Propagating Wave (1B)

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Maxwell Equations

$$\nabla \times \boldsymbol{E} = -\frac{\partial \mu \boldsymbol{H}}{\partial t} \qquad \nabla \cdot (\epsilon \boldsymbol{E}) = 0$$

$$\nabla \times \boldsymbol{H} = +\frac{\partial \epsilon \boldsymbol{E}}{\partial t} \qquad \nabla \cdot (\mu \boldsymbol{H}) = 0$$

$$\nabla = \frac{\partial}{\partial x}\boldsymbol{i}_x + \frac{\partial}{\partial y}\boldsymbol{i}_y + \frac{\partial}{\partial z}\boldsymbol{i}_z \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 \boldsymbol{E} = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} \xrightarrow{s(\boldsymbol{x}, t)} \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 s}{\partial t^2}$$

Wave Equation in Cartesian Coordinates

$$s(x, y, z, t) = A e^{j(\omega t - k_x x - k_y y - k_z z)}$$

$$= f(x) g(x) h(x) p(t) \quad \text{separable}$$

$$k_x^2 s(x, y, z, t) + k_y^2 s(x, y, z, t) + k_z^2 s(x, y, z, t) = \frac{\omega^2 s(x, y, z, t)}{c^2}$$

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

Monochrome Plane Wave (1)

$$s(x,y,z,t) = A e^{j(\omega t - k_x x - k_y y - k_z z)}$$ Fixed point

$$s(0,0,0,t) = A e^{j\omega t} = A\cos\omega t + A\sin\omega t$$ Monochrome

$$s(x,y,z,t) = A e^{j(\omega t - k_x x - k_y y - k_z z)}$$ Fixed time

$$s(x,y,z,t_0) = A e^{j(\omega t_0 - [k_x x + k_y y + k_z z])}$$

points (x,y,z) such that $k_x x + k_y y + k_z z = C$ Plane

$$s(x,y,z,t_0) = A e^{j(\omega t_0 - k_x x - k_y y - k_z z)}$$ has the same value

Monochrome Plane Wave (2)

$$s(x, y, z, t) = A e^{j(\omega t - k_x x - k_y y - k_z z)}$$

$$s(\boldsymbol{x}, t) = A e^{j(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}$$

planes of constant phase ➡ $\boldsymbol{k}\cdot\boldsymbol{x} = const$

$$s(\boldsymbol{x}+\delta\boldsymbol{x}, t+\delta t) = s(\boldsymbol{x}, t)$$

$$\omega(t+\delta t) - \boldsymbol{k}\cdot(\boldsymbol{x}+\delta\boldsymbol{x}) = \omega t - \boldsymbol{k}\cdot\boldsymbol{x}$$

$$\omega\delta t - \boldsymbol{k}\cdot\delta\boldsymbol{x} = 0$$

If truly a propagating wave

planes of constant phase move by $\delta\boldsymbol{x}$
as time advances by δt

Monochrome Plane Wave (3)

planes of constant phase → $\mathbf{k}\cdot\mathbf{x} = const$

perpendicular to $\mathbf{k}$

If truly a propagating wave — planes of constant phase move by $\delta\mathbf{x}$ as time advances by δt

$$s(\mathbf{x}+\delta\mathbf{x}, t+\delta t) = s(\mathbf{x}, t) \quad \rightarrow \quad \omega\,\delta t - \mathbf{k}\cdot\delta\mathbf{x} = 0$$

$\delta\mathbf{x}$ in the same direction $\mathbf{k}$ — minimum $|\delta\mathbf{x}|$

The direction of propagation $\quad \boldsymbol{\zeta}_0 = \dfrac{\mathbf{k}}{|\mathbf{k}|}$

in the same direction $\quad \mathbf{k}\cdot\delta\mathbf{x} = |\mathbf{k}||\delta\mathbf{x}|$

Monochrome Plane Wave (4)

planes of constant phase $\rightarrow$ $\boldsymbol{k} \cdot \boldsymbol{x} = const$

perpendicular to $\boldsymbol{k}$

$s(\boldsymbol{x}+\delta \boldsymbol{x}, t+\delta t) = s(\boldsymbol{x}, t)$ $\rightarrow$ $\omega \delta t - \boldsymbol{k} \cdot \delta \boldsymbol{x} = 0$

$\delta \boldsymbol{x}$ in the same direction $\boldsymbol{k}$ minimum $|\delta \boldsymbol{x}|$

The direction of propagation $\zeta_0 = \dfrac{\boldsymbol{k}}{|\boldsymbol{k}|}$

in the same direction $\boldsymbol{k} \cdot \delta \boldsymbol{x} = |\boldsymbol{k}||\delta \boldsymbol{x}|$

$$\omega \delta t = |\boldsymbol{k}||\delta \boldsymbol{x}|$$

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

$$\boldsymbol{k}^2 = \frac{\omega^2}{c^2}$$

$$\frac{\omega}{|\boldsymbol{k}|} = \boxed{\frac{|\delta \boldsymbol{x}|}{\delta t}}$$

$$c = \frac{|\delta \boldsymbol{x}|}{\delta t}$$

The speed of propagation of the plane wave

Maxwell Equations

$$A(t,\ t) = A_0 \cos(k x - \omega t)$$

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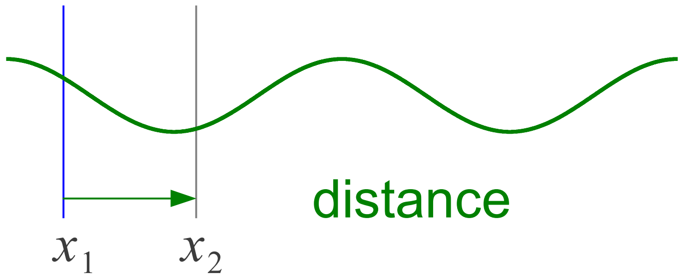
Maxwell Equations

$$A(t,\ t) = A_0 \cos(k\,x - \omega\,t)$$

Wavelength, Frequency

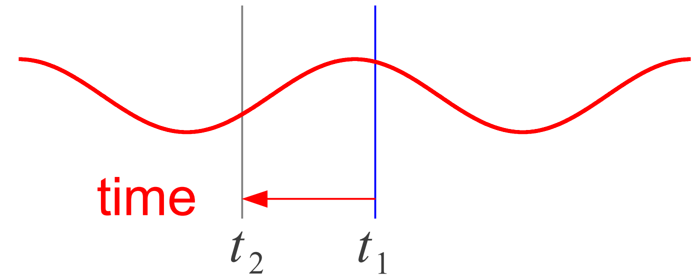
$A_0 \cos(k\,x - \omega\,t_0)$

At the snapshot of the time t_0



$A_0 \cos(k\,x_0 - \omega\,t)$

At the fixed site of the distance x_0



wavelength $\boxed{\lambda = \dfrac{2\pi}{k}}$

wave number $\boxed{k = \dfrac{2\pi}{\lambda}}$

frequency $f = \dfrac{\omega}{2\pi}$

period $\boxed{T = \dfrac{2\pi}{\omega}}$

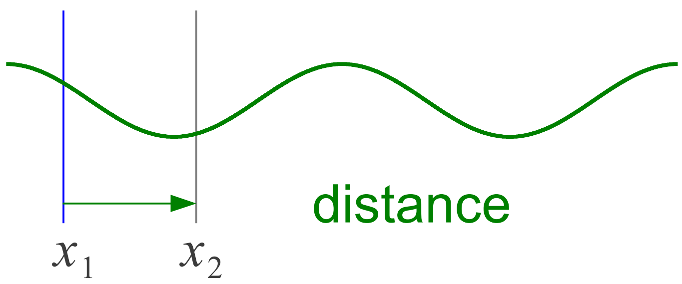
angular frequency $\omega = 2\pi f$

angular frequency $\boxed{\omega = \dfrac{2\pi}{T}}$

Wave Number, Angular Frequency

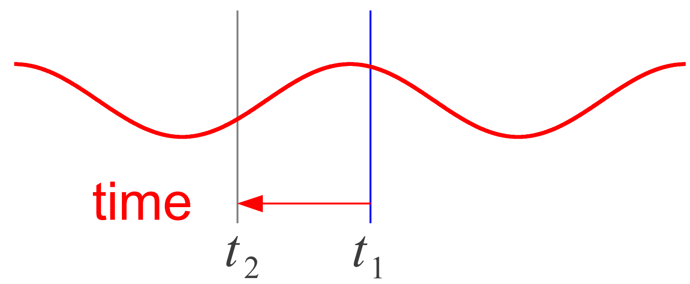
$$A_0 \cos(k\,x - \omega\, t_0)$$

At the snapshot of the time t_0



$$A_0 \cos(k\, x_0 - \omega\, t)$$

At the fixed site of the distance x_0



wave number $\quad k = \dfrac{2\pi}{\lambda}$

radians per unit distance

angular frequency $\quad \omega = \dfrac{2\pi}{T}$

radians per unit time

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] http://www.mathpages.com/, Phase, Group, and Signal Velocity
[4] R. Barlow, www.hep.man.ac.uk/u/roger/PHYS10302/lecture15.pdf
[5] P. Hofmann, www.philiphofmann.net/book_material/notes/groupphasevelocity.pdf
[6] D.H. Johnson, et. al., Arrary Signal Processing: concepts and techniques