Binary Angle Measurement (5A)

- Adaptive CORDIC
-

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

BAM Background

T.K. Rodrigues, “Adaptive CORDIC: Using Parallel Angle Recording to Accelerate Rotations”, IEEE Trans on Computers, 2010

Rotation of 25 degree

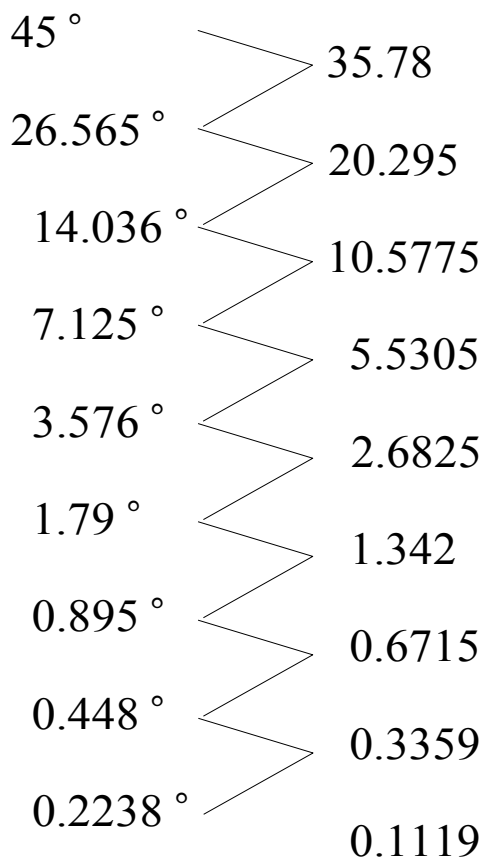
Original CORDIC

$$
\begin{aligned}
25^\circ \approx\ & +45^\circ \\
& -26.565^\circ \\
& +14.036^\circ \\
& -7.125^\circ \\
& -3.576^\circ \\
& +1.79^\circ \\
& +0.895^\circ \\
& +0.448^\circ \\
& +0.2238^\circ \\
\hline
=\ & 25.1268^\circ
\end{aligned}
$$

Angle Constants that is used

$$Q = \{45^\circ, 26.565^\circ, 14.036^\circ, 7.125^\circ, 3.576^\circ, 1.79^\circ, 0.895^\circ, 0.448^\circ, 0.2238^\circ\}$$

Range of Residual Angles around Angle Constant



$$\frac{(45+26.565)}{2}$$

$$[Z_{45^\circ}] = [35.78,\ 67.5]$$

$$[Z_{26.565^\circ}] = [20.295,\ 35.78]$$

$$[Z_{14.036^\circ}] = [10.5775,\ 20.295]$$

$$[Z_{7.125^\circ}] = [5.3505,\ 10.5775]$$

$$[Z_{3.576^\circ}] = [2.6825,\ 5.3505]$$

$$[Z_{1.79^\circ}] = [1.342,\ 2.6825]$$

$$[Z_{0.895^\circ}] = [0.6715,\ 1.342]$$

$$[Z_{0.448^\circ}] = [0.3359,\ 0.6715]$$

$$[Z_{0.2238^\circ}] = [0.1119,\ 0.3359]$$

Angle Recording Method

$\alpha \leftarrow \alpha_N$

$Z \leftarrow \theta$

while $(|Z| > \alpha_{min}/2)$ {

$\sigma = (Z \geq 0)\ ?\ +1 : -1;$

foreach $\alpha_i\ (\alpha_0, \alpha_1, \cdots, \alpha_N)$ {

if $(||Z| - \alpha_i| < ||Z| - \alpha_{max}|)$ {

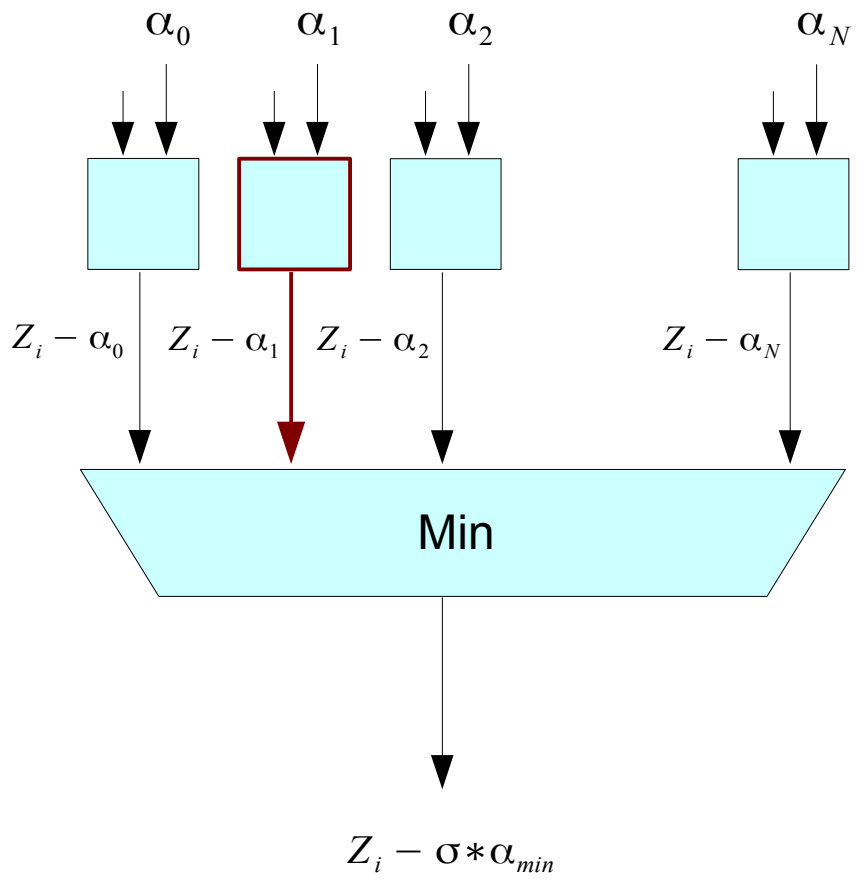
$\alpha_{min} = \alpha_i$

}

Store α_{max} *on* *adaptive−angle−list*

$Z = Z - \sigma * \alpha_{max}$

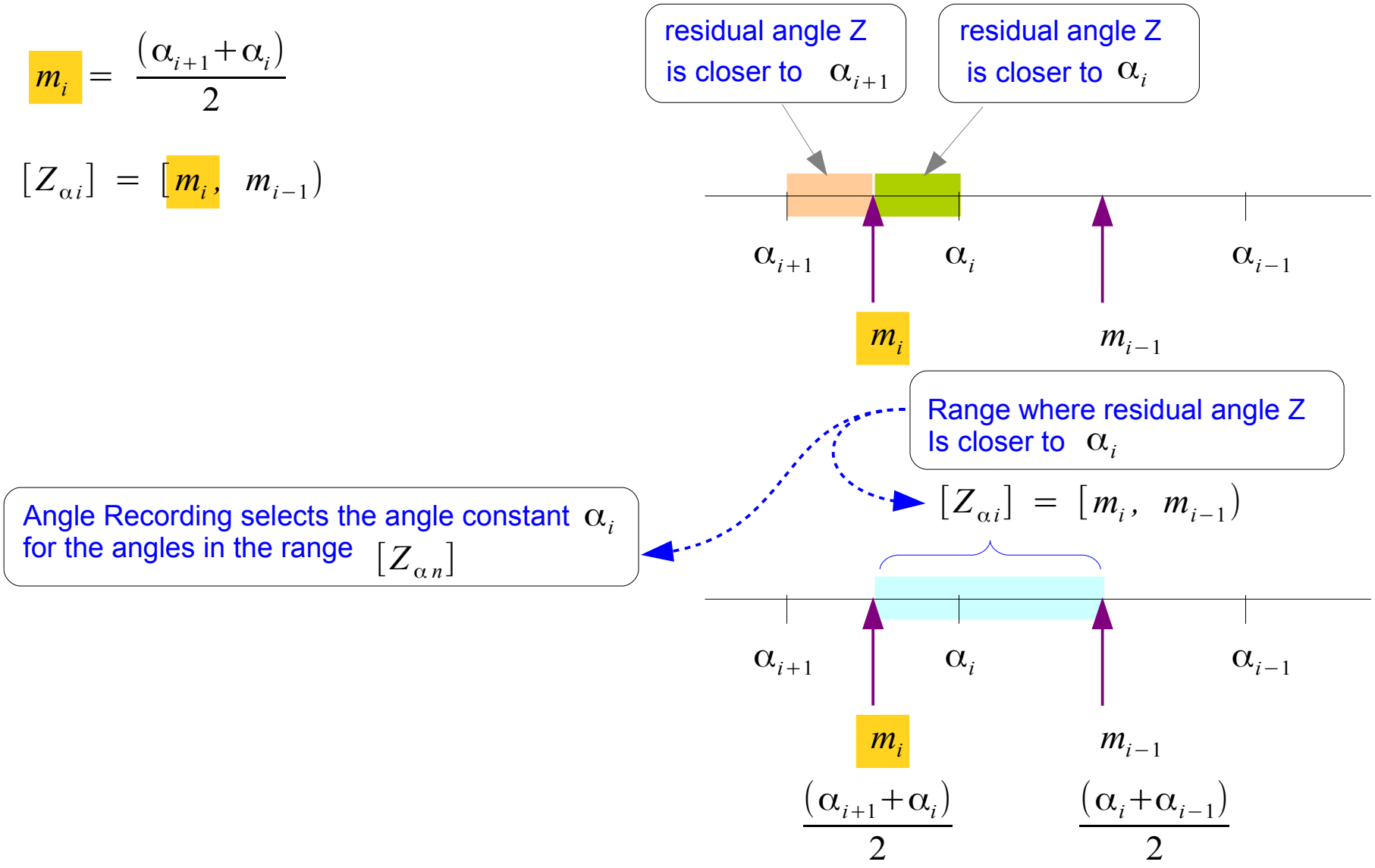
}



Range

$$m_i = \frac{(\alpha_{i+1} + \alpha_i)}{2}$$

$$[Z_{\alpha i}] = [m_i, \; m_{i-1})$$



Estimated Range

assume

step i → α_n → $[Z_{\alpha n}]$

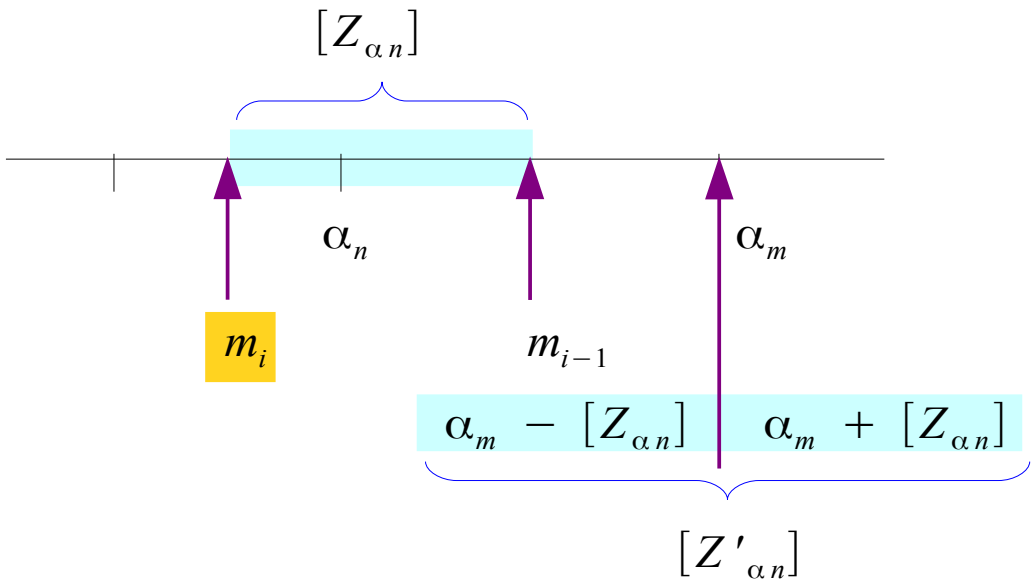
estimate

step $i-1$ → $\hat{\alpha}_m$ ⇢ $[Z'_{\alpha n}]$

$$[Z'_{\alpha n}]_{RHS} = \alpha_m + [Z_{\alpha n}]$$

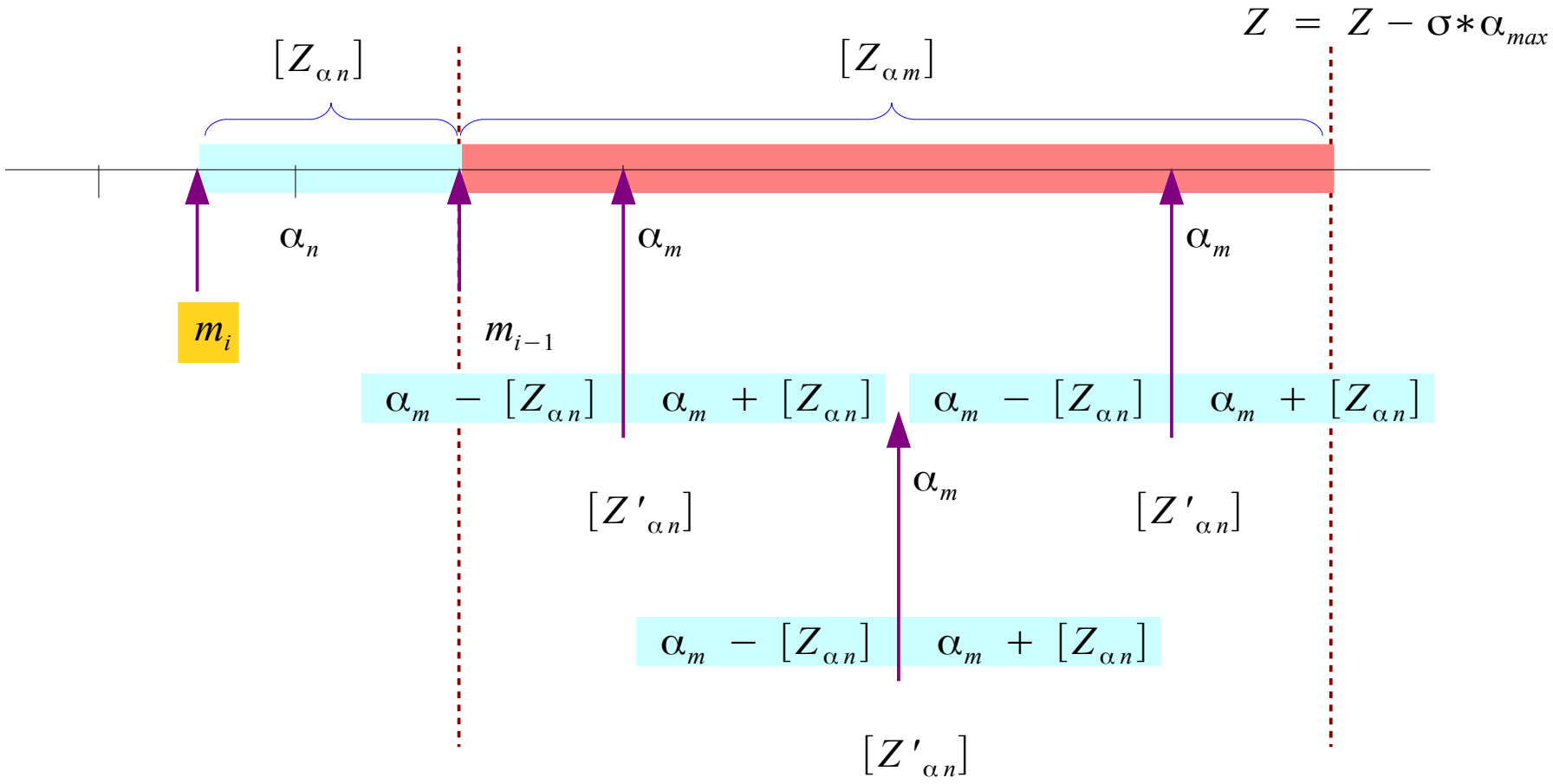
$$[Z'_{\alpha n}]_{LHS} = \alpha_m - [Z_{\alpha n}]$$

$$\begin{cases} [Z'_{\alpha n}]_{RHS} - \alpha_m \Rightarrow [Z_{\alpha n}] \\ [Z'_{\alpha n}]_{LHS} - \alpha_m \Rightarrow [Z_{\alpha n}] \end{cases}$$



$$Z = Z - \sigma * \alpha_{max}$$

Conditions of Estimated Range



Case 1) $[Z'_{\alpha n}]$ is contained etirely within $[Z_{\alpha m}]$

Case 2) $[Z'_{\alpha n}]$ straddles the boundary of $[Z_{\alpha m}]$

Case 3) $[Z'_{\alpha n}]$ lies completely outside $[Z_{\alpha m}]$

Example

$1.79^\circ \qquad 3.576^\circ \qquad 26.565^\circ$

$$[Z_{26.565^\circ}] = [20.295,\ 35.78]$$
$$[Z_{3.576^\circ}] = [2.6825,\ 5.3505]$$
$$[Z_{1.79^\circ}] = [1.342,\ 2.6825]$$

$$[Z_{1.79^\circ}] = [1.342,\ 2.6825]$$

$$[Z'_{1.79^\circ}]_{LHS} = 3.576^\circ - [Z_{1.79^\circ}] = [3.576-2.6825,\ 3.576-1.342] = [0.8935,\ 2.234]$$
$$[Z'_{1.79^\circ}]_{RHS} = 3.576^\circ + [Z_{1.79^\circ}] = [3.576+1.342,\ 3.576+2.6825] = [4.918,\ 6.2585]$$

$$= [4.918,\ 5.3505]$$

$$= [4.918,\ 5.3505]$$

$$[Z'_{3.576^\circ}]_{LHS} = 26.565^\circ - [Z_{3.576^\circ}] = [26.565-5.3505,\ 26.565-4.918]$$
$$[Z'_{3.576^\circ}]_{RHS} = 26.565^\circ + [Z_{3.576^\circ}] = [26.565+4.918,\ 26.565+5.3505]$$

Range Comparison

$\theta \in [21.2145,\ 21.647]$ or $\theta \in [31.483,\ 31.9155]$

Such rotation angle will use all three angle constants: 1.79° 3.576° 26.565°

A simple comparison operation

Without performing iteration sequentially

Find all combinations of angle constants

References

[1] http://en.wikipedia.org/
[2] CORDIC FAQ, www.dspguru.com
[3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
[4] J. S. Walther, A Unified Algorithm for Elementary Functions
[5] J. P. Deschamps, G. A. Bioul, G.D. Sutter, Synthesis of Arithmetic Circuits
[6] T.K. Rodrigues, "Adaptive CORDIC: Using Parallel Angle Recording to Accelerate Rotations", IEEE Trans on Computers, 2010