

Binary Angle Measurement (5A)

- Adaptive CORDIC
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BAM Background

T.K. Rodrigues, "Adaptive CORDIC: Using Parallel Angle Recording to Accelerate Rotations", IEEE Trans on Computers, 2010

Rotation of 25 degree

Original CORDIC

$$\begin{aligned} 25^\circ &\approx +45^\circ \\ &\quad -26.565^\circ \\ &\quad +14.036^\circ \\ &\quad -7.125^\circ \\ &\quad -3.576^\circ \\ &\quad +1.79^\circ \\ &\quad +0.895^\circ \\ &\quad +0.448^\circ \\ &\quad +0.2238^\circ \\ \hline &= 25.1268^\circ \end{aligned}$$

Angle Constants that is used

$$Q = \{45^\circ, 26.565^\circ, 14.036^\circ, 7.125^\circ, 3.576^\circ, 1.79^\circ, 0.895^\circ, 0.448^\circ, 0.2238^\circ\}$$

Range of Residual Angles around Angle Constant

45°	35.78	$\frac{(45 + 26.565)}{2}$	$[Z_{45^\circ}] = [35.78, 67.5]$
26.565°	20.295		$[Z_{26.565^\circ}] = [20.295, 35.78]$
14.036°	10.5775		$[Z_{14.036^\circ}] = [10.5775, 20.295]$
7.125°	5.5305		$[Z_{7.125^\circ}] = [5.3505, 10.5775]$
3.576°	2.6825		$[Z_{3.576^\circ}] = [2.6825, 5.3505]$
1.79°	1.342		$[Z_{1.79^\circ}] = [1.342, 2.6825]$
0.895°	0.6715		$[Z_{0.895^\circ}] = [0.6715, 1.342]$
0.448°	0.3359		$[Z_{0.448^\circ}] = [0.3359, 0.6715]$
0.2238°	0.1119		$[Z_{0.2238^\circ}] = [0.1119, 0.3359]$

Angle Recording Method

$\alpha \leftarrow \alpha_N$

$Z \leftarrow \theta$

while ($|Z| > \alpha_{min}/2$) {

$\sigma = (Z \geq 0) ? +1 : -1;$

foreach α_i ($\alpha_0, \alpha_1, \dots, \alpha_N$) {

if ($||Z| - \alpha_i| < ||Z| - \alpha_{max}|$) {

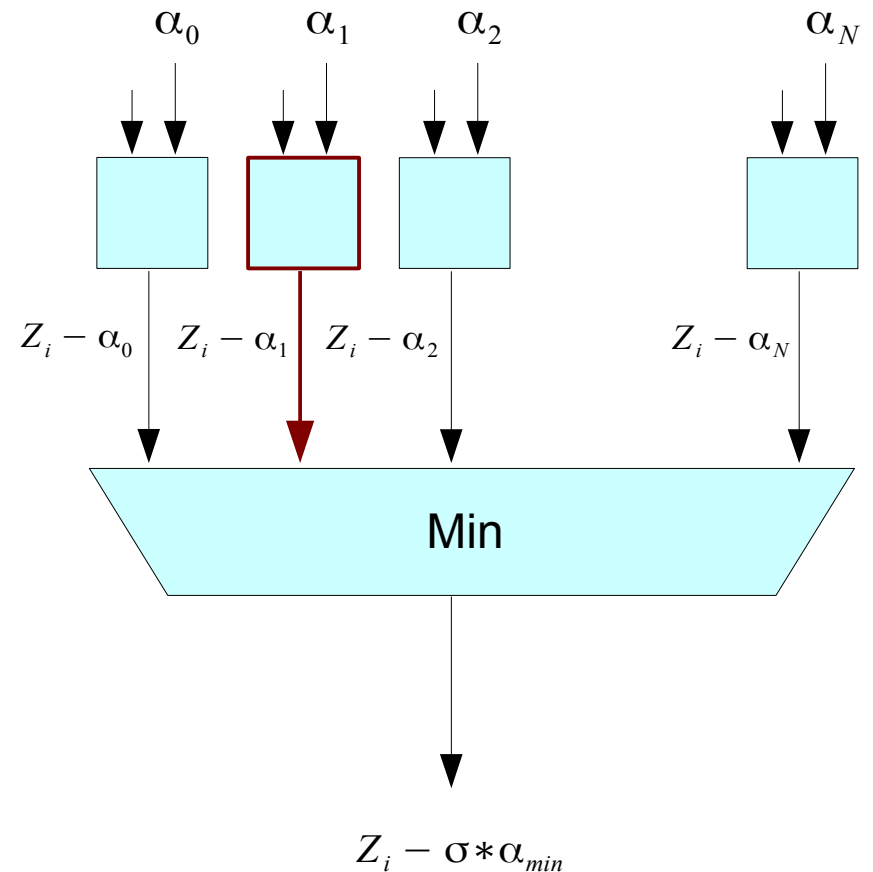
$\alpha_{min} = \alpha_i$

}

Store α_{max} *on adaptive-angle-list*

$Z = Z - \sigma * \alpha_{max}$

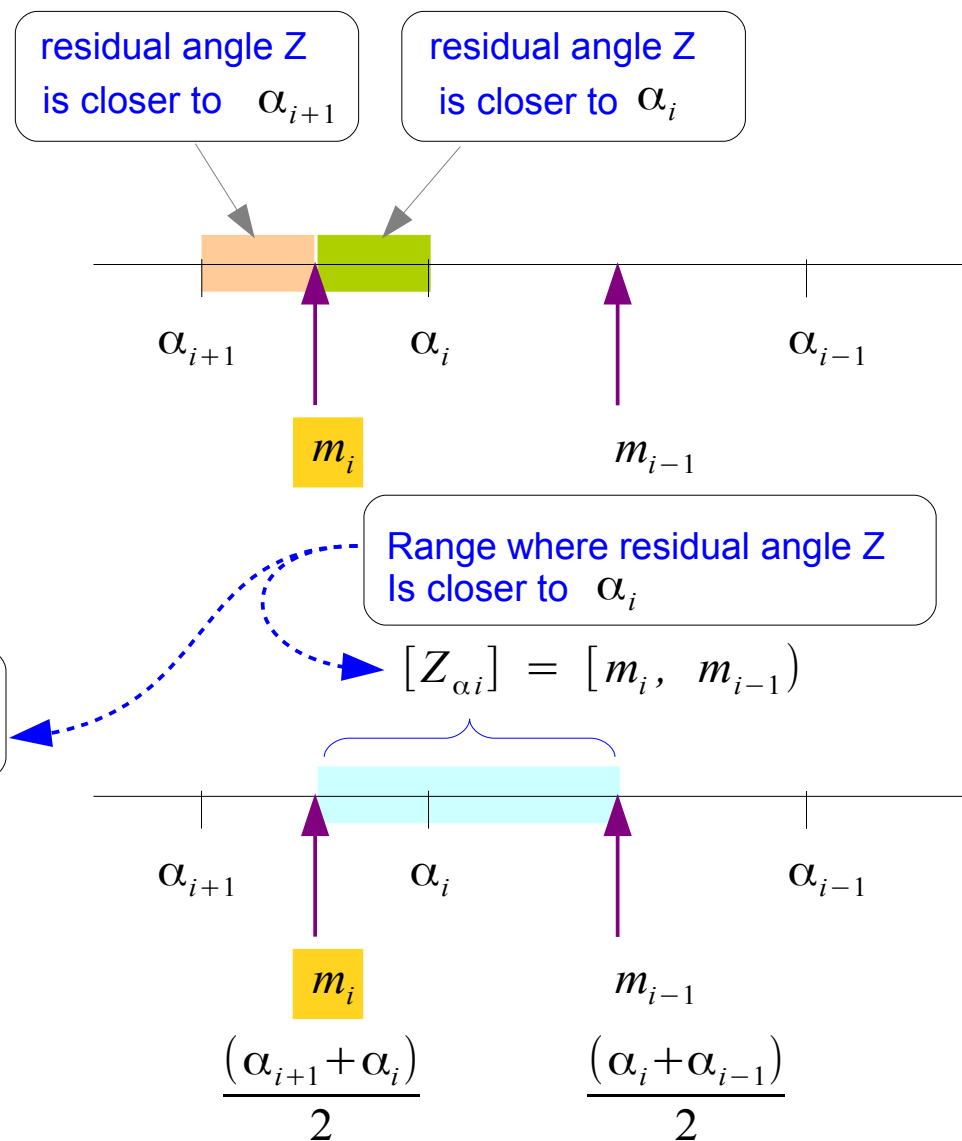
}



Range

$$m_i = \frac{(\alpha_{i+1} + \alpha_i)}{2}$$

$$[Z_{\alpha_i}] = [m_i, m_{i-1})$$



Estimated Range

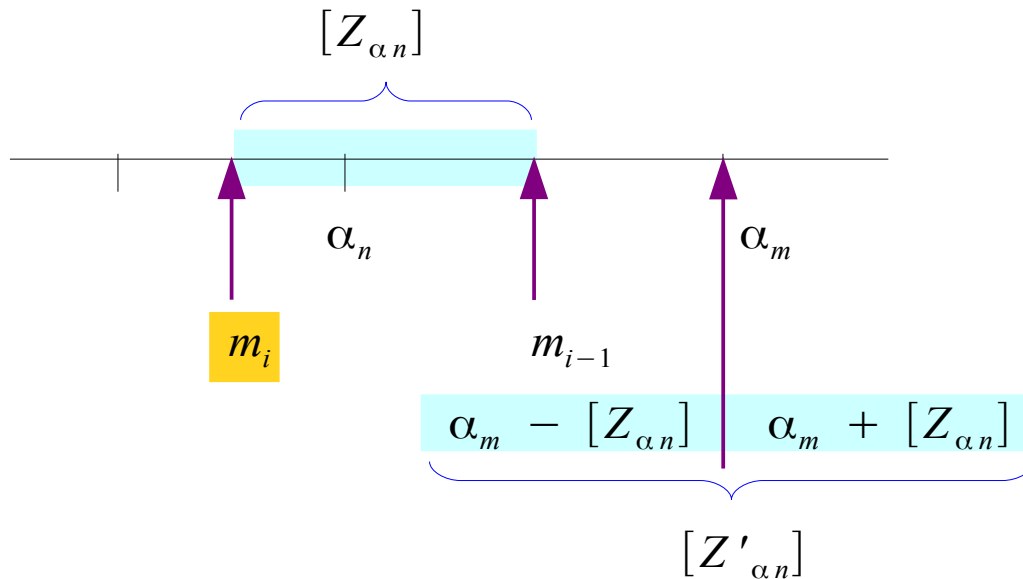
assume

step i \rightarrow α_n \rightarrow $[Z_{\alpha_n}]$

estimate \wedge

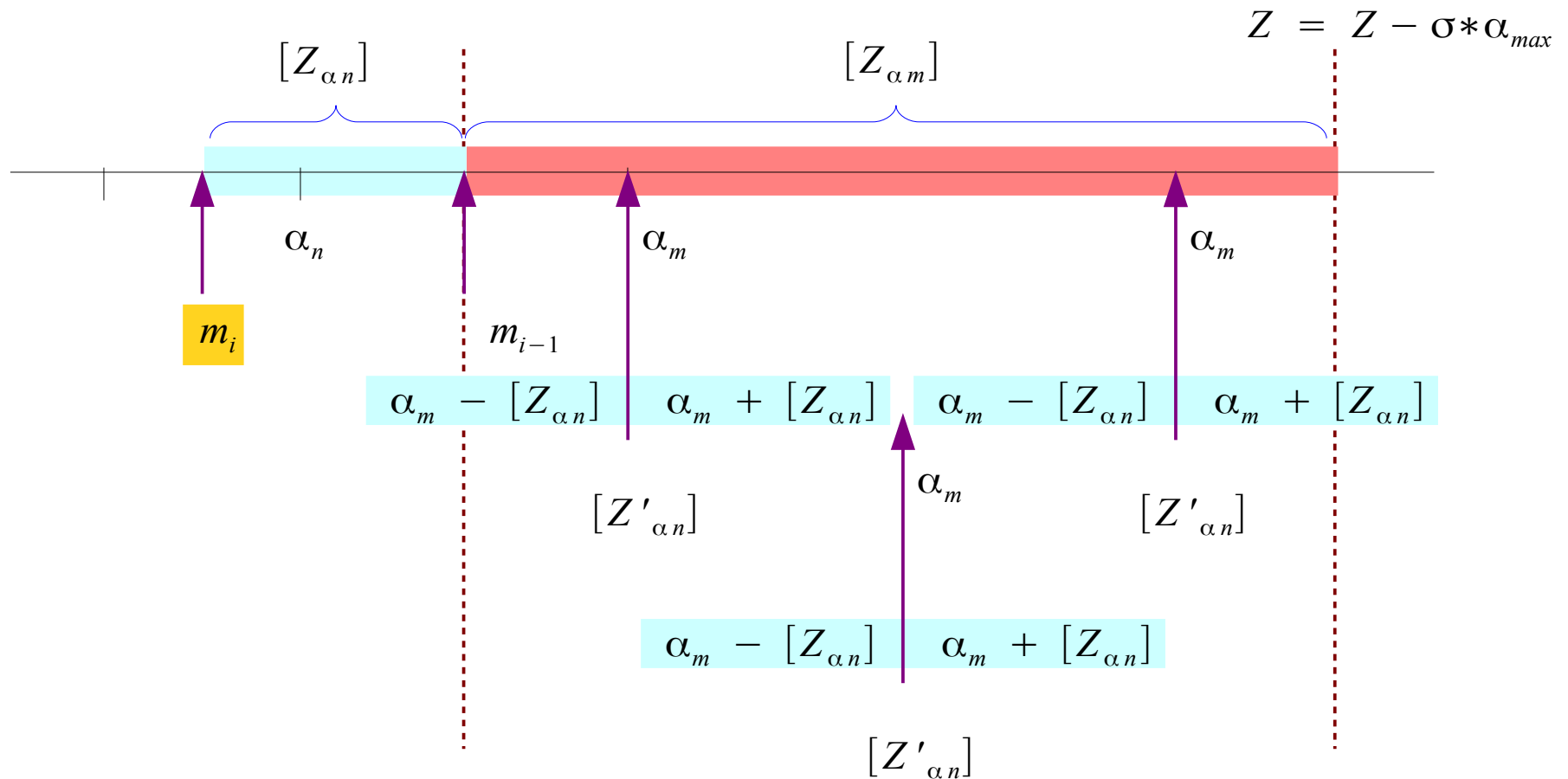
step $i-1$ \rightarrow α_m \rightarrow $[Z'_{\alpha_n}]$

$$\begin{aligned}
 [Z'_{\alpha_n}]_{RHS} &= \alpha_m + [Z_{\alpha_n}] \\
 [Z'_{\alpha_n}]_{LHS} &= \alpha_m - [Z_{\alpha_n}]
 \end{aligned}
 \quad \leftarrow \quad
 \begin{cases}
 [Z'_{\alpha_n}]_{RHS} - \alpha_m \Rightarrow [Z_{\alpha_n}] \\
 [Z'_{\alpha_n}]_{LHS} - \alpha_m \Rightarrow [Z_{\alpha_n}]
 \end{cases}$$



$$Z = Z - \sigma * \alpha_{max}$$

Conditions of Estimated Range



- Case 1) $[Z'_{\alpha_n}]$ is contained entirely within $[Z_{\alpha_m}]$
- Case 2) $[Z'_{\alpha_n}]$ straddles the boundary of $[Z_{\alpha_m}]$
- Case 3) $[Z'_{\alpha_n}]$ lies completely outside $[Z_{\alpha_m}]$

Example

$$\begin{array}{lll} 1.79^\circ & 3.576^\circ & 26.565^\circ \\ [Z_{26.565^\circ}] & = & [20.295, 35.78] \\ [Z_{3.576^\circ}] & = & [2.6825, 5.3505] \\ [Z_{1.79^\circ}] & = & [1.342, 2.6825] \end{array}$$

$$[Z_{1.79^\circ}] = [1.342, 2.6825]$$

$$\begin{aligned} [Z'_{1.79^\circ}]_{LHS} &= 3.576^\circ - [Z_{1.79^\circ}] = [3.576 - 2.6825, 3.576 - 1.342] = \cancel{[0.8935, 2.234]} \\ [Z'_{1.79^\circ}]_{RHS} &= 3.576^\circ + [Z_{1.79^\circ}] = [3.576 + 1.342, 3.576 + 2.6825] = \cancel{[4.918, 6.2585]} \\ &= [4.918, 5.3505] \end{aligned}$$

$$\begin{aligned} [Z'_{3.576^\circ}]_{LHS} &= 26.565^\circ - [Z_{3.576^\circ}] = [26.565 - 5.3505, 26.565 - 4.918] \\ [Z'_{3.576^\circ}]_{RHS} &= 26.565^\circ + [Z_{3.576^\circ}] = [26.565 + 4.918, 26.565 + 5.3505] \end{aligned}$$

Range Comparison

$$\theta \in [21.2145, 21.647] \quad \text{or} \quad \theta \in [31.483, 31.9155]$$

Such rotation angle will use all three angle constants: 1.79° 3.576° 26.565°

A simple comparison operation

Without performing iteration sequentially

Find all combinations of angle constants

References

- [1] <http://en.wikipedia.org/>
- [2] CORDIC FAQ, www.dspguru.com
- [3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
- [4] J. S. Walther, A Unified Algorithm for Elementary Functions
- [5] J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits
- [6] T.K. Rodrigues, "Adaptive CORDIC: Using Parallel Angle Recording to Accelerate Rotations", IEEE Trans on Computers, 2010