DLTI z-Transform

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Finding ZIR & ZSR Using Laplace Transform (1)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition $y[1]=3,\quad y[-2]=2$

input $x[n] = u[n]$

$$y[n] \quad \leftrightarrow \quad Y[z]$$

$$y[n-1] \quad \leftrightarrow \quad y[-1] + z^{-1}Y[z] = z^{-1}Y[z] + 3$$

$$y[n-2] \quad \leftrightarrow \quad y[-2] + z^{-1}y[-1] + z^{-2}Y[z] = z^{-2}Y[z] + 3z^{-1} + 2$$

$$x[n] \quad \leftrightarrow \quad X[z] = \frac{z}{z-1}$$

$$x[n-1] \quad \leftrightarrow \quad \frac{1}{z-1}$$

$$x[n-2] \quad \leftrightarrow \quad \frac{1}{z(z-1)}$$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}(z^{-1}Y[z] + 3) - \frac{1}{20}(z^{-2}Y[z] + 3z^{-1} + 2)$$

initial condition terms

Finding ZIR & ZSR Using Laplace Transform (2)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition $y[1]=3,\quad y[-2]=2$

input $x[n] = u[n]$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}(z^{-1}Y[z]+3) - \frac{1}{20}(z^{-2}Y[z]+3z^{-1}+2)$$

$$\frac{Y[z]}{z} = \frac{2z^2-z+3}{(z-1)(z-\frac{1}{3})(z-\frac{1}{4})} + \frac{(\frac{5}{4}z-\frac{3}{20})}{(z-\frac{1}{5})(z-\frac{1}{4})}$$

init cond terms

input terms

$$\frac{Y[z]}{z} = \frac{\frac{20}{3}}{(z-1)} + \frac{72}{(z-\frac{1}{5})} + \frac{\frac{230}{3}}{(z-\frac{1}{4})} - \frac{2}{(z-\frac{1}{5})} + \frac{\frac{13}{4}}{(z-\frac{1}{4})}$$

$$y[n] = \frac{30}{3} + 72\left(\frac{1}{5}\right)^n - \frac{230}{3}\left(\frac{1}{4}\right)^n - 2\left(\frac{1}{5}\right)^n + \frac{13}{4}\left(\frac{1}{4}\right)^n = \frac{20}{3} + 70\left(\frac{1}{5}\right)^n - \frac{881}{12}\left(\frac{1}{4}\right)^n$$

Zero State Resp

Zero Input Resp

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] B.P. Lathi, Linear Systems and Signals (2nd Ed)
[4] D. Sundararajan, A Practical Approach to Signals and Systems