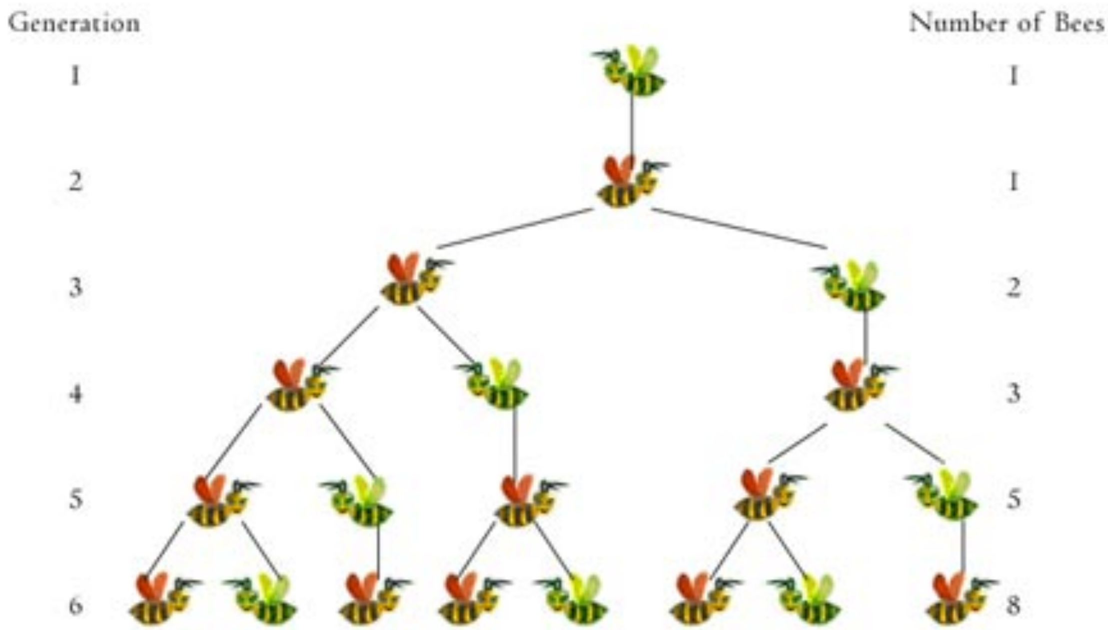
<u>Word Problem</u>: The Ancestry of Bees

When female bee lays an egg, if it is not fertilized by a male bee then the egg will hatch into a male bee. If it is fertilized then it will hatch into a female bee. So female bees have both a father and a mother. How would a genealogy tree of bees look?



When looking at the bee family tree, we see that every male can only father a female bee, but every female bee can mother either a male or a female. The pattern continues the same, generation after generation.

As more generations of bees are added onto the tree, the generational sizes increase exponentially, not linearly.

> ***Exponential growth*** *– rate or growth is based off of size of prior growth*
> ***Linear growth*** *– rate or growth are consistent and increase in same-sized increments*

Each progressive generation size depends on that of the generation before; however, this pattern is neither unique nor unpredictable. The name for this pattern is **the Fibonacci sequence**. While it is not exclusive to this word problem, the Fibonacci sequence does not appear in all tree diagrams; it is specific to 1-1, 1-2 geometric sequences. As is expressed along the right-hand side of the bee diagram above, the Fibonacci sequence shows the total number of bees in each generation and begins 1,1, 2, 3, 5, 8… and continues on infinitely. The sequence also can show the number of a specific gender in each generation. For instance, if we look at females beginning in the 2nd generation, the number of females progressively goes 1, 1, 2, 3, 5, and we could elaborate the tree to see the Fibonacci sequence would hold true to all following generations of female bees. Likely, the same pattern applies for the males beginning in the 3rd generation.

Where else can we see this sequence in mathematical context?

Remember the golden ratio, Φ?

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ddots}}}}$$

What is the fractional form of this by considering the first two terms of the continued fraction?

1+ (1/1) = 2/1

Considering the first three terms of the continued fraction?

1 + {1/[1+(1/1)]} = 3/2

Considering the first four terms of the continued fraction?

5/3

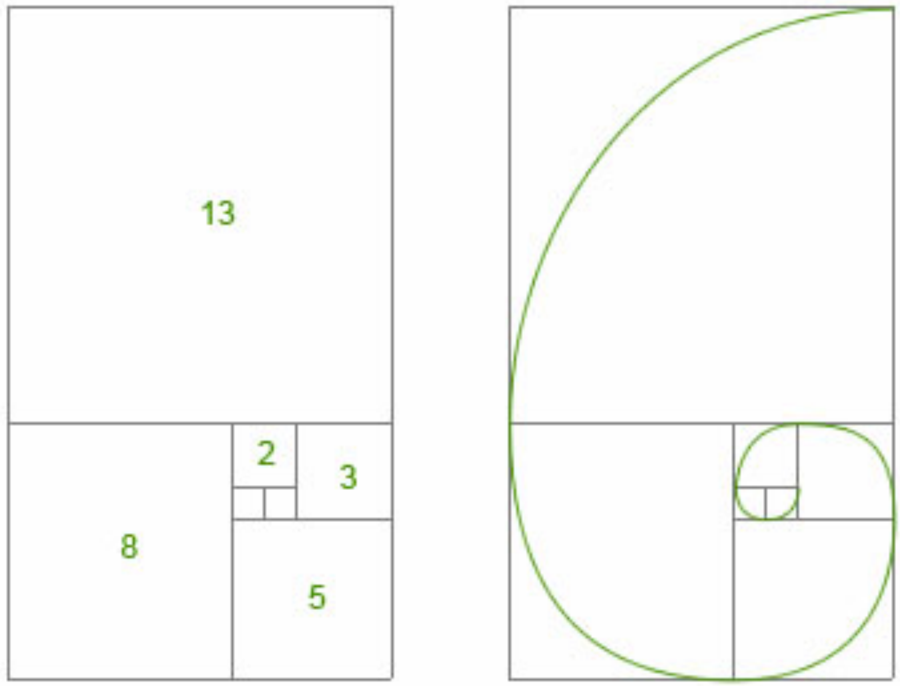
Considering the first five terms of the continued fraction?

8/5

Have we seen these numbers anywhere before? Perhaps in our **Fibonacci sequence**? Wow, how coincidental.

The Fibonacci sequence follows the golden ratio's formula of (n+1)/n, derived from our continued fraction form.

In a visual representation of the golden ratio, the same holds true.


13
2
3
8
5