Vectors (1A)

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Determinant

Determinant of order 2

$$\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \qquad \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Determinant of order 3

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\begin{bmatrix} a_1 & & \\ & b_2 & b_3 \\ & c_2 & c_3 \end{bmatrix} \qquad \begin{bmatrix} & a_2 & \\ b_1 & & b_3 \\ c_1 & & c_3 \end{bmatrix} \qquad \begin{bmatrix} & & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & & \end{bmatrix}$$

Determinant of order 3

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\begin{bmatrix} a_1 & & \\ & b_2 & b_3 \\ & c_2 & c_3 \end{bmatrix} \qquad \begin{bmatrix} & a_2 & \\ b_1 & & b_3 \\ c_1 & & c_3 \end{bmatrix} \qquad \begin{bmatrix} & & a_3 \\ b_1 & b_2 & \\ c_1 & c_2 & \end{bmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = + a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

Cross Product (1)

Determinant of order 3

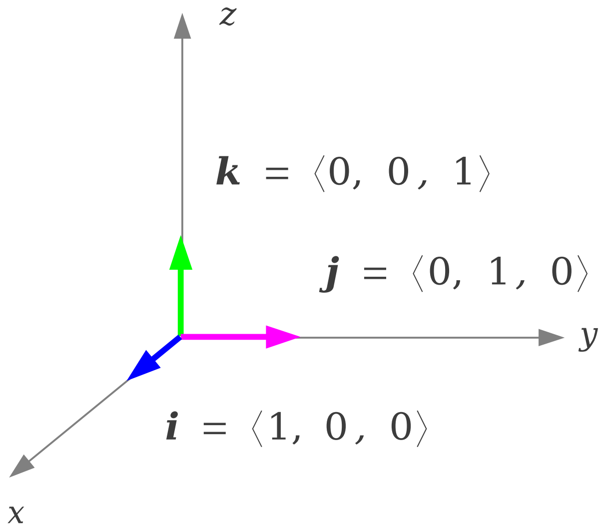
$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} \quad = \langle a_1,\ a_2,\ a_3 \rangle$$

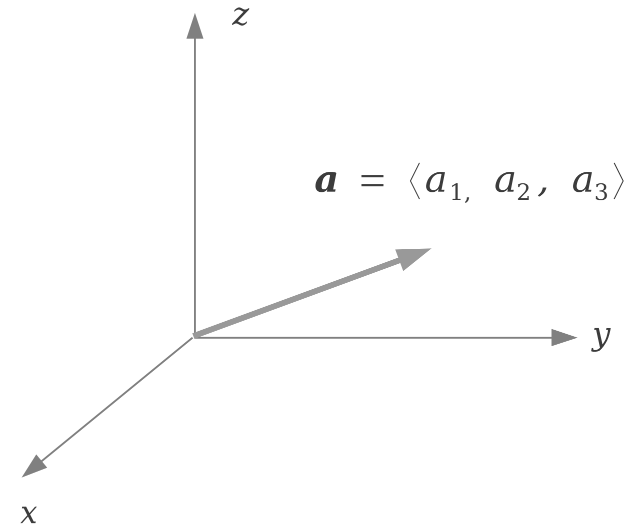
$$\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k} \quad = \langle b_1,\ b_2,\ b_3 \rangle$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \quad = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= (a_2 b_3 - a_3 b_2)\mathbf{i} - (a_1 b_3 - a_3 b_1)\mathbf{i} + (a_1 b_2 - a_2 b_1)\mathbf{i}$$

Cross Product (2)





$$\boldsymbol{i} \times \boldsymbol{j} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \boldsymbol{k}$$

normal to $\boldsymbol{i}$ & $\boldsymbol{j}$

$$\boldsymbol{j} \times \boldsymbol{i} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -\boldsymbol{k}$$

$$\boldsymbol{j} \times \boldsymbol{k} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \boldsymbol{i}$$

normal to $\boldsymbol{j}$ & $\boldsymbol{k}$

$$\boldsymbol{k} \times \boldsymbol{j} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\boldsymbol{i}$$

$$\boldsymbol{k} \times \boldsymbol{i} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \boldsymbol{j}$$

normal to $\boldsymbol{k}$ & $\boldsymbol{i}$

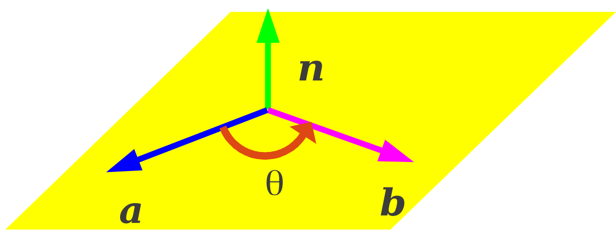
$$\boldsymbol{i} \times \boldsymbol{k} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\boldsymbol{j}$$

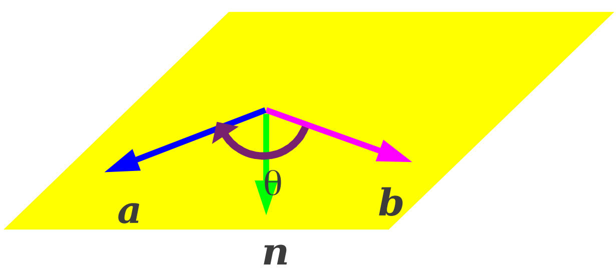
Right Hand Rule

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

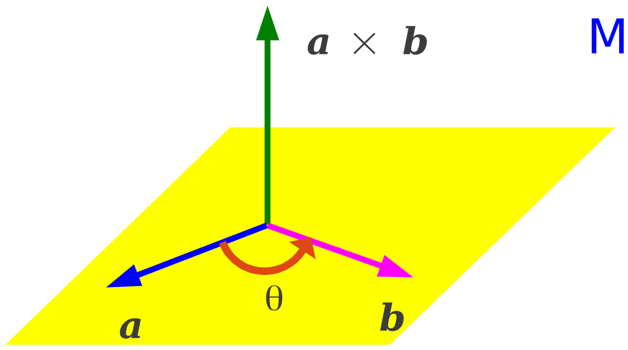
$$\mathbf{b} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

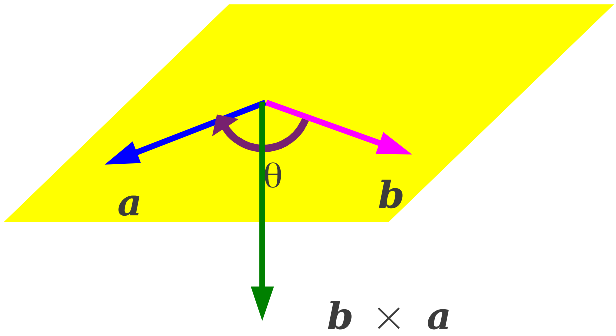
Normal direction $\mathbf{n}$





Magnitude = $\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \, \|\mathbf{b}\| \, \sin\theta$





Line Equations (1)

Vector Equation

$$\mathbf{r} = \mathbf{r}_2 + t\,\mathbf{a}$$

Parameter

Direction Vector

$$\mathbf{r}_2 = \langle x_2, y_2, z_2 \rangle$$
$$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$$

Parametric Equation

$$x = x_2 + t\,a_1$$
$$y = y_2 + t\,a_2$$
$$z = z_2 + t\,a_3$$

Component

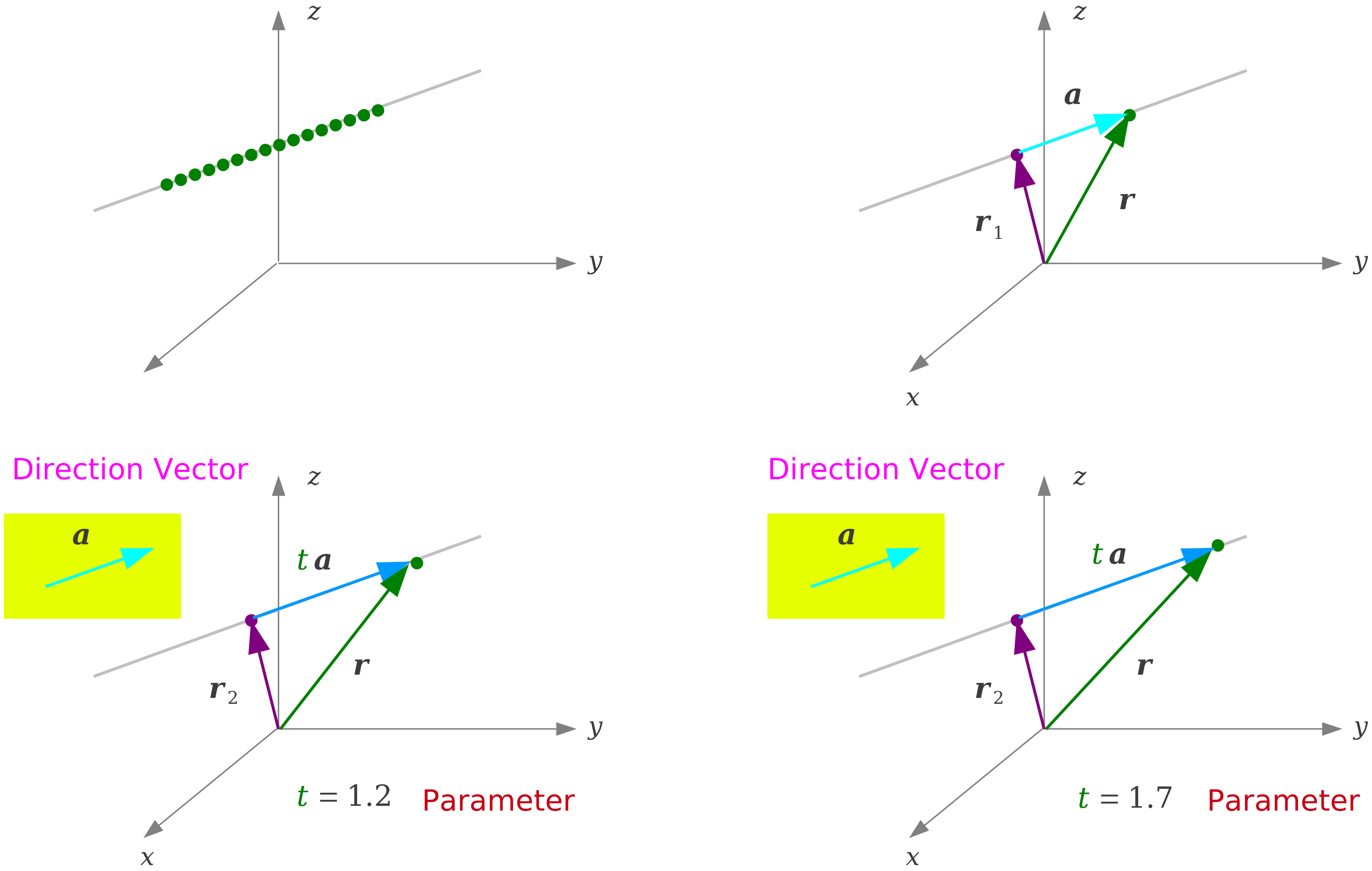
$$t\,a_1 = x - x_2$$
$$t\,a_2 = y - y_2$$
$$t\,a_3 = z - z_2$$

Symmetric Equation

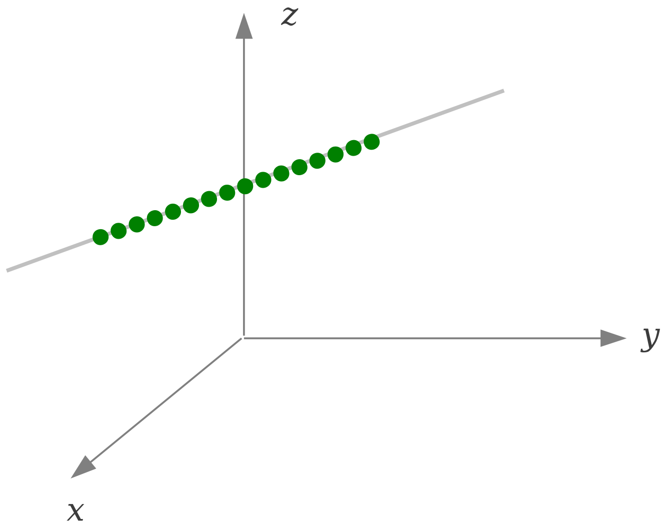
$$t = \frac{x - x_2}{a_1} = \frac{y - y_2}{a_2} = \frac{z - z_2}{a_3}$$

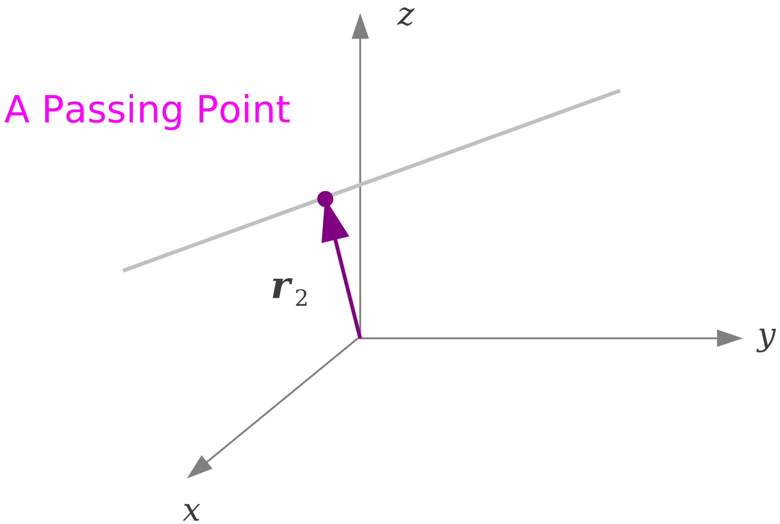
Elimination of parameter

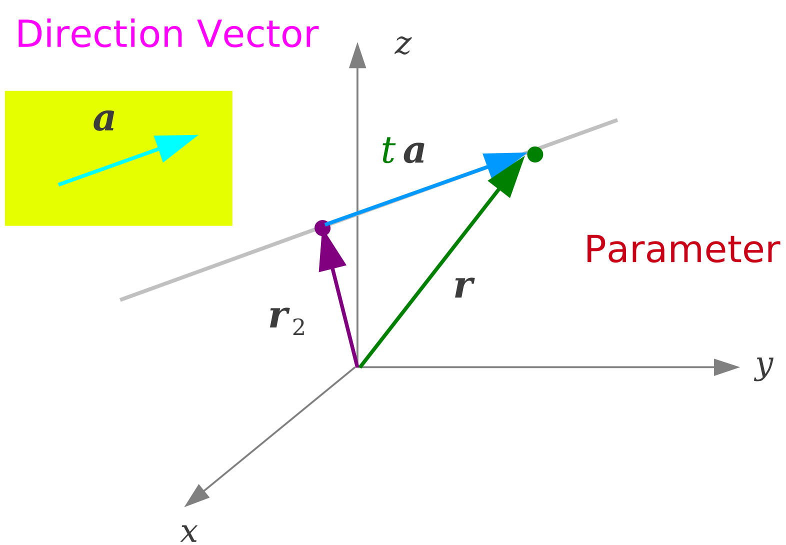
Line Equations (2)



Line Equations (3)







$$\boldsymbol{r} = \boldsymbol{r}_2 + t\,\boldsymbol{a}$$

$$\boldsymbol{r}_2 = \langle x_2,\ y_2,\ z_2 \rangle$$

$$\boldsymbol{a} = \langle a_1,\ a_2,\ a_3 \rangle$$

Plane Equations (1)

Vector equation

$$\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r}_1) = 0$$

Normal Vector

$$\boldsymbol{r} = \langle x, y, z \rangle$$

$$\boldsymbol{r}_1 = \langle x_1, y_1, z_1 \rangle$$

$$\boldsymbol{n} = \langle a, b, c \rangle$$

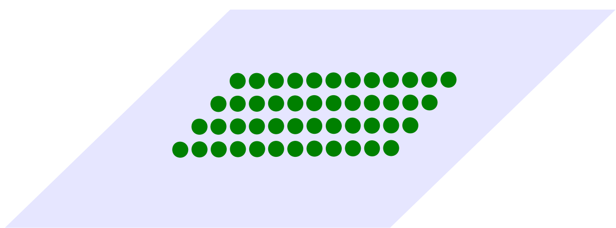
$$\boldsymbol{r} - \boldsymbol{r}_1 = \langle x - x_1, \; y - y_1, \; z - z_1 \rangle$$

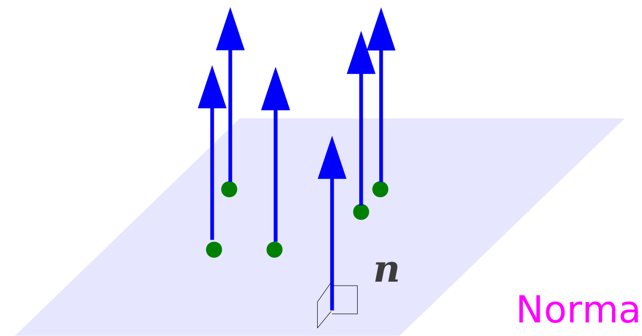
$$\boldsymbol{n} = \langle \quad a, \quad b, \quad c \rangle$$

Cartesian equation

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

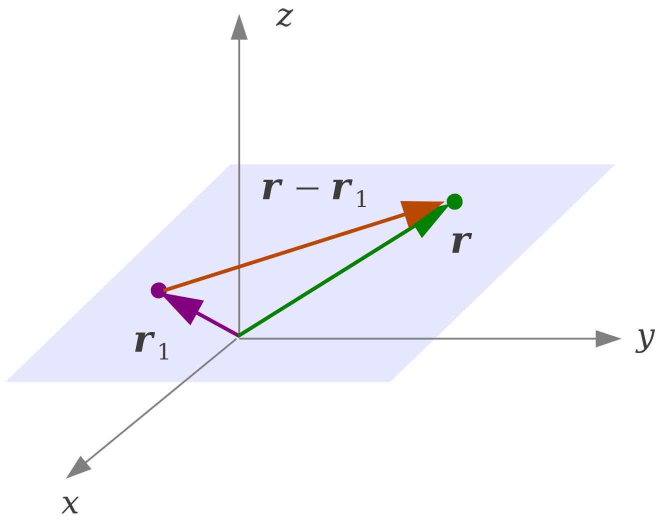
Plane Equations (2)

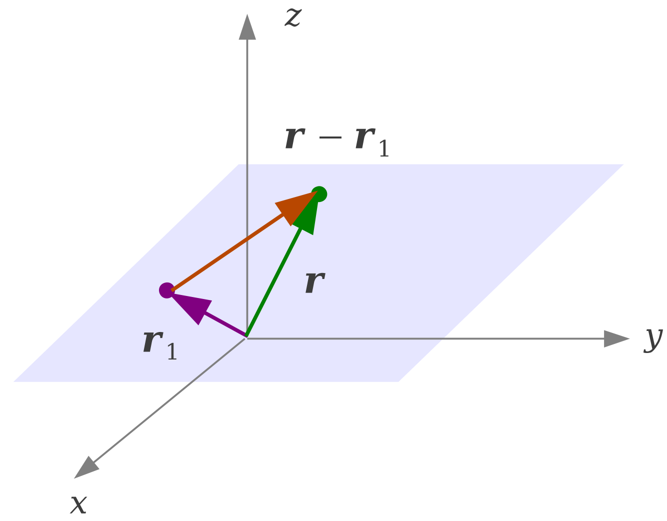




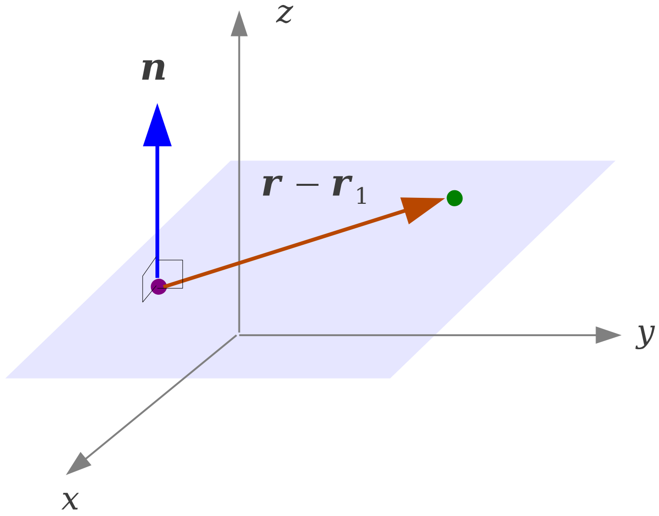
Normal Vector

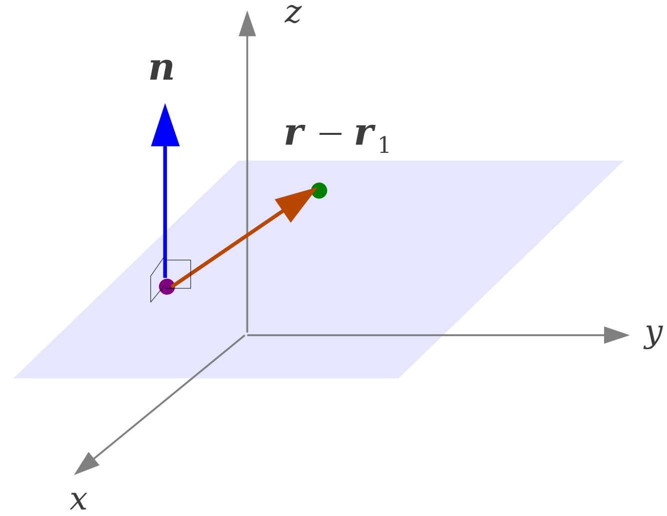
No Parameter

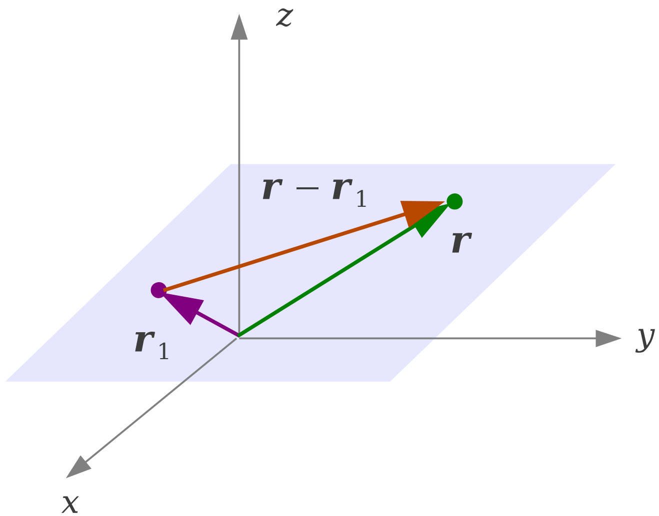




Plane Equations (3)







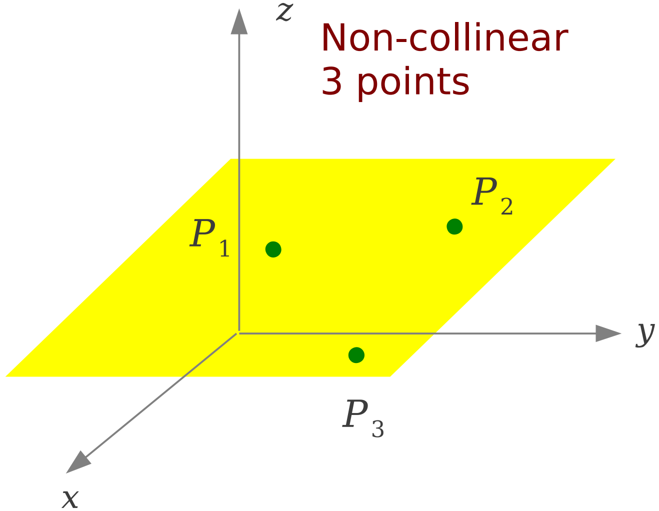
$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_1) = 0$$

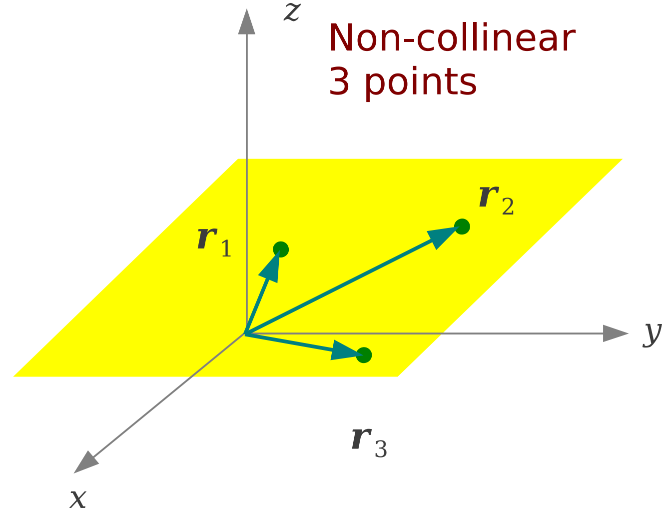
$$\mathbf{r} = \langle x, y, z \rangle$$

$$\mathbf{r}_1 = \langle x_1, y_1, z_1 \rangle$$

$$\mathbf{n} = \langle a, b, c \rangle$$

Normal Vector & 3 Points



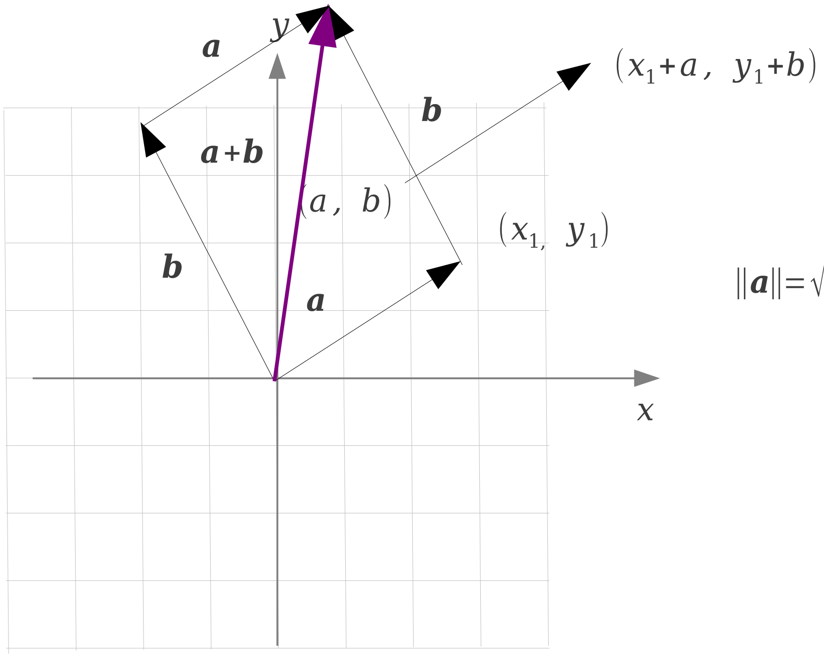


Graph of a plane

Line intersection of two planes

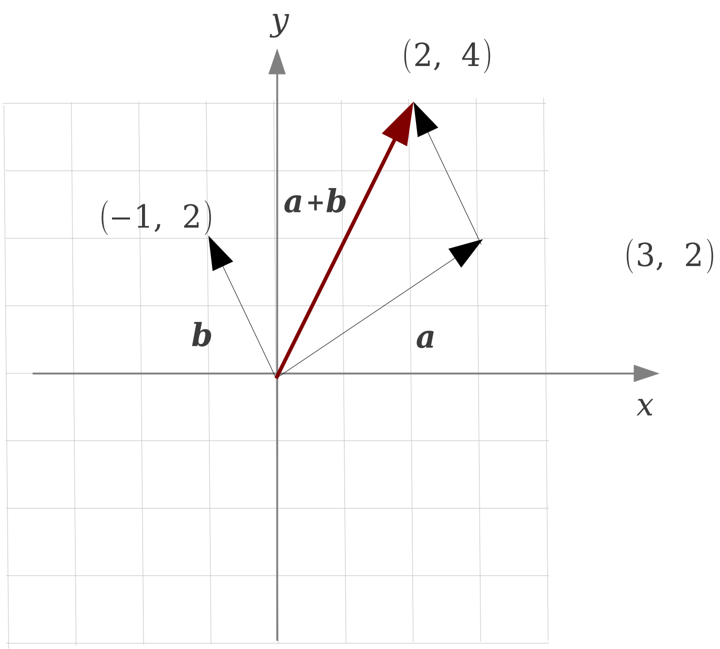
Point of intersection of a line and plane

Normal Vector & 3 Points

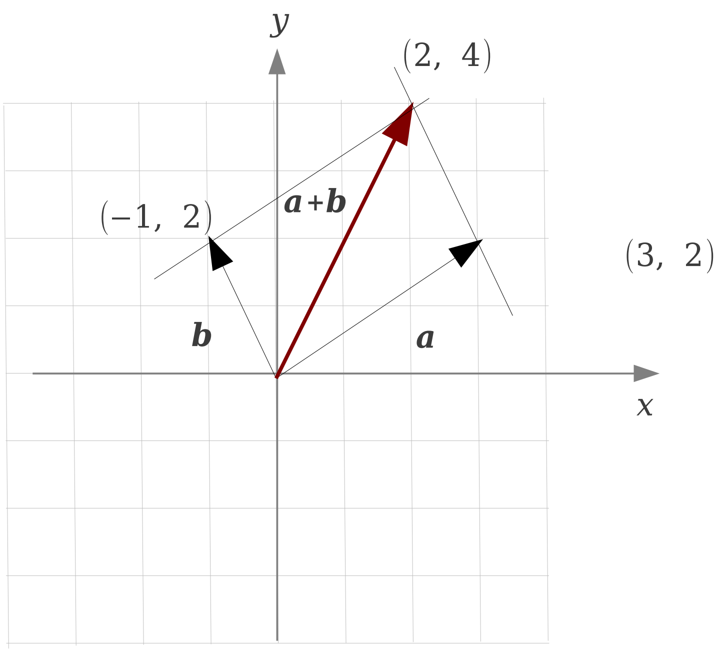


$$\|\boldsymbol{a}\| = \sqrt{x_1^2 + y_1^2}$$

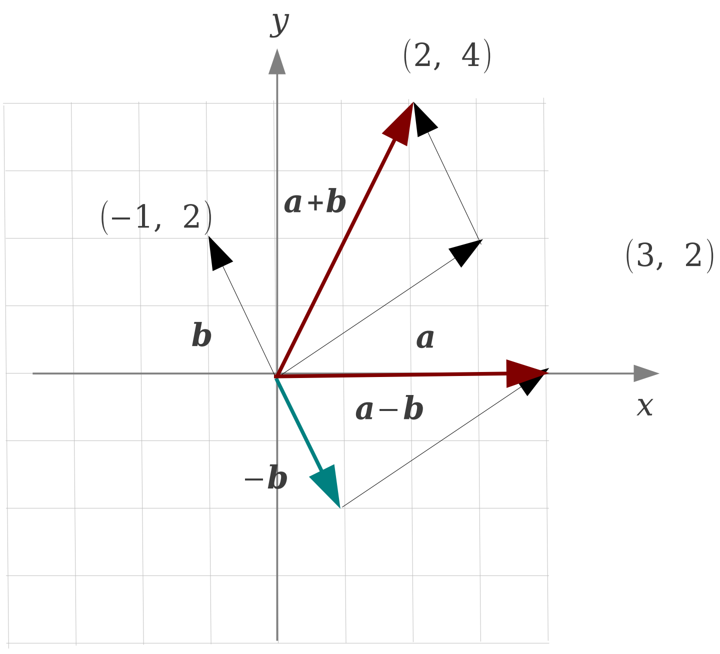
Normal Vector & 3 Points



Normal Vector & 3 Points

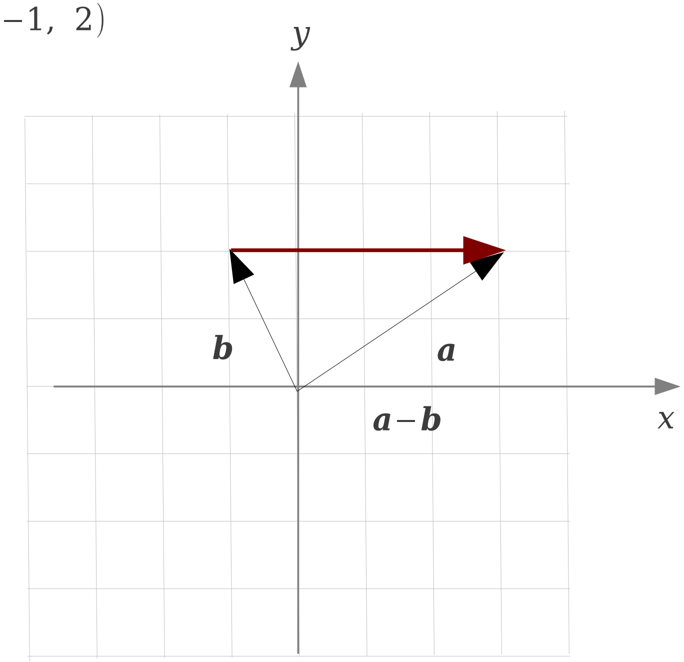


Normal Vector & 3 Points



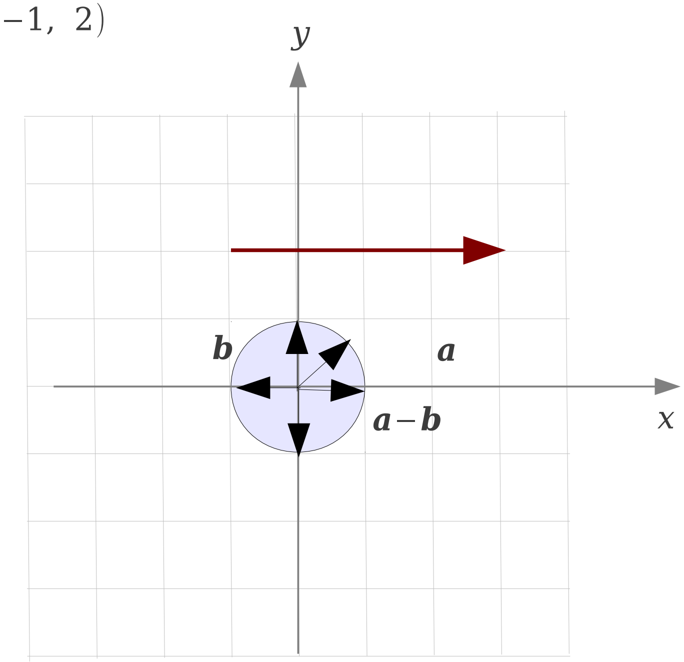
Normal Vector & 3 Points

$(-1,\ 2)$



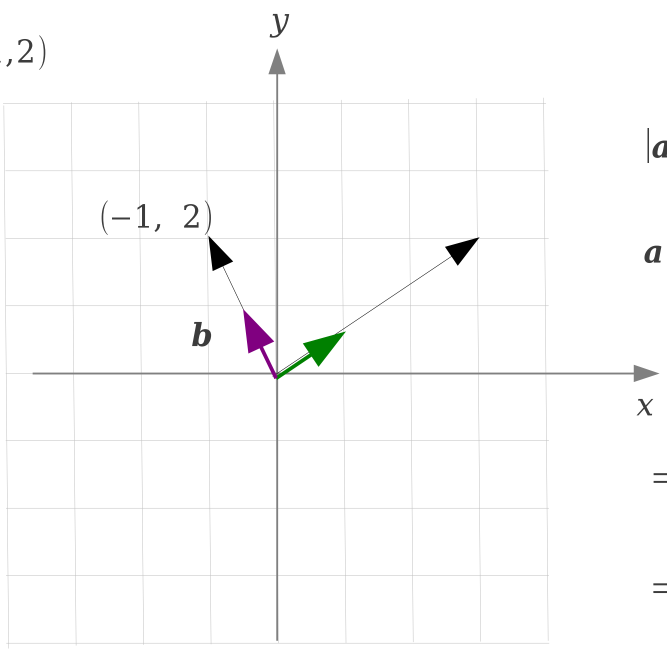
Normal Vector & 3 Points

$(-1,\ 2)$



Normal Vector & 3 Points

$\boldsymbol{b} = (-1, 2)$



$|\boldsymbol{a}| = \sqrt{3^2 + 2^2} = \sqrt{13}$

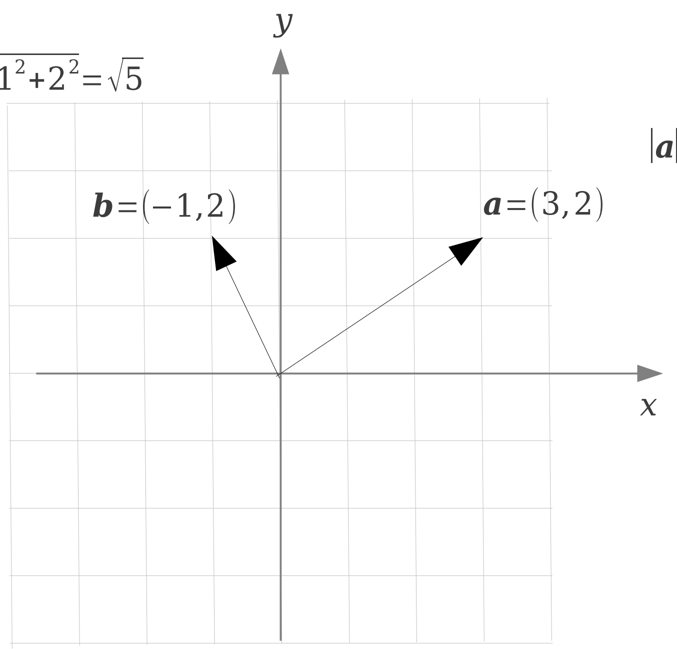
$\boldsymbol{a} = (3, 2)$

$= (\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}})$

$= \sqrt{\frac{3^2}{13} + \frac{2^2}{13}}$

Normal Vector & 3 Points

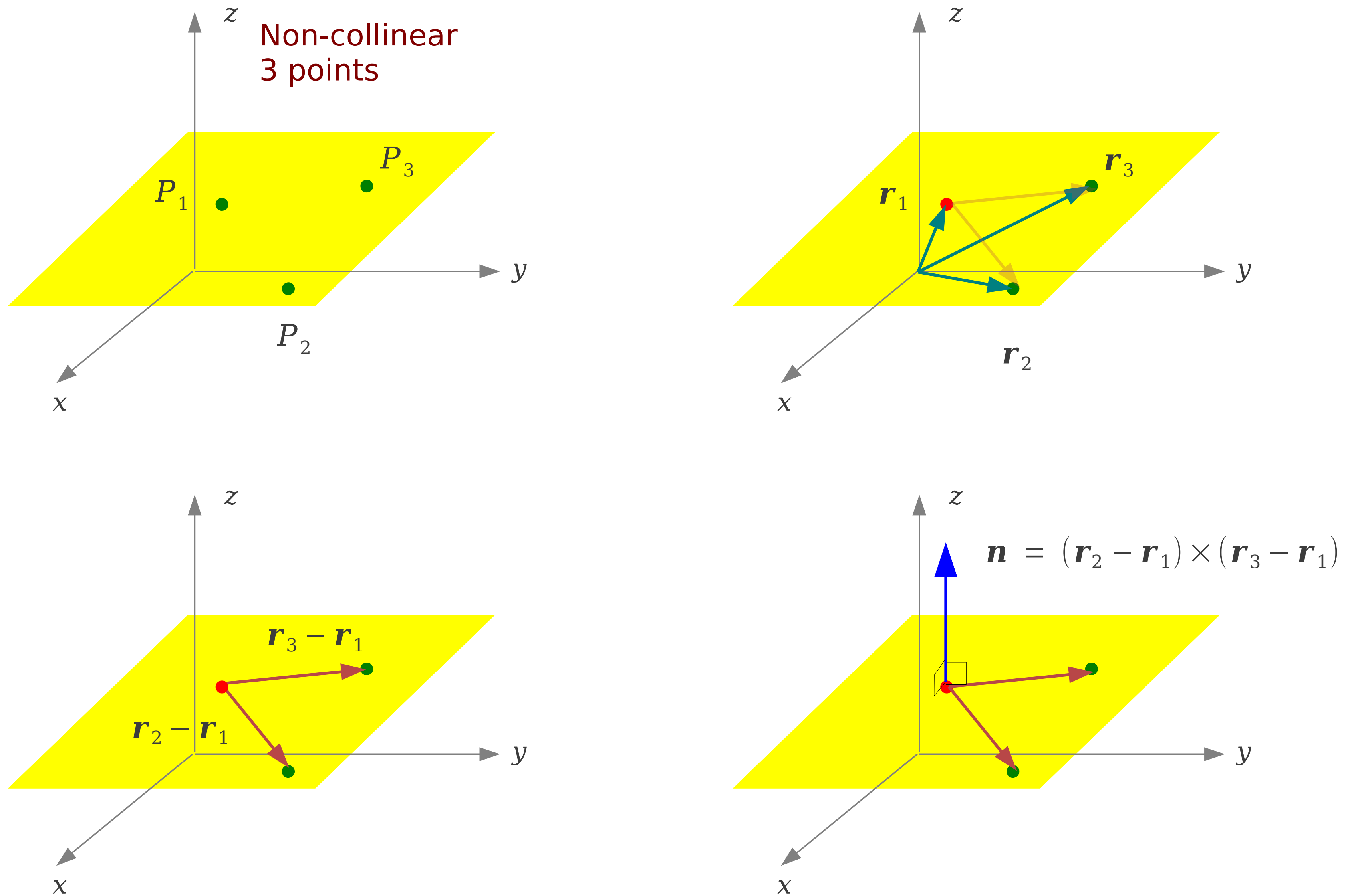
$|\boldsymbol{b}| = \sqrt{-1^2 + 2^2} = \sqrt{5}$



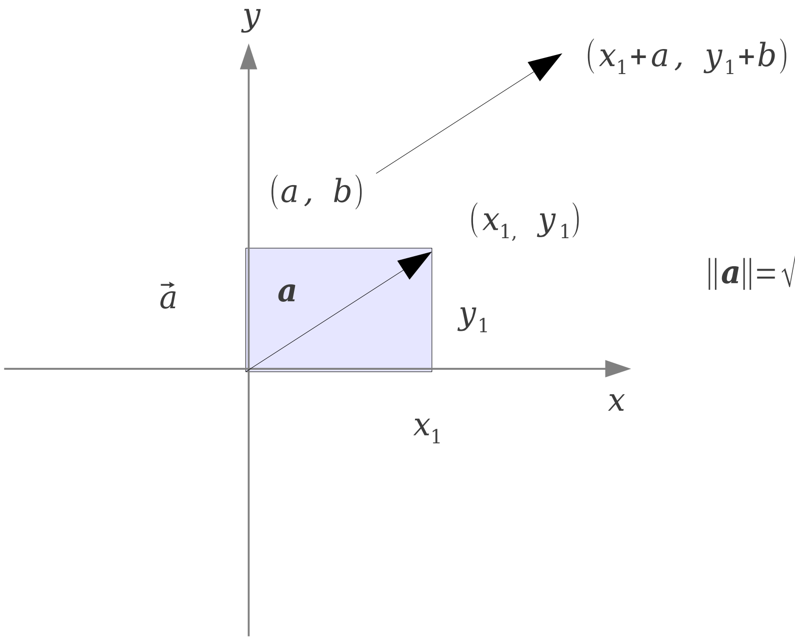
$|\boldsymbol{a}| = \sqrt{3^2 + 2^2} = \sqrt{13}$

$\boldsymbol{a} \cdot \boldsymbol{b} = 3 \cdot -1 + 2 \cdot 2 = -3 + 4 = 1$

Normal Vector & 3 Points

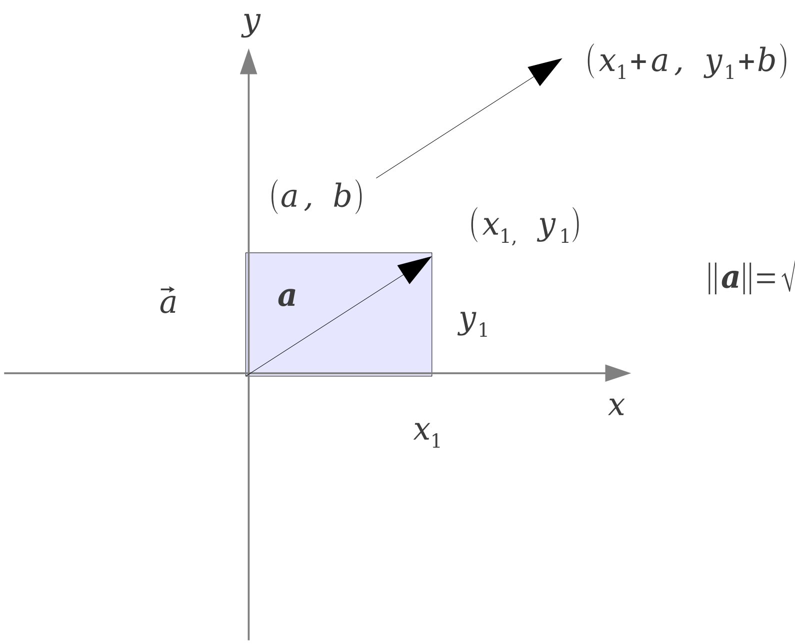


Normal Vector & 3 Points



$$\|\boldsymbol{a}\| = \sqrt{x_1^2 + y_1^2}$$

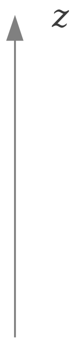
Normal Vector & 3 Points



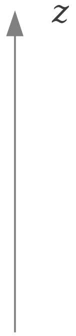
$\|\boldsymbol{a}\| = \sqrt{x_1^2 + y_1^2}$

Normal Vector & 3 Points

Non-collinear
3 points

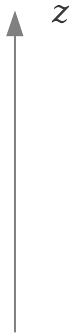


Normal Vector & 3 Points



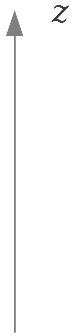
Normal Vector & 3 Points

Non-collinear
3 points



Normal Vector & 3 Points

Non-collinear
3 points



References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[4] D.G. Zill, “Advanced Engineering Mathematics”