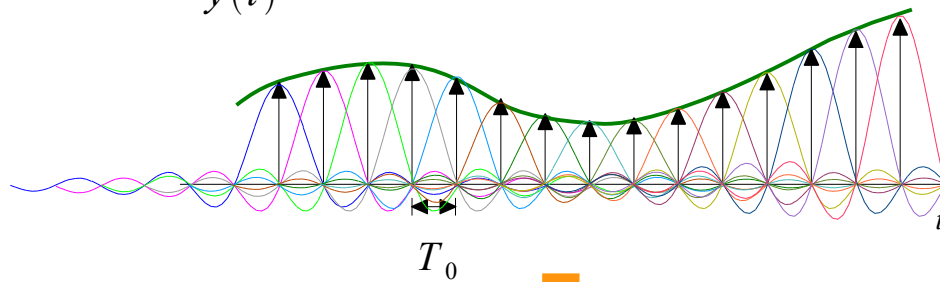
$$f_s = \frac{1}{T_0}$$



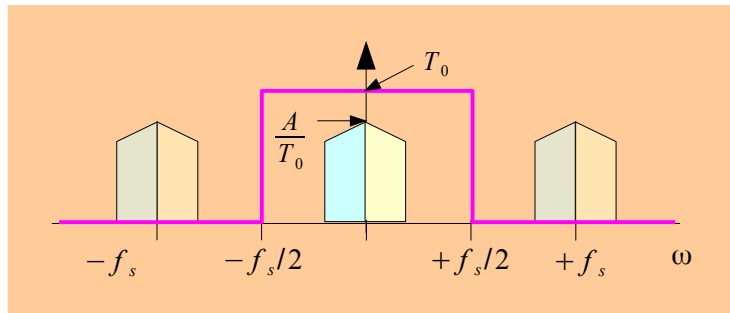
# Reconstructors in Frequency Domain

## Ideal Reconstructor

$$\hat{y}(t)$$

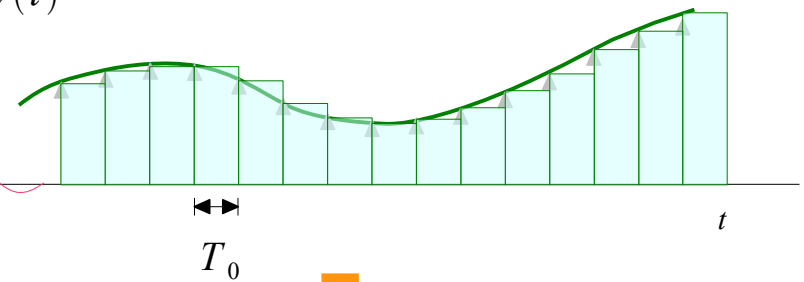


CTFT

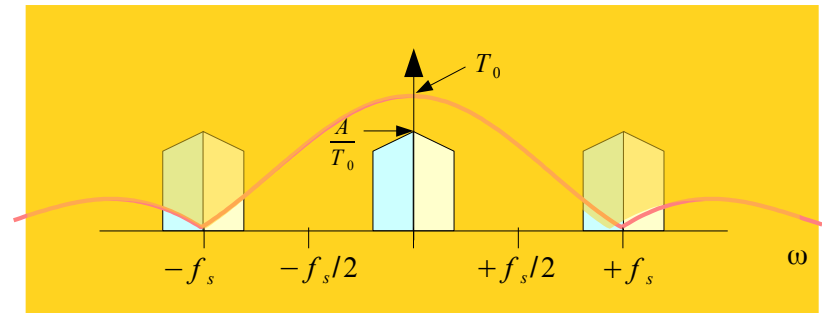


## Practical Reconstructor

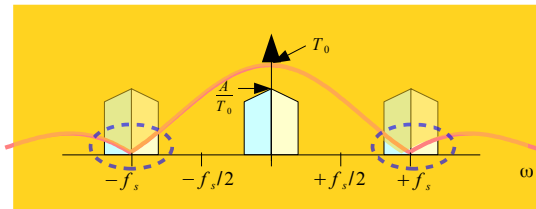
$$\hat{y}(t)$$



CTFT





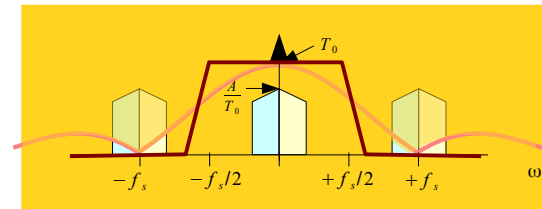


↑ surviving replica  
 ↑ Non-flat: Partially attenuated  
 ↑ surviving replica

Surviving spectral replicas can be removed by an additional lowpass filter

Anit-image Postfilter

Cutoff Frequency



$$f_{max} \leq \frac{f_s}{2}$$

Freq domain

(reconstructor + postfilter) to remove the spectral replicas as much as possible

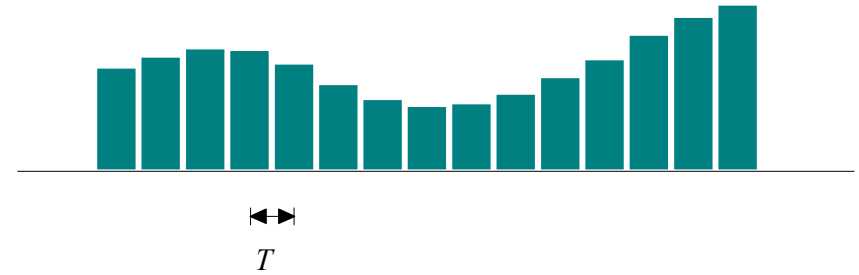
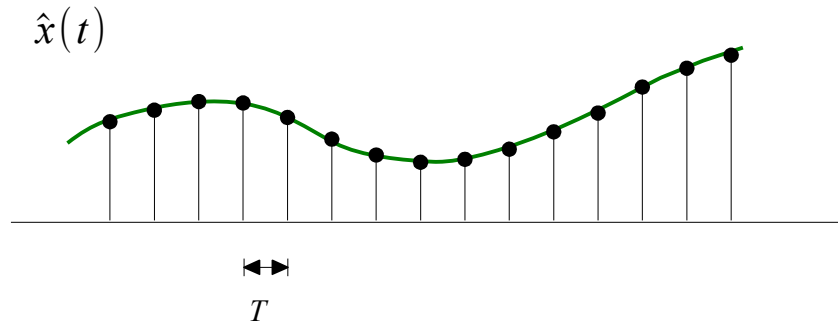
Time domain

Effect of rounding off the corners of staircase output making smoother

Two stage (Staircase Reconstructor + Postfilter) → simplicity of implementation of reconstructor : DAC – generating an analog output that remains constant during T

Emulate the ideal reconstructor

# Analog Reconstructor



$$\hat{y}(t) = \sum_{n=-\infty}^{+\infty} y(nT) \delta(t-nT)$$

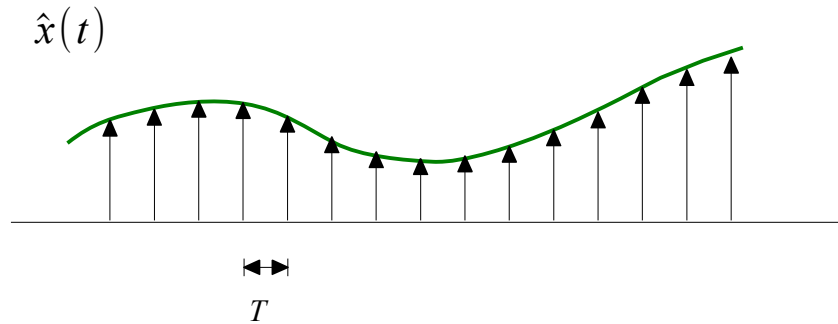
$$Y_a(f) = H(f) \hat{Y}(f)$$

$$y_a(t) = \int_{-\infty}^{+\infty} h(t-t') \hat{y}(t') dt'$$

$$\hat{Y}_a(f) = \frac{1}{T} \sum_{m=-\infty}^{+\infty} Y(f - m f_s)$$

$$y_a(t) = \sum_{n=-\infty}^{+\infty} y(nT) h(t-nT)$$

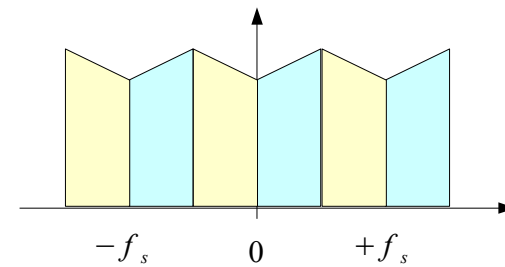
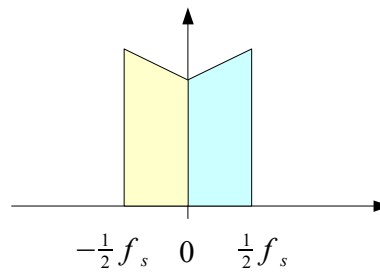
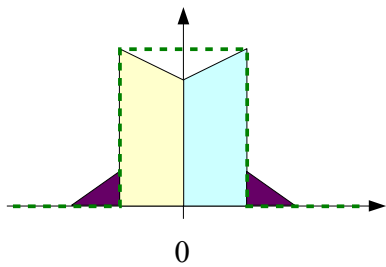
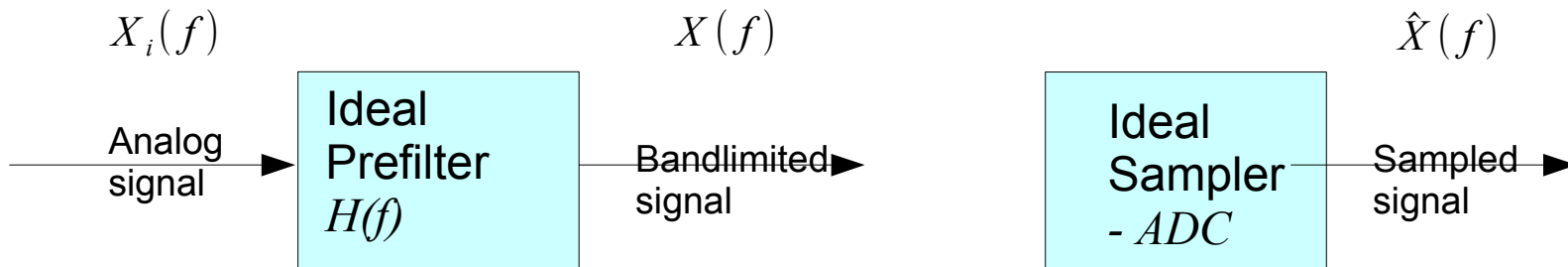
# Impulse Response of Ideal Reconstructor



$$\hat{Y}(f) = \frac{1}{T} Y(f) \quad -\frac{f_s}{2} \leq f \leq +\frac{f_s}{2}$$

$$y(t) = \sum_{n=-\infty}^{+\infty} y(nT) h(t-nT)$$

$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$



$\frac{2}{4}f_s$      $\frac{3}{4}f_s$      $f_s$





## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
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- [6] S.J. Orfanidis, Introduction to Signal Processing  
[www.ece.rutgers.edu/~orfanidi/intro2sp](http://www.ece.rutgers.edu/~orfanidi/intro2sp)